

What Instructional Factors do Prospective Secondary Teachers Attribute to their Learning?

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Although it is well known that motivational and cognitive resources influence secondary teachers' instructional quality, less is known about the tertiary instructional factors that influence secondary teachers' development of these resources. To address this gap, we report on factors that prospective secondary teachers attribute to their learning. We draw on survey responses of 70 prospective secondary teachers enrolled in mathematics courses for teachers using Mathematics of Doing, Understanding, Learning, and Educating for Secondary Schools (MODULE(S²)) materials in one of four content areas. We triangulate response themes with data from 300 prospective secondary teachers on their perceptions of instructional practices used in a mathematics course for teachers using the same suite of materials. Then, we compare these themes with literature documenting implementation of mathematics curricula in these courses. We argue that coordinating mathematics content, applications of mathematics to teaching practices, and tertiary instructional practices are key to success of these mathematics courses.

Keywords: Content Courses for Secondary Teachers, Mathematical Knowledge for Teaching, Instructional Practices

Although prospective teachers take many tertiary mathematics courses (e.g., Hill, 2011; Tatto & Bankov, 2018), many find their experiences in these courses irrelevant to secondary teaching (e.g., Goulding, Hatch, & Rodd, 2003; Zazkis & Leikin, 2010). One response to this problem is to incorporate applications of mathematics to teaching into content courses for secondary teachers (e.g., Álvarez et al., 2020a, 2020b; Bremigan et al., 2011; Buchbinder & McCrone, 2020; Hauk et al., 2017; Heid et al., 2015; Lischka et al., 2020; Sultan & Artzt, 2011; Wasserman et al., 2017). Emerging empirical scholarship on the impacts of such applications points to the promise of this solution (e.g., Buchbinder & McCrone, 2020; Wasserman & McGuffey, 2021). However, as with any curricular innovation, it is not just the curriculum materials that matter, but also how the curriculum materials are deployed (e.g., Cohen, 1990; Stein et al., 2007).

Our purpose in this report is to identify instructional factors in successful tertiary-level mathematics courses for prospective secondary teachers. By success, we mean that the prospective teachers develop their competence for teaching; and by competence, we include teachers' cognitive and motivational resources for teaching. We focus especially on the resources of mathematical knowledge for teaching, value commitments, and expectation of success in mathematics and teaching. We address the research questions: (RQ1) *What instructional factors do prospective secondary teachers attribute to their learning in a mathematics course for teachers?* (RQ2) *To what extent do tertiary instructors' instructional practices associate with prospective secondary teachers' increase in their expectation of future success?*

In the remainder of the report, unless otherwise noted, we use “teacher” to refer to prospective secondary mathematics teacher, “instructor” to refer to the instructor of a tertiary-level mathematics course, and “student” to refer to secondary students.

Background & Conceptual Perspective

Design and impact of mathematics courses for secondary teachers

A teacher’s instruction benefits from robust mathematical practice and knowledge of secondary content from an advanced perspective (Baumert et al., 2010; Sword et al., 2018), and course design has historically reflected this understanding (e.g., CUPM, 1961; CBMS, 2001; Murray & Star, 2013; Tucker et al., 2015).

However, results into the last decade also suggest that design principle for mathematics courses around *only* mathematics falls short: many teachers saw their mathematics courses as disconnected from their future teaching (e.g., Goulding et al., 2003; Zazkis & Leikin, 2010). Even when teachers enrich their understanding of the secondary mathematical concepts that they will teach, they may still exit tertiary mathematics believing that they may have become better mathematicians, but not better mathematics teachers (Wasserman & Ham, 2013).

In response to the inefficacy of mathematics courses for teachers, a number of scholars have advocated for the inclusion of *applications of mathematics to teaching* in these courses (Álvarez et al., 2020; Artzt et al., 2011; Bremigan et al., 2011; Heid et al., 2015; Lai, 2019; Lischka et al., 2020; Wasserman et al., 2017). By applications of mathematics to teaching, we mean opportunities for teachers to respond to secondary teaching scenarios using content addressed in the course. Two examples of such applications are shown in Figure 1. Bass (2005) and Stylianides and Stylianides (2010) argued that drawing on mathematical knowledge to address problems of teaching is enacting a form of applied mathematics. Their arguments are consistent with principles of a practice-based theory of professional education (e.g., Ball & Cohen, 1999).

Figure 1. Example snapshots of applications of mathematics to teaching

Two parts of a student's work on a task are shown below. Identify what the student has done well when analyzing the data, and areas to work towards understanding about analyzing univariate quantitative data. (B) Make a frequency table and histogram of the data for pen A. <table border="1" style="margin-right: 10px;"> <thead> <tr> <th>Interval</th> <th>Frequency</th> </tr> </thead> <tbody> <tr> <td>4.5-5.4</td> <td>14</td> </tr> <tr> <td>5.5-6.4</td> <td>5</td> </tr> <tr> <td>6.5-7.4</td> <td>6</td> </tr> <tr> <td>7.5-8.4</td> <td>9</td> </tr> </tbody> </table> (A) Find the mean and median weights of the chickens in pen A. Round to the nearest tenth if necessary. $4 + 5 + 6 + 7 + 8 = 30$ $30 / 5 = 6$ $\bar{x} = 6$ $4, 5, 6, 7, 8$ $n = 5$ $\frac{n+1}{2} = \frac{5+1}{2} = 3$ $\text{median} = 6$	Interval	Frequency	4.5-5.4	14	5.5-6.4	5	6.5-7.4	6	7.5-8.4	9	As students are working independently on rotations of a flag around a point, you observe two students with the following work completed. Student 1: Student 2: Record a video of yourself providing a response to both Student 1 and Student 2. Build on their thinking to help the student finish their thought, prompt the student to investigate an error, or help the student move forward in their thinking.
Interval	Frequency										
4.5-5.4	14										
5.5-6.4	5										
6.5-7.4	6										
7.5-8.4	9										

Pilot studies examining courses where such applications are used suggest that, in contrast to previous studies, teachers are able to articulate how course content connects to future teaching (e.g., Fukawa-Connelly et al., 2020; Wasserman & Galarza, 2018). Further, the 15 teachers in Buchbinder and McCrone’s (2020) study went on to incorporate task designs from their tertiary mathematics course into a lesson for secondary students while student teaching. Teachers in Wasserman and McGuffey’s (2021) study of 6 teachers attributed teaching moves to their experiences in a real analysis course featuring these applications. Attributions were predominantly of two kinds: experiences with applications, and instructional practices that were modeled by their real analysis instructor.

Instructional practices in middle and secondary grades and at the undergraduate level

Multiple studies with middle and secondary students suggest the beneficial impact of core teaching practices (e.g., Grossman et al., 2009) that maintain and elevate cognitive demand, and elicit and build on student thinking (e.g., Gates Foundation, 2012; Voss et al., 2011). Yet Banilower et al.'s (2013) national survey suggests that many secondary teachers may not teach in these ways. They found that only 55% of secondary teachers focus on developing students' mathematical practices, including mathematical justification. Banilower et al. do not suggest reasons for their findings. However, based on other literature, we do know that teachers' practices may depend on their self-efficacy (Zee & Koomen, 2016), values (e.g., Schoenfeld, 2010), and pedagogical content knowledge (Baumert et al., 2010).

Perhaps unsurprisingly, the principles of effective instruction for middle and secondary grades carry through to the undergraduate level. Undergraduate instructors are increasingly aware of benefits of eliciting undergraduates' reasoning and supporting undergraduates' collaboration (e.g., Laursen & Rasmussen, 2019). These teaching practices benefit student outcomes overall (e.g., Bressoud & Rasmussen, 2015).

Summary of perspective based on the above literature review

Altogether, we take the view that mathematics courses for teachers are a productive place to incorporate the kinds of teaching practices that have been shown to be effective at the middle and secondary level, where many candidates of secondary mathematics teacher education programs will end up teaching. These teaching practices are consistent with trends at the undergraduate level and benefit undergraduate students. Further, there is evidence that teachers may transfer instructional practices experienced at the tertiary to the settings of their future secondary teaching (Buchbinder & McCrone, 2020; Wasserman & McGuffey, 2021). Hence one way to move the needle on secondary teachers' practice may include tertiary instructors' modeling of effective teaching practices.

Data & Method

Study Context

Our report examines teachers' perceptions of their experiences in a mathematics course for teachers, during a term when the course used MODULE(S²) project materials in one of four different mathematical areas: algebra, geometry, mathematical modeling, and statistics. Materials for each area were intended to be used across one semester, and each area featured 6 extended applications of mathematics to teaching, termed "Simulations of Practice". The examples from Figure 1 are excerpts of these Simulations of Practice. These activities asked teachers to build on sample student thinking, and to generate questions that elicited student reasoning. Simulations were designed to apply the mathematics learned through the materials. All materials came in instructor-facing and teacher-facing versions, with the instructor-facing providing guidance for building on teacher thinking, and generating questions that elicited teachers' reasoning. Elsewhere we have analyzed teachers' pre- and post-term expectation of success and value of carrying out core teaching practices, and found mean increases in these variables across all content areas (see Lai et al. in these proceedings). Instructors received support from the project team in the form of summer workshops and meetings with materials developers throughout the academic year.

Participants

Data were drawn from responses of 368 teachers enrolled in tertiary mathematics courses using MODULE(S²) materials with 65 instructors at 54 different institutions across the United States and Canada. These participants consented to participation and completed various of the

instrument forms detailed below. We defined “completion” as completing the majority of questions on that form. Courses were taught at institutions ranging from regional public universities to large public research universities to small private colleges, and from those that served predominantly white populations to those that served predominantly minoritized populations.

Instruments, Analysis, and Phases of Data Collection

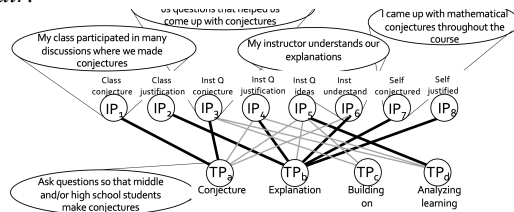
RQ1: Teacher Perceptions of Learning (RQ1). We distributed a survey at the end of the term where we asked teachers to identify factors that influenced their learning. These open-ended questions asked teachers, “What did you learn about doing [content area] as a result of this course? What was most helpful about this course for learning to do [content area]?” and “What did you learn about teaching [content area] as a result of this course? What was most helpful about this course for learning to teach [content area]?”

We examined factors influencing change in expectations of success or value in mathematics or mathematics teaching. We coded responses for *expectation of success*: confidence or facility in aspects of doing mathematics, learning mathematics, or teaching mathematics (Eccles & Wigfield, 2020); *value*: importance, benefit, worth, or enjoyment ascribed to aspects of doing mathematics, learning mathematics, or teaching mathematics (Eccles & Wigfield, 2020); and *course attribution*: attributing change in expectation of success or value to instruction, where instruction includes course activities, norms, or interactions (e.g., Cohen et al., 2003).

RQ2: Pre/Post Expectation of Success in Teaching Practices (TPs). We measured each teacher’s expectation of success for enacting selected teaching practices (TPs) in the area emphasized by their course. Our items for this construct use phrasing from Eccles et al.’s (1993) study. All items used a Likert scale from 0 (not at all) to 5 (very much) and read: “Suppose you are teaching middle or high school [content area] students [about key concept]. How well does this statement describe how you feel? ‘I would be comfortable [TP].’” Key concepts aligned to curriculum materials (e.g., covariational reasoning was an algebra key concept). We analyzed categorical shifts from pre- to post-test on the expectancy and value instruments across TPs and content areas using descriptive statistics of differences paired by teacher.

RQ2: Perception of Instructional Practices (IPs). We surveyed teachers’ perception of the extent to which they experienced instructional practices (IPs) similar to TPs. Item phrasing mirrored that of TPs, phrased from the perspective of learning (“I did ...”, “My class did ...”) and teaching (“My instructor ...”). We identified clusters of TPs that corresponded to or supported each other: for instance, an instructor or teacher’s capacity to build on learner thinking supports the capacity to ask questions that elicit conjectures. We used the Pearson correlation coefficient r to measure correlations between teachers’ expectation of success in a TP and IPs whose modeling corresponded to or supported the TP. For brevity, we do not list all TPs and IPs in full, but only a selection of them. These are shown in Figure 2.

Figure 2. Correlations explored between TPs and IPs. Bold line indicates a corresponding TP-IP pair. Gray line indicates supporting TP-IP pair.



Phases of Data Collection. Research occurred in two phases, before and after the first two years of the covid-19 pandemic. Phase One spanned the first two years of the project, with data

collected in two content areas per year. Phase Two spanned the fourth year of the project, with data collected in all four content areas. RQ1 analysis has only been completed with Phase Two data at time of writing. RQ2 analysis is reported with Phase One and Phase Two data.

Results

Instructional factors prospective secondary teachers attribute to their learning

Table 1 summarizes the percentage of teachers who made at least one statement regarding expectation of success, value, or course attribution to their learning, across their entire response to the Teacher Perception of Learning survey. This table also shows the number of total statements of expectancy, value, or course attribution.

Table 1. Coding results for responses to Teacher Perceptions of Learning.

	% participants mentioning expectation of success	% participants mentioning value	% participants mentioning attributions	# Expectation of success statement (Pos + Neg)*	# Value statements (Pos + Neg)*	# Attribution statements (Pos + Neg)*
Algebra ($n = 28$)	82.1%	60.7%	67.9%	37+0	25+2	27+2
Geometry ($n = 6$)	100.0%	83.3%	50.0%	12+0	7+0	5+0
Math Modeling ($n = 23$)	78.3%	95.7%	73.9%	30+0	48+2	25+5
Statistics ($n = 13$)	92.3%	76.9%	61.5%	21+1	14+2	9+3

*Pos = positive statement, neg = negative statement

Across all areas, teachers overall described increased facility in content knowledge and working with students. In algebra and geometry, multiple teachers cited increased knowledge of “why things work”, describing “deeper” levels of understanding (e.g., “Being challenged to dig deeper into these ideas will be helpful in my future career”). In statistics and mathematical modeling, multiple teachers described little previous knowledge of these topics, and feeling more confident about teaching the topic as a result of the course.

When looking across instructional factors that teachers reported as influential, the most common elements across all areas are applications of mathematics to teaching, or simulations of practice (22 mentions; e.g., “The videos we had to create where we looked at a student’s answer... get them to think where they might come up with the answer on their own without me giving them the answer I found very beneficial and helpful!”) and discussion with other teachers (35 mentions; e.g., “Having conversations with peers and being given time to absorb and reflect on ideas was really helpful.”) All the above factors were mentioned across all areas. Teachers also cited curricular structure and content (e.g., “Assignments that led us to figuring out an idea before giving the definition of that idea really helped make the definition impactful”), opportunities to develop conceptual understanding (e.g., “The thing that was most helpful was discussing why things work and seeing how it all connected”). For brevity, we only list themes with at least 5 mentions found in attribution statements. See Table 2.

Negative statements were comparatively rare, and differed by area. For instance, in Algebra, two teachers stated that the depth of content was inappropriate for secondary teaching. In Mathematical Modeling, all five negative attribution statements were from one instructor’s course and all described repeated experience with modeling as redundant. In Statistics, two stated that they still felt uncomfortable with statistical concepts, and one stated they had insufficient opportunity to apply statistics to teaching. In statistics and modeling, three described “unnecessary” inclusion of social justice issues.

Correlating changes in teachers' expectancy and instructional perception

Figure 3 indicates positive correlations between pairs of variables examined. Although all correlations are relatively small ($0.07 < r < 0.24$), that they are all positive indicates an overall pattern that larger increases in expectation of success can be predicted by teachers' more positive experience of their instructional environment, in particular, their perception of how much they experienced core teaching practices during the content course.

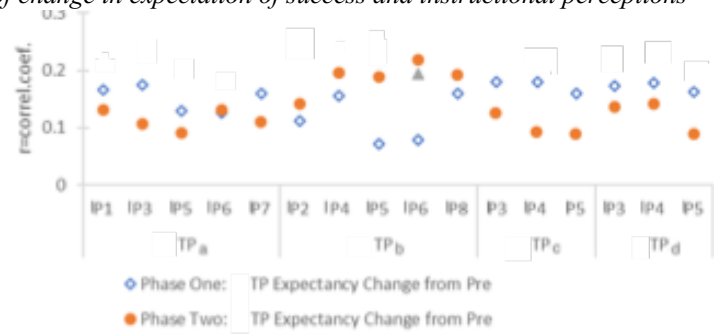
Table 2. Attribution categories for instructional factors

Attribution category	# statements (Pos + Neg)*	Content areas of mentions [◇]	Example
Discussions with other teachers (including instructor role in facilitation)	35+0	A, G, M, S	"In this class, the teacher was probably the most helpful. She did a great job pushing us to talk and discuss each problem. Then looking back, you can see the results of those discussions. Being able to do that myself will be a massive help."
Simulations of practice	22+0	A, G, M, S	"The videos we had to create where we looked at a student's answer... get them to think where they might come up with the answer on their own without me giving them the answer I found very beneficial and helpful!"
Course structure and content	7+4	A, G, M	"When I heard the phrase "in the future your students will ask you.", I never really thought about it, but after witnessing it first hand and with the exact same topics from class... I was shook and thankful that I have this class to teach me fundamental techniques and strategies to help with my future class. Thank you!!!!!!!!!!!!!! :)" "Although at some point it felt like I was doing the same assignment but with a different skin. What I mean by that is I felt like the assignment was the same for each new topic. The only difference was that we were given a new topic. I felt like there was nothing new to really learn after the first few weeks of the class."
Doing math modeling	11+0	M	"I think viewing and practicing modeling problems ourselves made it easier to see what modeling is and does."
Written materials	3+2	A, G, S	"The way that the book, and the class as a whole, took us step by step through each new content element was incredibly helpful." "The book was not helpful"

* Pos = positive, Neg = negative

◇ A = Algebra, G = Geometry, M = Mathematical Modeling, S = Statistics

Figure 3. Correlations of change in expectation of success and instructional perceptions



Discussion & Conclusion

We set out to examine the instructional factors that teachers attribute to their learning, and to what extent instructors' practices associate with teachers' increase in their expectation of future success. We found that discussions with other teachers and applications of mathematics to teaching were mentioned most as attributions to increased competence. In some ways, these results are not surprising. The findings bear out the working hypotheses of a practice-based theory of professional education: when teacher preparation is explicitly and intentionally linked to the practice of teaching, it is more likely to be effective.

Our findings are important because teachers attributed usefulness to specific features of curriculum and instruction. We strengthen results from existing smaller studies (e.g., Buchbinder & McCrone, 2020; Wasserman & McGuffey, 2021) by establishing the impact of instructional practices and applications of mathematics to teaching in four different content areas, with over 50 different instructors in over 50 different institutions.

Generalizations from this study are limited by the fact that piloting instructors volunteered to participate and had support, so they may have been more equipped to enact the curriculum as intended. The survey responses are based on teachers' self-report, and they may have felt compelled to respond more positively than they felt, or to not write as many negative comments. Nonetheless, given the sheer number of mentions of applications of mathematics to teaching, and discussion with other teachers, along with the overall small but positive associations between instructors' practices and teachers' expectation of future success in related practices, we conclude that applications of mathematics to teaching are a key innovation for secondary teachers to see the usefulness of the course content. We also conclude that instructors' practices do shape the impact of the course on teachers.

The practices identified by teachers in our study as well as in Wasserman and McGuffey's study, such as facilitating productive whole class discussions, are consistent with principles of inquiry-based mathematics education (e.g., Laursen & Rasmussen, 2019). Yet we emphasize that our conclusion is not that inquiry-based mathematics education is needed (though we believe this), but that practices of such instruction are part of a bigger picture. This picture includes both how the course taught and what is taught. Instructional practices are how a course is taught; what is taught includes the opportunities to learn from the curriculum, including activities such as the applications of mathematics to teaching. Our curriculum features intentional coordination of content with applications of mathematics to teaching. We propose that this coordination is an essential feature of curriculum for secondary teachers, if the course is to be perceived as useful, and to support the development of teachers' cognitive and motivational resources for teaching. Moreover, we posit that the instructional practices should be consistent with the images of teaching practice in applications of mathematics to teaching.

Looking forward, we see the need for further studies to document the impact of instructors' practices on teachers' development, for instance conducting case studies of courses with higher and lower mean gains in teachers' competence. With the hope that this curricular reform takes even greater hold, we also suggest studies into how instructors enact materials with applications of mathematics to teaching.

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