Performance of Mobile Cyber-Physical Agents with Discrete Trajectories for Sensing Spatio-Temporal Variables

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***Abstract*—In this paper, we study the performance of a cyber-physical system with agents moving according to random trajectories to sense an environmental variable with known spatio-temporal profile. We introduce two sensing methods to model the discrete time instants of measurements. We derive closed form expressions for the coverage probability defined as the probability that the environmental variable can be estimated at a random location within a certain tolerable error. We also derive joint coverage probability, which is the probability that a given location is covered jointly over multiple time instants. Our analysis also offers insights into the design of optimal trajectories of agents.**

I. Introduction

Modern cyber physical systems (CPSs) consists of a large number of wirelessly connected sensing agents actively coordinating and monitoring states of some environmental processes of interest [1]. A precise and timely knowledge of environmental processes is crucial for avoiding various natural hazards, *e.g.* forest-fires, air pollution, soil and water contamination. Since these sensing agents usually have a finite sensing region, it requires multiple sensing agents to provide a certain guarantee of coverage over a region of interest [2]. It is useful to note that most environmental variables including temperature, and humidity do not exhibit drastic variations over space or time [3], [4]. Hence one can expect significant redundancy in the sensed data if it is collected too frequently in time or too closely in space [5].

Past works have studied the spatial and/or temporal variation of various environmental variables. For example, the works [3] and [4] have studied this for soil and forest temperatures. Assuming that this variation profile is available for the environment variable, it is possible to estimate its value at various locations and times using the data measured over space and time. For example, [6] has shown an improvement in the coverage by utilizing spatial profile information. In [7], authors studied the coverage performance of a dynamic event that evolves with time (*e.g.* a forest fire expanding with time). These works were extended in [1] to include both spatial and temporal profile to improve coverage of a given region with the help of moving sensing agents. Assuming general trajectories of agents, in [1], we developed an analytical framework to derive a generic expression for the fraction of area that is covered assuming a certain error in estimation can be tolerated. Since trajectories were generic, the expression could not be simplified in [1]. As a result, the impact of various trajectories on the sensing performance has not been investigated in the past which is one of the goals of this paper.

In this work, we consider a CPS with mobile sensing agents deployed to sense an environmental variable with known spatio-temporal profile. We propose to model the agents' trajectories via a Poisson cluster process (PCP) with points denoting observation locations over time. Using the notion that an arbitrary location is treated covered if the environmental variable can be estimated at this location within a certain tolerable error, we obtain a closed form expression for the coverage probability. We also derive the probability that a location is covered jointly over a set of multiple time instants. Our analysis also provides insights on the design of optimal trajectories.

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II. System Model

In this work, we consider a CPS with mobile sensing agents to sense an environmental variable Θ represented as a spatio-temporal process $\Theta(t, \mathbf{x})$ which denotes its value at a location $\mathbf{x}$ at time t [1]. Let $\bar{\Theta}(t, \mathbf{x})$ denote its deviation from the mean value at a location $\mathbf{x}$ at time t, *i.e.* $\bar{\Theta}(t, \mathbf{x}) = \Theta(t, \mathbf{x}) - \mathbb{E}[\Theta(t, \mathbf{x})]$. We assume that the variation of Θ is bounded which means that it can vary by a finite value over a finite distance and time. In particular, for any two locations $\mathbf{x_1}$ and $\mathbf{x_2}$ and time instants t_1 and t_2, we have the following bound

$$\left|\bar{\Theta}(t_1, \mathbf{x_1}) - \bar{\Theta}(t_2, \mathbf{x_2})\right| \leq f\left(|t_1 - t_2|, \|\mathbf{x_1} - \mathbf{x_2}\|\right),$$

where $f(\cdot, \cdot)$ is termed the variation profile or the tolerance function of Θ. For example, it can take the following form [1]

$$f(|t_1 - t_2|, \|\mathbf{x_1} - \mathbf{x_2}\|) = A\left(e^{w\|\mathbf{x_1} - \mathbf{x_2}\| + v|t_1 - t_2|} - 1\right). \quad (1)$$

Here, v and w are the temporal and spatial variation rates of Θ, respectively, and A is the scaling coefficient. The parameters v and w determine how rapidly Θ changes with time and space, respectively.

We consider that the locations of the agents are distributed as a homogenous Poisson point process (PPP) $\Phi_{\mathrm{p}} = \{\mathbf{X}_i\}$ with intensity λ_{p}. To improve sensing coverage, the agents move to nearby locations and take measurement at those locations. The set of these locations along with their corresponding measurement time stamps T form the trajectory of the agent. We model these locations for each agents (*i.e.* its trajectory) via an independent finite PPP with the visit time serving as its marks. Let us denote the set of sensing locations for an agent at $\mathbf{X}$ (relative to $\mathbf{X}$) by $\mathbf{B}^{\mathbf{X}}$. Let $\mathbf{Y}_{ij}$ denote the jth observation location of ith agent relative to its initial location $\mathbf{X}_i$. The corresponding measurement time is τ_{ij}. For clarity, we call $\mathbf{X}_i$ as the *center* or the *deployment location*. On the other hand, $\mathbf{Y}_{ij}$'s where the measurement occurs are termed *observation locations*. Hence,

$$\mathbf{B}^{\mathbf{X}_i} = \{\mathbf{Y}_{ij}\}. \tag{2}$$

Further, the set of their absolute locations are given as

$$\Psi^{\mathbf{X}_i} = \{\mathbf{X}_i + \mathbf{Y}_{ij},\ \forall\ \mathbf{X}_i \in \Phi_{\mathrm{p}}\}. \tag{3}$$

The region physically covered by this CPS over time is given as

$$\Psi = \bigcup_{\mathbf{X}_i \in \Phi_{\mathrm{p}}} \mathbf{X}_i + \mathbf{B}^{\mathbf{X}_i}, \tag{4}$$

which is a clustered point process. In particular it is a PCP, which is a type of Neyman Scott process. In PCP, the daughter points are distributed as a PPP with average $\bar{m}$ number of points and each point having PDF $f_{\mathbf{Y}}(\cdot)$ to be at a location $\mathbf{y}$ with $\mathbf{y} \in \mathsf{E}$. Here E denotes the region in which the agent can move. For mathematical tractability we consider two special cases of PCP, namely Matérn cluster process (MCP) and Thomas cluster process (TCP). These two are introduced next.

MCP is a doubly Poisson cluster process whose daughter points are uniformly distributed in a ball of radius r_{d} centered at its parent point. The density of daughter point process is given by λ_{d}. The PDF of the relative location of a daughter point from its parent point is given as,

$$f_{\mathbf{Y}}(\mathbf{y}) = \begin{cases} \frac{1}{\pi r_{\mathrm{d}}^2} & \text{if } \|\mathbf{y}\| < r_{\mathrm{d}} \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

TCP is a doubly Poisson cluster process where the daughter points are distributed around its parent point according to a symmetric Gaussian distribution with variance σ^2. The number of points in each cluster is distributed as a Poisson random variable with mean $\bar{m}$ (for MCP, $\bar{m} = \lambda_d \pi r_d^2$). The PDF of the relative location of daughter point from its parent point is given as,

$$f_{\mathbf{Y}}(\mathbf{y}) = \frac{1}{2\pi\sigma^2} \exp\left(\frac{-\|\mathbf{y}\|^2}{2\sigma^2}\right),\ \mathbf{y} \in \mathbb{R}^2. \tag{6}$$

We consider two two sensing methods: slotted and unslotted as described below.

A. Slotted Sensing

In this sensing method, we divide the total time into slots each with a duration of T_s, termed as a *sensing window*. For each slot, a sensing agent moves to each of its sensing locations $\mathbf{Y}_{ij}$ within this time window at a random time τ_{ij} and takes a measurement. This process is repeated over sensing windows, *e.g.*, $(0,\, T_s)$, $(T_s,\, 2T_s)$, and so on.

At a given time t for the i-th sensor agent located at $\mathbf{x}$, its sensing locations $\Psi_{\mathbf{x}}$ can be divided into two components. The first is the group of locations where measurement has been done in the current slot. The second is the remaining locations for which we have to use measurements taken from the previous slot. We denote the first set of locations by the point process (PP) Ψ_{i1} and the second set of locations by the PP Ψ_{i2}. See Fig. 1 for an illustration. From independent thinning theorem, the two PPs are independent PPP.

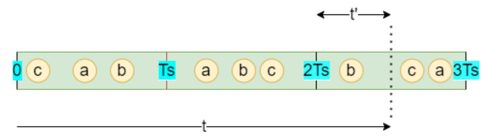

Fig. 1. Illustration of the slotted sensing model. Consider that an agent visits 3 different locations, namely a, b, and c at random times in a sensing window. We consider the slot $[2T_s, 3T_s]$, for this $U = 1$ for the location b and $U = 0$ for the locations a and c.

To each observation location, we assign independent marks (U) and (T). Here, the variable U is an indicator if the measurement has already occurred in the current slot. T denotes the difference between the current time and the time at which measurement was made for the specified observation location. Let us denote $t' = t \bmod T_s$, as the time elapsed since the start of the current sensing window, where t is the current time instant. Here, $U = 0$ indicates measurement from the current slot and $U = 1$ indicates measurement from the previous slot. Hence,

$$U = 0 \Rightarrow T \sim \text{Unif}\,[0, t']\,,\ \ U = 1 \Rightarrow T \sim \text{Unif}\,[t', T_s + t']\,.$$

In words, for each sensor agent, T is uniformly distributed in $[0, t']$ for sensing locations in Ψ_{i1} and in $[t', T_s + t']$ for sensing locations in Ψ_{i2}. At the current time instant t, the observation locations of the i-th sensing agent can be written as a marked PP

$$\Psi_{ti} = \{(\mathbf{y}, T, U) : \mathbf{y} \in \Psi^{\mathbf{x}_i}, U \in \{0,1\}, T\}\,.$$

Note that Ψ_{i1} and Ψ_{i2} are PPPs with densities $\lambda_{\mathrm{d}}(\mathbf{x})\frac{t}{T_s}$ and $\lambda_{\mathrm{d}}(\mathbf{x})\frac{(T_s - t)}{T_s}$, respectively.

B. Unslotted Sensing

We now consider unslotted sensing, where the agents do not repeat the sensing locations over slots. Each sensing location is a new point independent of its previous locations. For each sensor agent with center location $\mathbf{X}_i$, its observation instants can be modeled using a PPP Φ in $\mathsf{E}_i \times \mathbb{R}$, where E_i denotes the region in which the agent can move. For example, E_i is $\mathcal{B}(\mathbf{X}_i, r_{\mathrm{d}})$ for MCP and $\mathbb{R}^2$ for TCP. Note that observation location and the time together constitute a sensing instant. Hence, each point $z_{ij} = (\mathbf{y}_{ij}, \tau_{ij})$ of Φ gives an observation location $\mathbf{y}_{ij}$ and time τ_{ij}. Here, $\mathbf{y}_{ij}$ denotes the j-th sensing location of $\mathbf{x}_i$ and τ_{ij} denotes the time at which observation was taken at $\mathbf{y}_{ij}$. Hence,

$$\mathbf{B}^{\mathbf{X_i}} = \{(\mathbf{y}_{ij}, \tau_{ij}) : \mathbf{y}_{ij} \in \mathsf{E}_i, \tau_{ij} \in \mathbb{R}\}.$$

Let us consider a time interval $T = (t_1, t_2)$. The number of observation instants is given as

$$N(t_2 - t_1) = \Lambda(\mathsf{E} \times T) = \Lambda(\mathsf{E})\Lambda((t_1, t_2)),$$

where Λ denote the intensity measures in both dimensions- E and time. Let μ denote the sensing frequency such that

$$\Lambda((t_1, t_2) = \mu(t_2 - t_1)\ \forall t_1,\ t_2.$$

Further, let $\Lambda(E) = c$ denote the number of sensing instants in $1/\mu$ time duration. Now, the number of observation instants is given as

$$N(t_2 - t_1) = c\mu(t_2 - t_1).$$

Hence, for a fixed time interval of finite length t, the set of sensing locations of an agent can be seen as a finite PPP in E with $c\mu t$ as the total number of points, $\mathbf{y}_{ij}$'s as the locations in E_i and τ_{ij}'s as marks denoting their sensing time. Further, union of all sensing locations also constitutes a MCP/TCP under this model. Let us define $\kappa = \frac{1}{c\mu}$, which can be intuitively viewed as the average time between two successive measurements by a sensor agent.

From [1, Theorem 1], we know that for a sensing agent with center location $\mathbf{x}$ and an arbitrary trajectory $B(t)$, the event that uncertainty in Θ at a location (taken at the origin) and a given time instant is within η tolerance is equivalent to the event that the uncertainty offered by any of its past measurements is less than η. Given variation profile f, the uncertainty due to an observation at time $t - t'$ by the sensor agent with center at $\mathbf{x}$ is given as:

$$u_t(t') = f\left(t', \|\mathbf{x} + B(t - t')\|\right).$$

Then, the location is covered with η-tolerance by an agent centered at $\mathbf{x}$ if [1]

$$M_{\mathbf{x}}(t) = \min_{t' \geq 0} u_t(t') \leq \eta. \quad (7)$$

Further, from [1, Theorem-3], if the initial locations of the agents form a PPP and they are moving in a 2D region according to their trajectories $B_i(t)$, the probability that the value of Θ is known within η tolerance at an arbitrary location P at time t is given as,

$$\mu(t, \eta) = 1 - \exp\left(-\lambda_{\mathrm{p}} \int_{\mathbb{R}^2} \left(1 - \Sigma(\eta, t, \mathbf{x})\right) \mathrm{d}\mathbf{x}\right), \quad (8)$$

where, $\Sigma(\eta, t, \mathbf{x}) = \mathbb{E}\left[\mathbb{1}\left(M_{\mathbf{x}}(t) > \eta\right)\right]$. The inner expectation is with respect to random trajectory B.

III. Coverage Analysis

We define η-coverage probability as the time-averaged probability that at some random point of interest (which we take as origin without loss of generality) the environmental variable Θ is known within an error tolerance of η. The following theorems give the η-coverage for slotted and unslotted sensing.

Theorem 1. *If the agents are moving according to PPP-based discrete trajectories under slotted sensing, the η-coverage is given as (8) with (see Appendix A for the proof):*

$$\Sigma(\eta, t, \mathbf{x}) = \exp\left(-\bar{m}\frac{t'}{T_s}\int_{\mathbf{y}\in\mathbb{R}^2}\left(\frac{\Delta(\eta, \mathbf{x}, \mathbf{y})}{t'} f_{\mathbf{Y}}(\mathbf{y})\right)\mathrm{d}\mathbf{y}\right) \times \exp\left(-\bar{m}\frac{T_s - t'}{T_s}\int_{\mathbf{y}\in\mathbb{R}^2}\left(\frac{\Delta(\eta, \mathbf{x}, \mathbf{y})}{T_s} f_{\mathbf{Y}}(\mathbf{y})\right)\mathrm{d}\mathbf{y}\right), \quad (9)$$

where,

$$\Delta(\eta, \mathbf{x}, \mathbf{y}) = \frac{1}{v}\left(\ln\left(1 + \frac{\eta}{A}\right) - w\|\mathbf{x} + \mathbf{y}\|\right) \text{ and } t' = t \bmod T_s.$$

It is intuitive that an increase in $\bar{m}$ will improve the η-coverage. However, due to physical limitations, sensing at a location and movement between two locations take a non-zero time. The sensing window duration can be written as,

$$T_s = \pi {r_{\mathrm{d}}}^2/\nu + \bar{m}\kappa. \quad (10)$$

Here, ν is the area swapping speed of the sensor agent. For a fixed sensing window length, increasing the number of sensing locations would cause measurements to occur more closely in time which will maintain the freshness of data. At the same time, there would be a commensurate decrease in the sensing radius r_{d} as dictated by the above constraint equation. This would cause data to be observed from points closer to the center location of the agent, which might result in the loss of some spatial information. Because of these opposing factors, we expect to see a trade-off resulting in the existence of an optimum operating point.

Theorem 2. *If the agents are moving according to PPP based discrete trajectories under unslotted sensing, the η-coverage is given as (8) with (see Appendix B for proof):*

$$\Sigma(\eta, t, \mathbf{x}) = \exp\left(-\kappa^{-1}\int_{\mathbf{y}\in\mathbb{R}^2}\Delta(\eta, \mathbf{x}, \mathbf{y}) f_{\mathbf{Y}}(\mathbf{y})\mathrm{d}\mathbf{y}\right). \quad (11)$$

Note that this metric is independent of the current time.

Theorem 3 (Joint coverage)**.** *Under unslotted sensing, the probability that a point of interest* P *taken at origin is covered within η-tolerance threshold at all times $t \in \mathbb{T} = \{t_1, t_2, \ldots, t_r\}$ is given as (see Appendix C for proof):*

$$\mu(\mathbb{T}, \eta) = 1 + \sum_i (-1)^i \sum_{T_i \in \mathbb{T}_i} g(T_i), \quad (12)$$

where $\mathbb{T}_i$ is the set of all i-tuples of $\mathbb{T}$. For example, if $\mathbb{T} = \{t_1, t_2, t_3\}$, $\mathbb{T}_2 = \{\{t_1, t_2\}, \{t_1, t_3\}, \{t_2, t_3\}\}$.

$$g(T_i) = \exp\left(-\lambda_{\mathrm{p}}\int_{\mathbb{R}^2}\left((1 - e^{-\kappa^{-1}\int_{\mathbb{R}^2}(\Sigma(\eta, \mathbf{x}, \mathbf{y}, T_i) f_{\mathbf{Y}}(\mathbf{y})\mathrm{d}\mathbf{y})}\right)\mathrm{d}\mathbf{x}\right),$$

where,

$$\Sigma(\eta, \mathbf{x}, \mathbf{y}, T_i) = 1 + \sum_k (-1)^k \sum_{R_k \in T_i^k} \max(0, \Delta(\eta, \mathbf{x}, \mathbf{y}) + \min(R_k) - \max(R_k)),$$

where T_i^k is the set of all k-tuples of T_i. i.e. if $T_i = \{t_2, t_3, t_5\}$, $T_i^2 = \{\{t_2, t_3\}, \{t_2, t_5\}, \{t_3, t_5\}\}$.

IV. Numerical Results

In this section we verify our analytical results and present design insights.

Validation: Fig. 2 shows the mean probability that a point (without loss of generality, taken at origin) is covered within η tolerance threshold for slotted sensing with $T_s = 1.5$ s for MCP and TCP based trajectories. It presents both analytical and simulation results. By increasing the tolerance threshold, we observe an improvement in coverage. Fig. 3 shows the η-coverage probability for unslotted sensing. Similar to the slotted case, we observe better coverage by relaxing the tolerance threshold. Fig. 4 shows the probability that a random point

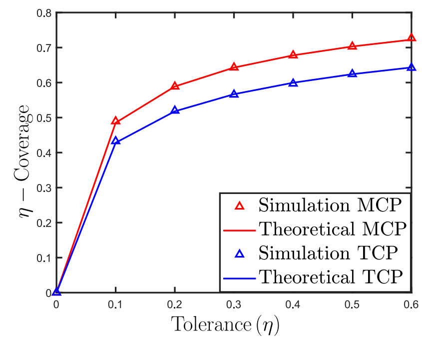

Fig. 2. Current time coverage simulation result for MCP and TCP trajectories in slotted sensing setup. Parameters: $T_s = 1.5$ s, $\lambda_{\mathrm{p}} = 0.05/\mathrm{m}^2$, $\bar{m} = 5$, $A = 6 \times 10^{-4}$, $v = 3\mathrm{s}^{-1}$, $w = 3\mathrm{m}^{-1}$, $r_{\mathrm{d}} = 3\,\mathrm{m}$, $\bar{m} = 5$.

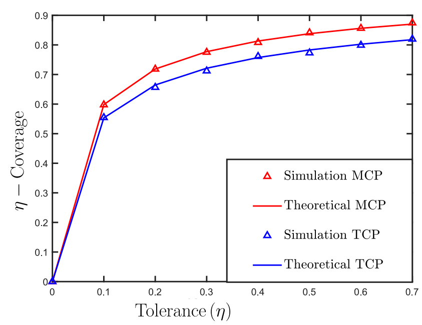

Fig. 3. Current time coverage simulation result for MCP and TCP trajectories in unslotted sensing setup. Parameters: $\lambda_{\mathrm{p}} = 0.1/\mathrm{m}^2$, $A = 6 \times 10^{-4}$, $v = 3\mathrm{s}^{-1}$, $w = 3\mathrm{m}^{-1}$, $r_{\mathrm{d}} = 3\mathrm{m}$, $\sigma = 1$, $\kappa = 0.5$ s.

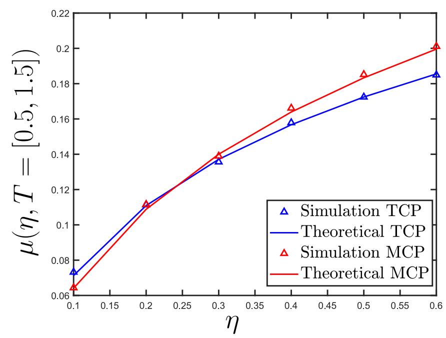

Fig. 4. Joint coverage simulation result for MCP and TCP trajectories in unslotted sensing setup. Parameters: $\lambda_{\mathrm{p}} = 0.02/\mathrm{m}^2$, $A = 6 \times 10^{-4}$, $v = 3\mathrm{s}^{-1}$, $w = 3\mathrm{m}^{-1}$, $r_{\mathrm{d}} = 3\mathrm{m}$, $\kappa = 0.5$ s, $\sigma = 1$, times of interest $T = [0.5, 1.5]$ s.

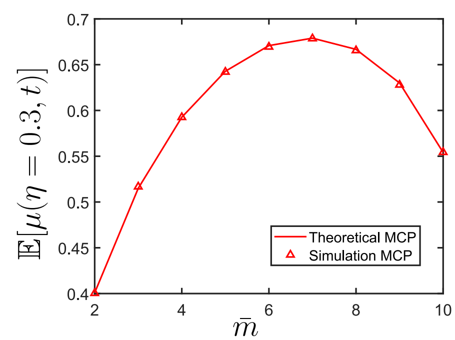

Fig. 5. Tradeoff between number of sensing locations and sensing radius for a slotted MCP system. Parameters: Sensing window $T_s = 1.5$ s, $\lambda_{\mathrm{p}} = 0.05/\mathrm{m}^2$, $A = 6\times10^{-4}$, $v = 3\mathrm{s}^{-1}$, $w = 3\mathrm{m}^{-1}$, $\kappa = \frac{12}{85}\mathrm{s}$, $\nu = \frac{34\pi}{3}\mathrm{m}^2/\mathrm{s}$.

of interest (taken as origin) is covered at multiple specified time instants in an unslotted sensing system with TCP and MCP based trajectories. The close agreement of simulation and theoretical results also verifies the analysis.

Impact of number of sensing locations $\bar{m}$: Fig. 5 shows the variation of η-coverage with respect to $\bar{m}$ (denoting the number of observation locations) in each slot by varying $\bar{m}$ while keeping the slot duration T_s constant according to (10). We can see that coverage first increases with $\bar{m}$ as an increase in the number of observations reduces the chance of observations going stale in time. An increase in $\bar{m}$ beyond a certain point degrades coverage by reducing the area agents can span *i.e,* trajectory radius r_{d}. Therefore, the observation starts becoming stale over distance. This result shows that there is an optimum value of $\bar{m}$ and hence, trajectory radius of agents.

Conclusions

In this paper, we considered a CPS with agents moving according to random trajectories. We demonstrated that by using the spatio-temporal profile of the environmental variable, one can avoid redundancies in sensor deployment. We also obtained coverage expressions to quantify the probability with which the data can be ensured to be available under a certain error tolerance level at specified time instant(s). Their are numerous possible extensions of this work. Finding joint coverage under the slotted sensing model is an interesting challenge. One can also extend the idea of joint coverage to all time coverage which could ensure the uninterrupted functioning of the CPS.

Appendix A

Using (7) the event $M_{\mathbf{x}}(t) > \eta$ is equivalent to $\min_j u_t(T_j) > \eta$, where j traverse over all observation points of the agent. Note that this forms the daughter PP of the sensing agent. Here, T_j is the time difference between the current time and the time at which observation was taken at the j-th daughter point.

$$f(T_j, \|\mathbf{x}_i + \mathbf{y}_j\|) > \eta, \;\; \forall j.$$

Recall that Ψ_i denote the daughter point process for the i-th cluster center. The inner expectation (8) can be solved as

$$\mathbb{E}\left[\mathbb{1}\{M_{\mathbf{x}}(t) > \eta\}\right] = \mathbb{E}\left[\prod_{\mathbf{y}\in\Psi_i} \mathbb{E}_T\left[\mathbb{1}\left(f(T, \|\mathbf{x}+\mathbf{y}\|) > \eta\right)\right]\right].$$

Recall that Ψ_{i1} and Ψ_{i2} denote the set of locations at which measurements have been taken in the current sensing window and the set of locations which have not been visited in the current sensing window, respectively. The mean number of points in each PP is given as

$$\bar{m_1} = \bar{m} \times \frac{t'}{T_s}, \; \bar{m_2} = \bar{m} \times \frac{T_s - t'}{T_s}.$$

As mentioned previously, t denotes the current time, t' denotes the time relative to the sensing window start, and T_s denotes the length of the sensing window, $t = k \times T_s + t'$: $t' \in [0, T_s]$

$$\mathbb{E}\left[\mathbb{1}\{M_{\mathbf{x}}(t)>\eta\}\right]=\mathbb{E}\left[\prod_{\mathbf{y}\in\Psi_{1i}}\mathbb{E}_T\left[\mathbb{1}\left(f(T,\|\mathbf{x}+\mathbf{y}\|)>\eta\right)\right)\right]$$
$$\times\,\mathbb{E}\left[\prod_{\mathbf{y}\in\Psi_{2i}}\mathbb{E}_T\left[\mathbb{1}\left(f(T,\|\mathbf{x}+\mathbf{y}\|)>\eta\right)\right]\right].$$

Now the following are equivalent,

$$f(T,\|\mathbf{x}+\mathbf{y}\|)>\eta\equiv\Delta(\eta,\mathbf{x},\mathbf{y})<T.$$

If $\mathbf{y}\in\Psi_{1i}\Rightarrow T\sim\text{Unif}[0,t']$ then $\mathbb{E}\left[\mathbb{1}\left(\Delta(\eta,\mathbf{x},\mathbf{y})\right)<T\right]=1-\frac{\Delta(\eta,\mathbf{x},\mathbf{y})}{t'}$ Similarly, if $\mathbf{y}\in\Psi_{2i}\Rightarrow T\sim\text{Unif}[t',T_s+t']$ then $\mathbb{E}\left[\mathbb{1}\left(\Delta(\eta,\mathbf{x},\mathbf{y})\right)<T\right]=1-\frac{\Delta(\eta,\mathbf{x},\mathbf{y})-t'}{T_s}$

$$\Rightarrow\mathbb{E}\left[\mathbb{1}\{M_{\mathbf{x}}(t)>\eta\}\right]=$$
$$\mathbb{E}\left[\prod_{\mathbf{y}\in\Psi_{1i}}\left(1-\frac{\Delta(\eta,\mathbf{x},\mathbf{y})}{t'}\right)\right]\mathbb{E}\left[\prod_{\mathbf{y}\in\Psi_{2i}}\left(1-\frac{\Delta(\eta,\mathbf{x},\mathbf{y})-t'}{T_s}\right)\right].$$

Using the probability generating functional (PGFL) of PCPs,

$$\begin{aligned}&\Sigma(\eta,t,\mathbf{x})=\mathbb{E}\left[\mathbb{1}\{M_{\mathbf{x}}(t)>\eta\}\right]\\&=\exp\left(-\bar{m}\times\frac{t'}{T_s}\times\int_{\mathbf{y}\in\mathbb{R}^2}\left(\frac{\Delta(\eta,\mathbf{x},\mathbf{y})}{t'}\times f_{\mathbf{Y}}(\mathbf{y})\right)\mathrm{d}\mathbf{y}\right)\times\\&\exp\left(-\bar{m}\times\frac{T_s-t'}{T_s}\times\int_{\mathbf{y}\in\mathbb{R}^2}\left(\frac{\Delta(\eta,\mathbf{x},\mathbf{y})}{T_s}\times f_{\mathbf{Y}}(\mathbf{y})\right)\mathrm{d}\mathbf{y}\right).\end{aligned}$$

Appendix B

Assume sensing starts at time $-T_o$ with $T_o\to\infty$. Here we need to consider all past sensing locations of the agent at $\mathbf{x}$ over time $[-T_o,t)$. Recall that for a time interval of length $(t+T_o)$ the set of sensing locations of the agent is a finite PPP Ψ_i with total number of points $\bar{m}=c\mu(t+T_o)=\frac{t+T_o}{\kappa}$. Hence,

$$\begin{aligned}&\mathbb{E}\left[\mathbb{1}\{M_{\mathbf{x}}(t)>\eta\}\right]=\mathbb{E}\left[\prod_{\mathbf{y}\in\Psi_i}\mathbb{E}_T\left[\mathbb{1}\left(f(T,\|\mathbf{x}+\mathbf{y}\|)>\eta\right)\right)\right]\\&=\mathbb{E}\left[\prod_{\mathbf{y}\in\Psi_i}\left(1-\frac{\Delta(\eta,\mathbf{x},\mathbf{y})}{T_o+t}\right)\right]\\&\Rightarrow\Sigma(\eta,t,\mathbf{x})=\mathbb{E}\left[\mathbb{1}\{M_{\mathbf{x}}(t)>\eta\}\right]\\&=\exp\left(-\bar{m}\int_{\mathbf{y}\in\mathbb{R}^2}\left(\frac{\Delta(\eta,\mathbf{x},\mathbf{y})}{T_o+t}\times f_{\mathbf{Y}}(\mathbf{y})\right)\mathrm{d}\mathbf{y}\right)\\&=\exp\left(-\kappa^{-1}\int_{\mathbf{y}\in\mathbb{R}^2}\left(\Delta(\eta,\mathbf{x},\mathbf{y})\times f_{\mathbf{Y}}(\mathbf{y})\right)\mathrm{d}\mathbf{y}\right).\end{aligned}$$

Appendix C

The event E that the origin is covered at all time instants in $\mathbb{T}$ is given as,

$$E=\bigcap_{t_m\in\mathbb{T}}\left(\bigcup_{\mathbf{x}_{ij}\in\Psi}\mathbb{1}\left[f(\mathbf{x}_{ij},|t_{ij}-t_m|)<\eta\right]\right)$$
$$P(E)=\mu(\mathbb{T},\eta)=\mathbb{E}\Bigg[\prod_{t_m\in\mathbb{T}}\{1-\prod_{\mathbf{x}_{ij}\in\mathbf{\Psi}}\Big(\mathbb{1}\left[f(\mathbf{x}_{ij},t_{ij}-t_m)>\eta\right]]\times$$
$$\mathbb{1}(t_{ij}>t_m)+\mathbb{1}(t_{ij}<t_m)\Big)\}\Bigg].$$

Note that, here we are considering only measurements taken before the time of interest t_m. Here t_{ij} denotes how long back (from current time $t=0$) measurement occurred at the position $\mathbf{x}_{ij}$.

$$\begin{aligned}&\mu(\mathbb{T},\eta)=\mathbb{E}\Bigg[\prod_{t_m\in\mathbb{T}}\Big(1\\&-\prod_{\mathbf{x}_{ij}\in\Psi}\left(\mathbb{1}\left[t_{ij}>t_m+\Delta(\eta,\mathbf{x},\mathbf{y})\right]+\mathbb{1}\left[t_{ij}<t_m\right]\right)\Big)\Bigg]\\&=\mathbb{E}\left[\prod_{t_m\in\mathbb{T}}\left(1-\prod_{\mathbf{x}_{ij}\in\Psi}\left(1-\mathbb{1}\left[t_m<t_{ij}<t_m+\Delta(\eta,\mathbf{x},\mathbf{y})\right]\right)\right)\right].\end{aligned}$$

Let $h(t_m)=\prod_{\mathbf{x}_{ij}\in\Psi}\left(1-\mathbb{1}\left[t_m<t_{ij}<t_m+\Delta(\eta,\mathbf{x},\mathbf{y})\right]\right)$

$$\begin{aligned}&\mu(\mathbb{T},\eta)=\mathbb{E}\Bigg[1-\sum_{\{t_1\}\in\mathbb{T}_1}h(t_1)+\sum_{\{t_1,t_2\}\in\mathbb{T}_2}h(t_1)h(t_2)-\ldots+\\&\qquad(-1)^r h(t_1)h(t_2)\ldots h(t_r)\Bigg]\\&=\mathbb{E}\left[1+\sum_k(-1)^k\sum_{\mathbf{T}_k\in\mathbb{T}_k}g(\mathbf{T}_k)\right]=1+\sum_k(-1)^k\sum_{\mathbf{T}_k\in\mathbb{T}_k}\mathbb{E}\left[g(\mathbf{T}_k)\right],\end{aligned}$$

where, $g(\mathbf{T}_k)=h(T_1)h(T_2)\ldots h(T_k)$ and $\mathbf{T}_k$ is represented as $\mathbf{T}_k=[T_1,T_2,\ldots,T_k]$. Solving further we get

$$\begin{aligned}&\mathbb{E}\left[g(\mathbf{T}_k)\right]=\mathbb{E}\Bigg[\prod_{\mathbf{x}_{ij}\in\mathbf{\Psi}}\Big(\left(1-\mathbb{1}\left[T_1<t_{ij}<T_1+\Delta(\eta,\mathbf{x}_{ij})\right]\right)\ldots\\&\left(1-\mathbb{1}\left[T_k<t_{ij}<T_k+\Delta(\eta,\mathbf{x}_{ij})\right]\right)\Big)\Bigg]\\&=\mathbb{E}\Bigg[\prod_{\mathbf{x}_{ij}\in\mathbf{\Psi}}\Big(1+\sum_r(-1)^r\sum_{R_k\in\mathbf{T}_k^r}\mathbb{1}\Big[\max(R_k)<t_{ij}\\&\qquad<\min(R_k)+\Delta(\eta,\mathbf{x}_{ij})\Big]\Big)\Bigg]\\&=\mathbb{E}_{\mathbf{\Psi}}\left[\prod_{\mathbf{x}_{ij}\in\mathbf{\Psi}}\left(1+\sum_r(-1)^r\sum_{R_k\in\mathbf{T}_k^r}\frac{\Delta(\eta,\mathbf{x}_{ij})+\min(R_k)-\max(R_k)}{T_o}\right)\right]\\&=\exp\Bigg(-\lambda_{\mathrm{p}}\int_{\mathbb{R}^2}\Bigg(1-\exp\Bigg(-\kappa^{-1}\int_{\mathbb{R}^2}\Bigg[\sum_r(-1)^r\sum_{R_k\in\mathbf{T}_k^r}\\&\max\Big(0,\Delta(\eta,\mathbf{x},\mathbf{y})+\min(R_k)-\max(R_k)\Big)\Bigg]f_{\mathbf{Y}}(\mathbf{y})\mathrm{d}\mathbf{y}\Bigg)\Bigg)\mathrm{d}\mathbf{x}\Bigg).\end{aligned}$$

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