The Potential Impact of Opportunities to Apply Mathematics to Teaching on Prospective Secondary Teachers' Competence

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In the past two decades, there has been a trend in materials for mathematics courses for prospective secondary teachers: more opportunities for teachers to "apply mathematics to teaching". That is, materials increasingly highlight how mathematical knowledge learned in the course can be useful in secondary teaching, and provide opportunities for teachers to harness this knowledge in simulations of teaching. There is little known about the effects of this curricular reform on teachers' competence. In this report, we use data from the Mathematics of Doing, Understanding, Learning, and Educating for Secondary Schools MODULE(S^2) project to examine the potential impact of using such curricular materials. The data include over 300 prospective secondary teachers' responses to 3 sets of Likert pre-/post-term surveys addressing: mathematical knowledge for teaching; expectancy for enacting selected core teaching practices; and valuing of enacting these practices. We found mean increases across the survey results. We conclude with directions for future research on the impact of this curricular reform.

Keywords: Content Courses for Secondary Teachers, Mathematicians, Mathematical Knowledge for Teaching, Expectancy, Value

Introduction & Background

Many secondary teachers do not find their tertiary mathematics experiences usable in secondary mathematics teaching (e.g., Goulding, Hatch, & Rodd, 2003; Winsløw & Grønbæk, 2014; Zazkis & Leikin, 2010). In response to this problem, multiple scholars, including within the RUME community, have advocated for mathematics courses for prospective secondary mathematics teachers to provide explicit opportunities to connect university mathematics with secondary mathematics teaching (e.g., Álvarez et al., 2020; Lai, 2019; Wasserman et al., 2017).

In these scholars' view, the disconnection problem in the US may be a consequence of historical attempts to emphasize tertiary-secondary connections through *mathematics* (e.g., The Panel on Teacher Training, 1971) at the expense of attention to *mathematics teaching practice.* To rectify this situation, secondary mathematics teacher educators have in the past decade or so developed materials that articulate links to teaching practice (e.g., Álvarez et al., 2020a, 2020b; Bremigan et al., 2011; Buchbinder & McCrone, 2020; Hauk et al., 2017; Heid et al., 2015; Lischka et al., 2020; Sultan & Artzt, 2011; Wasserman et al., 2017). An instructional device common to each of these materials is *applications of mathematics to teaching* – where teachers respond to a description of a secondary teaching situation by leveraging material learned in a tertiary course (cf. Bass, 2005; Stylianides & Stylianides, 2010).

Since the advent of this curricular reform, there have been relatively few studies describing the potential impact of the use of materials featuring applications of mathematics to teaching. As far as we know, there are two exceptions (Buchbinder & McCrone, 2020; Wasserman and McGuffey, 2021) which though promising, also featured fifteen and six teachers only. Hence advancement of research and practice in secondary mathematics education faces a problematic gap: we do not know much about the impact of a growing curricular trend in materials for mathematics courses for secondary teachers.

The purpose of this report is to take a first step in addressing this gap, by examining changes in teachers' competence. As explained below, we view competence as including mathematical knowledge for teaching, expectation of success of enacting core teaching practices, and valuing of core teaching practices. The research questions guiding this report are:

Over the course of a semester-long experience featuring applications of mathematics to teaching that were coordinated with mathematical content learned,

1. *To what extent did teachers' mathematical knowledge for teaching change?*
2. *To what extent did teachers' expectation of success for carrying out selected core teaching practices change?*
3. *To what extent did teachers' valuing of these selected core teaching practices change?*

We note that while statistics and mathematics are distinct disciplines, in secondary education, "mathematics" is often seen as including statistics, a practice that we will follow in this paper. Unless otherwise indicated, we use "teacher" to mean a prospective secondary mathematics teacher, "student" to mean secondary student, and "instructor" to mean a university instructor.

Conceptual Perspective

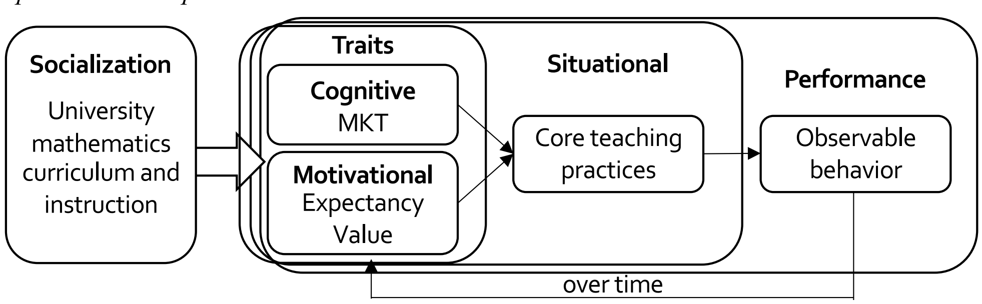
We follow Blömeke et al.'s (2015) notion of competence for teaching to include cognitive, motivational, and situational resources. Cognitive and motivational resources are also known as teacher traits; we see these as malleable. Cognitive resources include mathematical knowledge for teaching (MKT; Ball et al. 2008; Thompson & Thompson, 1996), the knowledge entailed in carrying out recurrent work of teaching. Within motivation, we focus on expectancy and value. Expectancy is the expectation of success at enacting a task in a particular situation (Wigfield & Eccles, 2000). Value is the subjective value of enacting the task, and it encompasses utility, enjoyment, and personal fulfillment (see Eccles & Wigfield, 2020, for a review).

Situational resources include teachers' understandings and conceptions of core teaching practices. These practices are those that benefit students in equitable ways; are learnable by teachers; and have the potential to promote teaching improvement over time (Grossman et al., 2009). Example core teaching practices include generating questions and discussion that promote students' reasoning, and learning about student understanding using their explanations.

We follow Eccles and Wigfield's (2020) framing that one's choices and performance feed iteratively into one's expectancy and value, and vice versa. If one experiences successful performance in mathematics and its applications to teaching, one is more likely to try again in the future. Conversely, if one perceives failure, one might desire to avoid core teaching practices.

From this perspective, an important role of teacher education is socialization: providing opportunities for teachers to learn and succeed, so as to encourage them to choose to engage in core practices in the future. Figure 1 summarizes our conceptual perspective.

Figure 1. Perspective on Competence



Data & Method

Study Context

Our report examines change in teachers' competence over the course of using MODULE(S^2) project materials in four different mathematical areas: algebra, geometry, mathematical modeling, and statistics. Materials for each area were intended to be used across one semester, and each featured 6 extended prompts. These extended opportunities specified a secondary student-level task, a goal for the scenario involving engaging students in a mathematical practice, and in most cases text or images of sample work drawn whenever possible from secondary student work obtained from secondary classroom teachers consulting for the project. Teachers were to respond to half of the prompts by writing a narrative describing how they would use the given responses to set up a whole class discussion, and the other half by videotaping themselves responding to sample student contributions as if they were personally responding to a student in their own class. For all responses, teachers were asked to make specific reference to the students' thinking, elicit student conceptions, and pose questions to advance mathematical understanding. All materials came in instructor-facing and teacher-facing versions, with the instructor-facing providing guidance for enacting core teaching practices in their own instruction.

Participants

Participants were 368 teachers enrolled in tertiary mathematics courses using MODULE(S^2) materials with 65 instructors at 54 different institutions across the United States and Canada. These participants consented to participation and completed various of the instrument forms detailed below. We defined "completion" as completing the majority of questions on that form.

Courses were taught at institutions ranging from large public research universities to small private colleges to regional public universities, and from those that served predominantly white populations to those that served predominantly minoritized populations.

Phases of Study and Instruments

Research occurred in two phases, before and after the first year of the covid-19 pandemic. Phase One spanned the project's Years 1 and 2, with data collected in two content areas per year. Phase Two spanned the project's Year 4, with data collected in all four content areas.

MKT measures. At the beginning and end of the term using MODULE(S^2) materials, we measured teachers' content knowledge for teaching. These include items from the Exponential, Quadratics, and Linear assessment (Howell et al., 2016); Geometry Assessments for Secondary Teaching (GAST; Mohr-Schroeder et al., 2017); Anhalt and Cortez's (2016) instrument on preparation for teaching modeling; and sample items based on the Levels of Conceptual Understanding of Statistics repository (LOCUS, n.d.), and an assessment item developed by Randall Groth (personal communication). To analyze change in teachers' content knowledge for teaching, we used Cohen's *d* to quantify effect size for knowledge change in each area. As of writing, we scored only assessments from Phase One data collection, so we only report those results.

Expectancy and value measures. At the beginning and end of the term, we measured each teacher's expectancy and value for enacting selected core teaching practices (SCTPs) in the area emphasized by their course. In Phase One, we used the same form in pre- and post-term administrations. After analyzing these results, and in view of conversations with piloting faculty, we posited that the large pre- ratings potentially obscured changes in expectancy and value for participants. In particular, faculty reported that teachers enrolled in their class remarked on how much they learned that they did not know could be learned. In Phase Two, to take this perception into account, we modified the post-term form to include a retrospective pre-rating for each item on the post-term measures. Moreover, in Phase Two, conversations with piloters and among the research team suggested adding an SCTP. See Table 1 for SCTPs included in each phase. See Figures 2 and 3 for a sample Expectancy item with a retrospective pre- rating; the post-term Value forms featured similar items. Underlined portions indicate parts specific to a content area.

We analyzed categorical shifts from pre- to post-test on the expectancy and value instruments across SCTPs and content areas using descriptive statistics of differences paired by teacher. We are interested in *the effect size* of any changes, rather than simply asking *whether* a difference exists. We use Cohen's d to quantify effect size for expectancy and value for enacting each SCTP; it measures the practical significance of the observed difference by accounting for variability in the difference. Common benchmarks for interpreting Cohen's d are 0.2 for a small but non-negligible effect, 0.5 for a medium effect, 0.8 for a large effect, and 1.3 for a very large effect (Sullivan & Feinn, 2012).

Table 1. Selected core teaching practices emphasized in materials used by study participants

Code	Definition	Phase One	Phase Two
SCTP1	Ask questions so that middle and/or high school students make conjectures.	✓	✓
SCTP2	Ask questions and lead discussions that help middle and/or high school students come up with mathematical explanations.	✓	✓
SCTP3	Ask questions that help middle and/or high school students make connections between different representations of the same idea		✓
SCTP4	Ask questions so that middle and/or high school students understand how to build on their thinking and what to revise.	✓	✓
SCTP5	Analyze middle and/or high school students' responses to understand their reasoning	✓	✓

Figure 2. Sample items used in Phase One pre- and post-term and Phase Two pre-term only

Expectancy item and set up, with content area and key concept emphasized	Value item and set up, with content area emphasized (no key concepts cited)
Suppose you are teaching middle or high school <u>algebra</u> students how to <u>think about functions in terms of how changes in the value of one variable may impact the value of the other variable.</u> How well does this statement describe how you feel?	How much do you personally agree with the following ideas about teaching <u>algebra</u> in middle or high school?
I would be comfortable regularly asking questions so that middle or high school students make conjectures.	*I think it is important to regularly ask questions so that middle or high school students make conjectures.*
Not at all … Very much 0 1 2 3 4 5	Not at all … Very much 0 1 2 3 4 5

Figure 3. Sample item in Phase Two Expectancy Measure, post-term only

Expectancy item and set up, with content area and key concept underlined		
Suppose you are teaching middle or high school <u>algebra</u> students how to <u>think about functions in terms of how changes in the value of one variable may impact the value of the other variable.</u>		
Looking back, how well did these statements describe you at the beginning of the course, AND now at the end of the course?		
	BEGINNING of course	NOW, at END of course
I would be comfortable regularly asking questions so that middle or high school students make conjectures.	Not at all … Very much 0 1 2 3 4 5	Not at all … Very much 0 1 2 3 4 5

Results

Changes in mathematical knowledge for teaching (MKT)

Table 2 indicates that all areas exhibited a mean increase in teachers' MKT. All effect sizes are above the threshold for large practical significance (Cohen's $d > 0.8$). All post scores are less than 50% of the maximum score, and pre- and post- means across all areas are roughly comparable. As context for interpreting this finding, we turn to validation studies of measures for content knowledge for teaching geometry, where there has been significant work at the secondary level with reported raw scores. Mohr-Schroeder et al. (2017) validated GAST forms to measure mathematical knowledge for teaching geometry with a sample of predominantly practicing teachers. In contrast, our participants were strictly prospective teachers. On full GAST forms, the mean score was 20 points out of 30 possible. Mohr-Schroeder et al. did not compare prospective and practicing teachers' performance in their sample. However, in reports of their results with a different instrument (MKT-G), Milewski et al. (2019) reported that most prospective teachers' scores were comparable to those in the lower half of practicing teachers' scores. In view of these results, it is not surprising that the teachers in our sample scored, overall, in a lower range than those in Mohr-Schroeder et al.'s sample, and apparently in a similar range over all areas. Finally, we note that the indicated sample sizes include only those teachers who completed both pre and post forms. Because not all piloters administered the post form, participant numbers are lower in some areas.

Table 2. Paired difference results for MKT, reported in terms of percentage of maximum score on each area assessment

Area	Pre-Mean	Post-Mean	Mean Δ	SD_{Δ}	Cohen's *d*
Algebra (*n*=9)	20.8%	39.2%	18.3%	13.5%	1.36
Geometry (*n*=63)	25.6%	35.9%	10.3%	10.1%	1.02
Mathematical Modeling (*n*=20)	31.3%	44.4%	13.1%	17.9%	0.73
Statistics (*n*=40)	26.6%	42.2%	15.6%	16.4%	0.95

Changes in expectancy and value for enacting selected core teaching practices (SCTPs)

Tables 3-8 summarize our findings. For brevity, the table does not include all 32 + 40 combinations of expectancy and value (Phase One: 4 SCTPs × 4 content areas ×{expectancy, value}; Phase Two: 5 SCTPs × 4 content areas ×{expectancy, value}), but we discuss their characteristics now. In Phase One, all *expectancy* effect sizes among SCTPs are non-negligible (Cohen's $d > 0.2$). Eight of the 16 indicate at least medium practical significance (Cohen's $d > 0.5$), and 1 indicates large practical significance (Cohen's $d > 0.8$). Among effect sizes for *value*, 9 indicate non-negligible practical significance, with most of them being for SCPT1 and SCTP2.

Retrospective pre-ratings were overall lower than actual pre-ratings, confirming our hypothesis that teachers, when looking back, now believe they perceived less expectancy and value at the beginning of term, as compared to their perceptions at the actual beginning of term. The lowest mean expectancy *retrospective* pre-rating for any SCTP in any content area was 2.58, compared to 3.20 for *actual* pre-ratings. The lowest mean value *retrospective* pre-rating for any SCTP in any content area was 3.46, compared to 4.28 for *actual* pre-ratings.

All effect sizes for *actual difference in pre/post expectancy* are non-negligible, 16 indicate medium practical significance, and 8 indicate large practical significance. Effect sizes for actual pre/post increases in expectancy in Phase Two are slightly larger than those in Phase One; this may be due to improvements to the materials, or simply to differences in students and instructors. All effect sizes for *retrospective difference in pre/post expectancy* are non-negligible and in fact are above the threshold for large practical significance and 15 indicate very large practical significance (Cohen's $d > 1.3$). Among effect sizes for *actual difference in pre/post value*, 14 indicate small practical significance, and 1 indicates medium practical significance. All effect sizes for *retrospective difference in pre/post value* are above the threshold for medium practical significance, and 11 indicate large practical significance.

Table 3. Phase One paired difference results for pre/post expectancy, with min and max Cohen's d across SCTPs					
Area	Pre-Mean	Post-Mean	Mean Δ	min Cohen's *d*	max Cohen's *d*
Algebra (*n*=54)	3.840	4.293	0.453	0.288 (SCTP5)	0.498 (SCTP1)
Geometry (*n*=50)	3.583	4.040	0.457	0.292 (SCTP1)	0.446 (SCTP2)
Mathematical Modeling (*n*=26)	3.421	4.320	0.899	0.675 (SCTP2)	0.839 (SCTP1)
Statistics (*n*=44)	3.317	4.119	0.802	0.502 (SCTP4)	0.613 (SCTP2)

Table 4. Phase One paired difference results for pre/post value, with min and max Cohen's d across SCTPs					
Area	Pre-Mean	Post-Mean	Mean Δ	min Cohen's *d*	max Cohen's *d*
Algebra (*n*=54)	4.384	4.565	0.181	0.075 (SCTP5)	0.336 (SCTP2)
Geometry (*n*=50)	4.455	4.510	0.055	0.088 (SCTP5)	0.206 (SCTP1)
Mathematical Modeling (*n*=26)	4.289	4.548	0.259	0.196 (SCTP5)	0.340 (SCTP2)
Statistics (*n*=44)	4.489	4.705	0.216	0.094 (SCTP4)	0.450 (SCTP2)

Table 5. Phase Two paired difference results for pre/post expectancy, with min and max Cohen's d across SCTPs					
Area	Pre-Mean	Post-Mean	Mean Δ	min Cohen's *d*	max Cohen's *d*
Algebra (*n*=69)	3.50	4.23	0.74	0.76 (SCTP1)	0.91 (SCTP5)
Geometry (*n*=27)	3.44	4.37	0.92	0.67 (SCTP3)	0.83 (SCTP4)
Mathematical Modeling (*n*=26)	3.35	4.36	1.02	0.71 (SCTP2)	0.99 (SCTP1,3)
Statistics (*n*=68)	3.45	4.06	0.62	0.42 (SCTP2, 5)	0.5 (SCTP1)

Table 6. Phase Two paired difference results for pre/post value, with min and max Cohen's d across SCTPs					
Area	Pre-Mean	Post-Mean	Mean Δ	min Cohen's *d*	max Cohen's *d*
Algebra (*n*=70)	4.60	4.75	0.15	0.12 (SCTP2)	0.28 (SCTP1)
Geometry (*n*=27)	4.60	4.82	0.22	0.22 (SCTP3)	0.54 (SCTP2)
Mathematical Modeling (*n*=26)	4.43	4.69	0.26	0.20 (SCTP3)	0.46 (SCTP1)
Statistics (*n*=67)	4.47	4.65	0.18	0.07 (SCTP3)	0.40 (SCTP1)

Table 7. Phase Two paired difference results for retrospective pre/post expectancy, with min and max Cohen's d across SCTPs					
Area	Pre-Mean	Post-Mean	Mean Δ	min Cohen's *d*	max Cohen's *d*
Algebra (*n*=54)	2.72	4.23	1.52	1.61 (SCTP5)	1.68 (SCTP1,4)
Geometry (*n*=50)	2.75	4.37	1.58	1.51 (SCTP1)	1.56 (SCTP3)
Mathematical Modeling (*n*=26)	2.79	4.36	1.56	1.76 (SCTP3)	1.91 (SCTP4)
Statistics (*n*=44)	2.95	4.06	1.11	1.20 (SCTP3)	1.29 (SCTP1)

Table 8. Phase Two paired difference results for retrospective pre/post value, with min and max Cohen's d across SCTPs					
Area	Pre-Mean	Post-Mean	Mean Δ	min Cohen's *d*	max Cohen's *d*
Algebra (*n*=54)	4.384	4.75	0.96	0.79 (SCTP5)	0.99 (SCTP3)
Geometry (*n*=50)	4.455	4.82	0.81	0.56 (SCTP3)	0.88 (SCTP4)
Mathematical Modeling (*n*=26)	4.289	4.69	1.11	0.83 (SCTP5)	1.11 (SCTP4)

Statistics (*n*=44)	4.489	4.65	0.62	0.67 (SCTP3)	0.82 (SCTP2)

Discussion & Conclusion

We sought to examine changes in teachers' competence before and after a semester-long experience featuring applications of mathematics to teaching. Our results address a gap in the literature concerning the impact of curricular reform on teachers' competence. We analyzed data about teachers' cognitive and motivational resources, specifically MKT, and expectation of success and value for enacting core teaching practices when teaching a particular content area. Overall, we found that teachers' MKT, expectation of success, and value increased after this semester-long experience.

A significant strength of our results is that they come from teachers across a variety of institutions, with different instructors, across four distinct content areas. Our results on motivation are limited in that, like most studies in this area, we rely on self report. Teachers may have felt pressure to respond more positively than the reality. Nonetheless, the overall increase in all variables, across all areas, indicates that it is plausible that expectation of success and value did improve, even if it is to a lesser extent than reported. Hence, we theorize that teachers benefit when using materials that coordinated mathematical learning opportunities with applications of mathematics to teaching, and these benefits come in the form of increased competence for teaching.

Any conclusions that can be drawn from these results are limited in two crucial ways, which also point to directions of future research. First, correlation is not causation. Although we designed the materials to enhance teachers' competence, the data here do not say what factors exactly supported the mean improvement over time we found across our data. We cannot say whether the benefits derive from the materials' design, the confluence of taking the content course alongside a pedagogically-focused methods course, a time effect that when teachers take similar pre/post forms they will generally improve, or another factor. Beyond technical studies of how teachers' knowledge appears to change or not simply by having been exposed to an assessment of MKT, we also suggest that examining potential mechanisms for change is also important. Future studies, for instance, could investigate whether certain instructional practices are associated with changes in teachers' competence; whether teachers' knowledge changes as a result of propositional knowledge or change in mathematical practice; or how opportunities to learn mathematical content and applications of mathematics to teaching interplay.

Second, we have not addressed directly the problem that many teachers themselves have been documented to find content courses not useful. We and other scholars believe that teachers would find applications of mathematics to teaching useful, but we cannot conclude this based on the analysis reported here. We are currently in the process of analyzing the data from MODULE(S^2) project to address this gap.

Finally, we observed that teachers' retrospective pre- ratings of their motivation were generally lower than their actual ratings from the beginning of term. This finding suggests practical and methodological questions. The practical question is: What changes their perspective? The methodological question is: in what areas of competence would self-reports change retrospectively, and what are the affordances and limitations of retrospective ratings?

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