Accuracy Self-Estimation in an 8-Dimensional Quantum State Identification at a Telecom Wavelength

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Abstract: We experimentally obtain single-shot confidences of a state estimation measurement with M=8 arbitrary sets of non-orthogonal states. The 8-dimensional vector of confidences represents the best knowledge of the input state for each individual quantum measurement.

1. Introduction

Identifying randomly distributed quantum states from a known set of states is an important application of quantum measurements. Perfect identification of nonorthogonal states is fundamentally impossible. It has been shown that quantum measurements can significantly surpass classical measurements and, in some cases, asymptotically approach the fundamental identification accuracy. Most of the time, theoretical and experimental efforts are focusing on statistical results, i.e. how often identification errors occur, as opposed to how accurate each experimental outcome is and if one outcome is any more certain than the other [1-3]. Here we build on our previous research [4-6] and report that confidences that can be extracted from a continuous quantum measurement can serve that purpose using the example of discriminating M = 8 coherent states that differ in (1) frequency, (2) phase, and (3) both frequency and phase. We collect a large group of data and compare single-shot confidence estimates to the ensemble-averaged probabilities of error and show that they yield identical results. This is the first experimental measurement of confidences obtained with long (M>4) alphabets and a range of modulations.

2. Continuous quantum measurement and measurement records

We take explicit advantage of the fact that the continuous measurement of the input coherent state is made over time T, where T is the duration of the input state. As input, we use modulated laser light at ~1550nm. The state identification is based on the adaptive displacement of the input with a local oscillator (LO), also a coherent state. The receiver tries to guess the input state and adjusts LO such that the correct guess displaces the input to the vacuum. A single photon detector provides feedback: if even a single photon is detected, the LO is not properly displacing the input, and the hypothesis needs an adjustment (Fig. 1). In that way, this measurement can surpass the shot noise limit of an ideal classical receiver. We report that our setup identifies states with an error rate that is nearly 4 times lower compared to an idealized classical measurement (this figure depends on the energy per input state and the modulation type), see Table 1. For each trial, our experimental setup produces a full measurement record that is comprised of (a) the history of applied displacements and (b) time stamps of photon detections. This record can be used to calculate Bayesian probabilities that the input is in one of M possible input states.

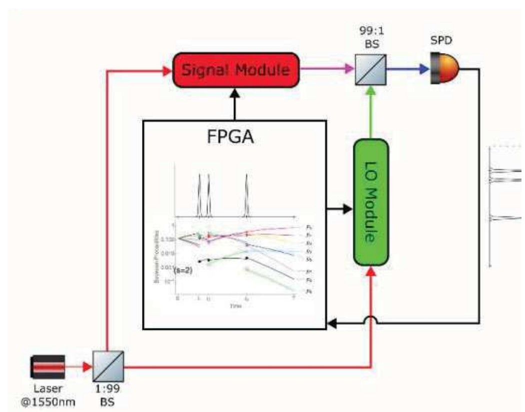

Fig. 1 Experimental setup and the example of a measurement record. A schematic diagram of the adaptive displacement-based state identification experiment used to determine the state of the single input pulse from the set of M=8 possible coherent states with arbitrary (but known) modulation. The inset demonstrates typical experimental output, each measurement yields a measurement record that is comprised of applied displacements and recorded single photon detections. As a result, a set of 8 probability estimates p_i is found and retained.

Bayesian probability values comprise a vector of confidences (in our case, this vector is 8-dimensional). The inset of Fig. 1 demonstrates the example of the evolution of the vector of confidences and its final value at time T. In our experiment, we obtain 8-dimensional confidence vectors and experimentally verify that the components of this vector, the Bayesian probability estimations based on a single measurement, are in good agreement with probabilities observed after averaging over a large ensemble of measurements (Kholmogorov probabilities). In other words, the vector of confidences is the best post-measurement knowledge of the input state. Therefore, single-shot confidences can be used for post-measurement evaluation of the quality (uncertainty) of each act of measurement. Because the detection of photons from a coherent state is stochastic, each act of measurement yields a unique post-measurement uncertainty, unpredictable prior to measurement, but known after each measurement so long as the measurement record is retained.

3. Results and discussion

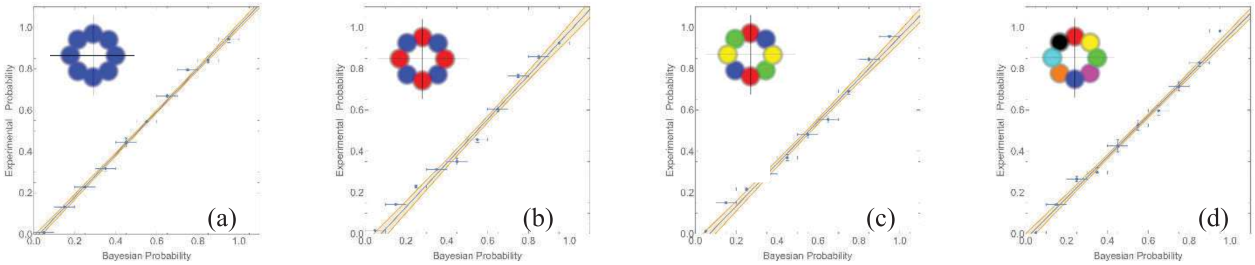

Fig. 2 Experimentally measured single-shot confidence vs the experimental ensemble average probability of successful state discrimination obtained for faint flat-top laser pulses input prepared in one of M=8 states that differ (a) by initial phase; (b) by a combination of the initial phase and carrier frequency, where M_f=2, M_p=4 (c) same as (b), but M_f=4, M_p=2, (d) by carrier frequency. Vertical error bars correspond to 1 standard deviation, horizontal error bars show histogram bin size. Straight line – best linear fit, curved lines – the uncertainty of the best linear fit. M_f (M_p) is the number of different carrier frequencies (initial phases) used in a hybrid phase/frequency alphabet.

To evaluate the universal physical meaning of confidences, we define 4 modulation protocols and establish M = 8 alphabets. We use phase modulation (known as phase shift keying), coherent frequency modulation, and hybrid (phase and frequency) modulation methods [2,3]. For each modulation, we collect a large ensemble of measurements. Next, we find how often the true state of the input matches a measured state component of the confidence vector whose single-shot probability falls into a certain range. That is, for all 8 components of the confidence vector p_i, we define the 10%-wide bins, and we compute the number of successful and unsuccessful state identifications. The ensemble-averaged experimental probability of successful identification is $q(p)=N_{correct}/(N_{correct}+N_{incorrect})$, where $N_{correct,incorrect}$ are the number of correct (incorrect) detections. We plot this experimental probability as a function of the value of the Bayesian estimate (p), for all four modulation methods (Fig. 2). As a next step, we use linear regression and fit all experimental data to a straight line. In the ideal case, the single-shot confidence values are equal to ensemble-averaged probability values, which follow from quantum theory, so we expect fit coefficients a=1, b=0, where $q(p)=ap+b$.

Modulation	8-PSK	M_f=2, M_p=4	M_f=4, M_p=2	8-CFSK
a	1.03	1.06	1.02	0.99
b	-0.02	-0.08	-0.07	-0.02
Best $\mathrm{SER_e}/\mathrm{SER_{SNL}}$, dB	-2.9	-2.2	-3.2	-5.8
Optimal $\langle n \rangle / \log(M)$	2.5	1.5	1.3	2.2

Table 1 Experimentally obtained regression values that establish the relation between single-shot confidence estimate from a quantum measurement and the experimental probability of a correct identification found from an ensemble of experimental data for different M=8 modulations. PSK – phase shift keying, CFSK – coherent frequency shift keying. In all experiments, we achieve sub-shot-noise state identification. The measured signal error rate $\mathrm{SER_e}$ is compared to that at the shot noise limit $\mathrm{SER_{SNL}}$. Negative values surpass the shot noise limit. Lower values are better. The best SER ratio is achieved at the optimal energy per bit $\langle n \rangle/\log(M)$, measured in photons/bit.

We provide fit parameters found via regression in Table 1. Experimentally, fit parameters are close to the expected values, therefore we verify that the ensemble average discrimination error probabilities observed for an ensemble of single-shot measurements are nearly equal to the observed single-shot confidence estimations.

In this work, we verified that the vector of confidences is the best post-measurement knowledge of the input state for long (M=8) alphabets of states (c.f. [6]) and does not depend on the modulation method. The modulation methods studied here are important for traditional and quantum-measurement-enhanced telecommunications. In the context of communications, the confidence values that are identical to ensemble averages can be used for quantum-enabled error correction.

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