(Re)defining Developmental Mathematics: A Critical Examination of the Research Literature

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Although "developmental math" is widely discussed in higher education circles, exactly what developmental math encompasses is often underdeveloped. In this theoretical report, we use a sample of highly cited works on developmental math to identify common characterizations of the term "developmental math" in the literature. We then interrogate and problematize each characterization, particularly in terms of whether they serve equity-related goals such as access to college credentials and math learning. We close by proposing an alternative characterization of developmental math and discuss the theoretical implications. We see this as a first step towards conversations about how developmental math could be conceptualized.

Keywords: Developmental Math; Equity; Time Capital; Postsecondary; College Level

Developmental math has been a regular focus of education research for decades. Understanding these courses is important as they disproportionately enroll students from historically marginalized backgrounds (e.g., Chen, 2016) and many students in these courses never complete their required math sequence (Bailey et al., 2010). We posit that despite the heavy focus on developmental math in higher education research and policy, the way that "developmental math" is defined or characterized in the research literature often runs contrary to equity concerns and begs the question of what developmental math actually is. Using definitions drawn from a comprehensive literature search, we consider how developmental math has been defined in the research literature, with the aim of problematizing some common characteristics of these definitions. We refrain from initially defining developmental math explicitly so as to let the term itself drive our sampling and analytical procedures. We devote the majority of this report towards developing a framework for understanding characteristics used to define developmental math and the different conditions each characteristic privileges. We close with a discussion about how we might define this term moving forward, and how this may impact equity goals.

Theoretical Framework: Approaching Equity from Credential vs. Learning Orientations

There is a growing recognition that developmental math and institutional structures around it are a consequence of and often reproduce structural inequalities in K-12 education (e.g., Larnell, 2016; Ngo & Velasquez, 2020). That is, the developmental population is *created* through often unexamined institutional norms and values. We aim to make the values and associated equity implications that are implied by various definitions of developmental math explicit, with the goal of proposing a way to move forward with how we understand and define developmental math.

Historically, developmental math was created to provide access to college math classes by providing instruction on content that has traditionally been considered prerequisite to advanced study, such as algebra, that students may not previously have had the opportunity to master, or that they have forgotten after a gap in enrollment between high school and college (e.g., Dotzler, 2003). Providing access to advanced study in math is motivated by two distinct, but related, equity goals: providing access to (a) college credentials (e.g., degree progression, retention), which we refer to as a *credential orientation*, and (b) math learning (e.g., procedural/conceptual knowledge), which we refer to as a *learning orientation*. Both orientations center equity, but have different outcomes of interest, even as they are interdependent (Figure 1). Considering only one at a time may have negative equity consequences. Problematically, research demonstrates that inequities exist in both developmental students' access to college credentials (e.g., Boatman & Long, 2018; Crisp & Delagado, 2014; Sanabria et al., 2020; Xu & Dadgar, 2018) and their access to rich and meaningful math learning or instruction (e.g., Givvin et al., 2011; Goldrick-Rab, 2007; Hammerman & Goldberg, 2003; Stigler et al., 2010; Webel & Krupa, 2015).

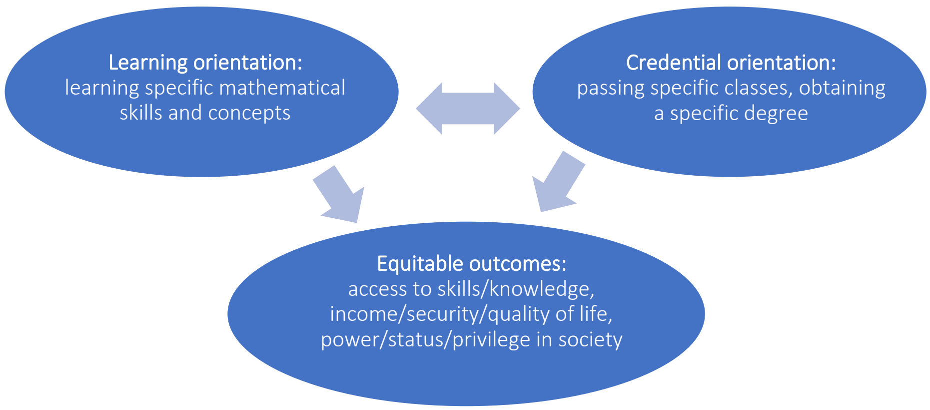

Figure 1. *Framework Relating Learning and Credential Orientations to Equity Approaches in Developmental Math*

Method

We conducted a comprehensive literature search for original reports related to developmental math. This work is part of a larger project. Here we focus only on how developmental math was defined or characterized in highly cited empirical reports or peer-reviewed journal articles.

Sample

To identify articles related to developmental math, we searched EBSCOhost for journal articles or reports published between 2000 and 2020 (inclusive) with abstracts that included the word "developmental" or "remedial", a word with the stem "math", and one of the following: college(s), universit(y/ies), post-secondary, postsecondary, or undergraduate(s). We removed duplicates, brief reports of full reports that were also included in the sample, and any articles that were not about developmental math instruction, classrooms, curricula, instructors, or students enrolled in U.S. colleges or universities. This resulted in 446 reports: 281 (62%) peer-reviewed journal articles and 168 (38%) non-journal reports. Of the 281 peer-reviewed articles, 66 (23%) were published in journals usually consumed by a math education audience. The remaining articles were published in journals with a more general education audience or that target an educational research subdomain, with the higher education audience the most prominent (47%).

Because we were interested in analyzing the most "influential" definitions in the literature, we used citation frequency as a rough proxy for a report's "influence" in order to determine which reports to code. Towards this end, in June 2022 we used Google Scholar to find the number of times each report in our sample had been cited (26 reports did not show up on Google Scholar, in which case we entered 0). We calculated the Annual Citation Rate (ACR; total number of citations divided by the report's age, in years) for each report. We formed our final subsample of influential reports by including any report that had the top 10 highest ACR and/or the top 10 total number of citations in one of the following categories: peer-reviewed journal article aimed at a math education audience, peer-reviewed journal aimed at a general education audience, and non-journal reports. We sampled from different groups to be able to contrast the definitions used by different stakeholders. There was overlap between articles with top ACRs and top overall number of citations, so the final sample included 36 records (8% of the total sample). The list of the 36 coded reports is available at Open Science Framework. Here we focus on two groups of reports within this sample: those geared at a math education audience and those not published in math education journals, hereafter referred to as “non-math education reports”.

Analysis

Development of Coding Scheme. Using the constant comparison method (Lincoln & Guba, 1985), we developed an emergent coding scheme to capture the most common characterizations observed across the literature (Table 1). These were *Level* (e.g., described as “not-college-level”), *Credit* (e.g., described as “not-for-credit”), and *Content* (e.g., described in terms of the content covered or the course names). The three characterizations are necessarily intertwined, but capture different implicit orientations about developmental math, discussed in the next section.

Table 1. Coding scheme for characterizations of developmental math

Code & Description	Example
Level: Developmental math courses cover content that is not “college-level”, sometimes described as secondary school level.	“Broadly speaking, the term ‘developmental education’ connotes a set of policies and practices designed for students who are underprepared to do college-level work in a given area. The goal of this experience is to give students the knowledge, skills, and habits that will help them be successful in the college-level version of the course (Bailey et al., 2016). The growing use of developmental education reflects an increasingly normative transition from high school to college, which while predicated on completion of secondary schooling, does not necessarily imply adequate preparation for what is deemed ‘postsecondary’ work.” (Valentine, et al., 2017)
Credit: Developmental math courses are non-credit courses.	“With their open-door admission policy, community colleges serve a population with diverse needs and a wide range of skills. In order to prepare this diverse population for college-level courses, community colleges offer non-credit developmental courses in math, reading, and writing.” (Ashby, et al., 2011)
Content: Courses cover specific math topics, typically substantially similar to second year school algebra or below.	“Remedial math includes basic arithmetic, pre-algebra, beginning algebra, intermediate algebra, and geometry. College-level math includes all courses that address topics of a skill-level equal to, or greater than, college algebra.” (Bahr, 2008).

During the code development process, we noticed that sometimes characterizations of developmental courses were mentioned, but that developmental math was never explicitly defined. A lack of an explicit definition suggests the assumption that the reader has a shared understanding of what is being discussed, which may be problematic given the complex nature of developmental math. To capture such instances, we coded whether the characterizations were part of an explicit definition, an implicit definition (in which some characteristics of developmental math were described but no explicit definition was given), or whether there was no clear attempt to define developmental math (either explicitly or implicitly).

Coding Developmental Math Characterizations. Each report was coded by two coders. Initial agreement across all codes was between 81% and 92%. Further norming was undertaken to reach consensus for all codes. Codes for the characterization of developmental math were not mutually exclusive. Codes for the nature of the definition were mutually exclusive.

Results

Table 2 gives the coding distribution in the sample overall and for our subsamples of interest. "Not college-level" was the most common characterization of developmental math (92%), followed by mathematical content (67%) and then "not-for-credit" (53%). Characterization choice appears field related. Non-math education reports used "not-college-level" (100%) and "non-credit-bearing" (70%) characterizations more often than math education (77% and 23%, respectively). In contrast, math education research favored characterizing developmental math by content (85%) compared to non-math education research reports (57%). While "non-credit" and "not-college-level" characterizations might be presumed to be linked, these characterizations only co-occurred in 53% of reports. In addition, while the specific math content might be presumed to determine whether a course is "college-level", these characteristics did not always co-occur, especially in math education literature where 41% of papers that characterized developmental math in terms of content did not characterize the course as "not-college-level".

Table 2. Summary of characterizations of developmental math in research articles in the sample

	Overall		Math-education		Non-math education	
	n	%	*n*	%	*n*	%
College-level	33	91.7	10	76.9	23	100
Not for credit	19	52.8	3	23.1	16	69.6
Mathematical content	24	66.7	11	84.6	13	56.5
College-level & not for credit	19	52.8	3	23.1	16	69.6
College-level & mathematical content	21	58.3	8	61.5	13	56.5
All three	12	33.3	3	23.1	9	39.1
n	36		13		23	

In terms of implicit versus explicit definitions, 7 (19%) reports did not provide either an explicit or implicit definition; only 9 (25%) provided an explicit definition. The distribution of the nature of the definition was similar between non-math education and math education reports.

Problematizing Characterizations of Developmental Math

The characterizations of developmental math courses that we identified (level, credit, and content) align somewhat with the credential and learning goal orientations for developmental math discussed in the Theoretical Framework. However, the extent to which these characterizations measure the intended credential and learning goals is unclear. While there is much discussion in the literature about whether existing developmental math courses serve students, we could find no substantial discussion about whether or not existing *definitions* of developmental math serve these goals. Here we attempt to address that gap.

Level Characteristic

The most commonly used characteristic to describe developmental math was "not-college-level", which often co-occurred with both "not-for-credit" and content characteristics (and by extension both credential and learning orientations). But what constitutes “college-level” math content was often left undefined and unexamined. Doing so invites a deficit framing of students in developmental classes by implicitly suggesting that developmental students lack necessary skills for engaging in college-level math work (e.g., Larnell, 2016). Indeed, characterizing developmental math courses as “not-college-level” suggests an assumption that students can only access college-level math by first repeating high-school course content; however, this assumption is problematic (e.g., Stigler et al., 2010) and unsupported by evidence.

Credit Characteristic

Characterizing developmental math courses as “not-for-credit” was particularly common in non-math education reports (and by extension, higher education research literature), which is consistent with a credential orientation that stresses degree progress. However, credits may not be a good measure of degree progress, and measuring credits in isolation de-couples equity from any direct relationship to learning outcomes. For example, there are many credit-bearing courses (e.g., precalculus) that carry credits but do not always “count” towards STEM degrees.

Previous scholarship has critiqued the non-credit characteristic of developmental courses for introducing stigma (e.g., Larnell, 2016) and not directly furthering degree progress (e.g., Logue et al., 2016). However, an oft-neglected issue with “not-for-credit” characterizations is that non-credit developmental courses have *inequitable time costs* for students. Students must still invest time in the material, regardless of whether credit is awarded. Simply attaching credits to all courses does not necessarily address this inequity. For example, co-requisite models often merge non-credit and credit-bearing math courses to produce a single for-credit course with additional non-credit hours attached (Meiselman & Schudde, 2021; Ran & Lin, 2022). While this may reduce the number of terms needed to access “college-level” math, it does not reduce the *time capital* students invest in the non-credit portions of the course. This exacerbates existing time inequities, as Black, Hispanic, female, and “non-traditional” students are more likely to both take developmental math (Chen, 2016) and to have disproportionately less time capital to invest in college (Wladis et al., 2018, 2021a, 2021b, 2022; Conway et al., 2021).

Critical structures, such as financial aid, do not give developmental students, or others with low time capital or high academic time demands, more time for college. Instead, higher time costs, which are the consequence of structural inequities, are borne by individual students (e.g., Wladis et al., 2018). Real systemic change requires an attempt to equalize time inequities, rather than focusing solely on credits, which requires shifting our definition of developmental math.

Content Characteristic

Characterizing developmental math courses as those that cover particular mathematical content was typically linked to whether that content was considered “college-level” or not. However, which courses signal the transition to “college-level” varies, ranging from Intermediate Algebra (e.g., Logue et al., 2016) to Calculus I (e.g., Hsu & Gehring, 2016). Sometimes “college-level” appears to be defined as “not-secondary-level”, but this is also contradictory, as many classes often considered “college-level” are regularly taught in high school: for example, courses above Algebra II (typically called Intermediate Algebra in college) almost universally carry college credit, yet 70% of students who enrolled directly from high school into college had taken a course above Algebra II in high school (IES, HSLS:09).

The content characteristic appears to be motivated by a learning orientation, yet learning objectives for these courses were systematically underspecified. Typically reports characterized content by the specific objects of study (e.g., linear equations) or specific course titles. But these characterizations tell us little about how students might be expected to engage with those mathematical objects. This is inconsistent with how courses are designated as “college-level” in other disciplines: for example, students might study Shakespeare in the secondary or postsecondary context, yet college Shakespeare courses are not typically classified as “developmental”[1]. This is presumably because it is not the specific *object* of study that determines the “level”, but rather *how students engage with that object*. Mathematicians also do not classify “level” based on the objects of study, but by how one reasons about them: both first grade arithmetic and number theory focus on the integers, but at radically different levels.

Developmental algebra courses have typically taken the same teaching/learning approaches as $8^{th}/9^{th}$ grade Algebra I (Givvin et al., 2011; Grubb et al., 1999; Mesa et al., 2014; Stigler et al., 2010). However, college students are developmentally different from 14-year-olds: they have typically already passed an Algebra I class (e.g., Ngo & Velasquez, 2020); they have more sophisticated reasoning skills, independence, and life experience (e.g., Mesa et al., 2014); and they often excel in academic areas in which computational math skills are not a prerequisite. Yet, existing developmental algebra courses rarely leverage these strengths. Research suggests that developmental algebra students are capable of engaging in more rigorous reasoning, justification, generalization, and abstraction about core algebraic objects, without requiring extensive prerequisite courses in computation (e.g., Givvin et al., 2011). Thus, it is possible to offer no-prerequisite algebra courses at the college-level, if we re-define “college-level” as being measured by the *way in which students engage with those algebraic objects*. In fact, offering such courses is critical if we are to provide students access to college-level math: students need opportunities to develop the higher-order-math skills that will be necessary for upper-level math courses, rather than spending time repeating procedural processes that they already practiced in high school. This, along with other features of our problematization and interrogation of other definitional characteristics of “developmental math” in the literature, leads us to propose one potential new definition of developmental math.

New Definition of Developmental Math

In this section we present one potential new definition of developmental math, where developmental is no longer the complement to college-level, but rather a *subtype* of college-level. In particular, we propose to define developmental math as follows:

> **Definition: Developmental math courses** in college are courses that (1) require **no extensive prerequisite knowledge of algebra** (or other computational skills beyond arithmetic), (2) provide students with **immediate access to college-level math** (defined based on the kinds of reasoning/justification, abstraction/generalization, and particular conceptions expected of students), and (3) provide students with **necessary time resources** for learning college-level math, based on their individual math time demands and access to time capital more generally.

This reconceptualization addresses several issues associated with current characterizations of developmental math. First, by positioning developmental courses around college-level skills that do not require algebra, this conceptualization is an asset-based approach that focuses on leveraging college students’ strengths (such as their developmental maturity). Doing so also provides better access for *every* student to develop higher-order mathematical skills from the

[1] Special thanks to Carolyn James for the specific use of Shakespeare as a metaphor.

start, which are necessary to succeed in advanced math courses. Second, positioning these courses as college-level also eliminates stigma and practical problems associated with offering courses not-for-credit. Lastly, this conceptualization explicitly recognizes and addresses systemic *time inequities* that have been ignored by existing structures.

Reconceptualizing what developmental math *is* also requires reconceptualizing how success in developmental math is assessed. As previously discussed, measures of success require both a credential and learning orientation. Towards these goals, measures of success in a reconceptualized developmental math framework could include: 1) whether the courses meet the criteria stipulated in the developmental math definition above; 2) what students are actually learning in these courses (e.g., using concept inventories or other validated assessments [e.g., Carlson et al., 2010; Peralta et al., 2020; Wladis et al., 2018]); and 3) whether students are given the time resources needed to succeed in these courses (e.g., validated measures of time capital/demands [e.g., Wladis et al., 2018; Conway et al., 2021]). We note that if these three measures are met, we should also naturally see other positive outcomes (e.g., grades, persistence, general and STEM-specific degree progress and attainment). However, more traditional measures alone do not necessarily ensure that the first three measures are being met, and thus that equity in terms of access, learning, and time capital is being attained.

We recognize that our proposed definition requires a dramatic shift in how developmental math is implemented and thought about. If adopted, it also requires a renegotiation of undergraduate mathematics at large. This is not an easy proposition and would likely be the work of a generation of researchers and practitioners working together. However, substantial evidence suggests that existing definitions and implementations of developmental math are both inequitable and ineffective. We contend that dramatic reconceptualizations are necessary.

Conclusion

In this paper we have proposed to re-frame the definition of developmental math around two critical areas: (1) the extent to which developmental courses provide students access to true "college-level" math content (defined by the types of reasoning, abstraction and conceptions in which students engage, rather than computational skill), and (2) the extent to which developmental math courses provide students access to the time resources needed to address inequities in other resources (e.g., financial, prior educational access). These two foci of our revised definition align with the traditional learning and credential orientations, but in new ways that we have not seen represented in the literature. Our hope is that other scholars will build on this perspective, to further improve on our implementations of developmental math, so that it can better serve the equity goals developmental math was designed to support.

This new definition alone is not enough to solve all of developmental math's equity challenges. Research and advocacy are needed in many other areas, including: classroom and college climate, implicit bias, math anxiety and trauma, and specific teaching methods (e.g., Larnell, 2016; Martin, 2009; Mesa et al., 2014). This new perspective is just one important shift that we see as critical to more equitably serving developmental math students.

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