Multilevel Code Designs for mmWave Networks

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***Abstract*—Millimeter-wave (mmWave) networks support a large variety of applications, particularly delay-sensitive applications by providing high-speed communication. A well-recognized challenge in mmWave communications is that mmWave links are susceptible to blockage and thus, communication may get disrupted. In this paper, we design and evaluate *low-complexity proactive* transmission mechanisms for mmWave networks that are resilient to such disruptions. Our mechanisms build on the multipath environment and on the existence of accurate models for link blockage probabilities in mmWave networks. We propose the deployment of symmetric multilevel codes across paths to achieve an attractive trade-off between the average information rate and a graceful performance degradation. Our numerical evaluations show that our proposed coding schemes indeed provide a graceful performance degradation compared to alternative schemes (such as erasure correcting codes), while significantly reducing the code complexity compared to traditional multilevel code designs.**

I. Introduction

Millimeter-wave (mmWave) communication networks have currently been deployed to support a large variety of applications. Since they can offer high-speed communications, they are particularly important for delay-sensitive applications such as cyberphysical and control systems [1]–[3]. However, a well-recognized challenge is that mmWave links are susceptible to blockage and hence, communication may get disrupted.

In this paper, we design and evaluate *low-complexity proactive* transmission mechanisms for mmWave networks that are resilient to link blockages. Broadly speaking, there are two mechanisms for reliability: (i) proactive mechanisms, which allocate sufficient resources in advance so that we can still offer communication guarantees at no additional delay even if the network resources get diminished due to blockage; and (ii) reactive mechanisms, which identify network disruptions using feedback mechanisms and adapt to them. Reactive mechanisms are more resource efficient but they incur delay while collecting feedback and implementing network reconfigurations. Differently, proactive mechanisms are better suited to offer reliability guarantees for delay-sensitive applications. Given that mmWave networks can provide high throughput, in this paper we focus on proactive mechanisms which sacrifice throughput but achieve operation with no additional latency.

Our mechanisms build on the following opportunities: (i) over mmWave networks, we may have multiple paths connecting a source and a destination; this would create a multipath environment; and (ii) in mmWave networks, we can accurately estimate the link blockage probabilities[1] in advance through existing models [4]–[7]. However, leveraging these opportunities is not straightforward.

The first challenge is that link blockage leads to the "permanent" unavailability of paths in the timescale of a delay-sensitive communication, and thus we cannot simply "average out" these events. For instance, consider a set of 6 paths that all have blockage probability 0.3. This implies that when we use this network, with probability 0.32 only 4 of the paths (and we do not know which ones) will be operational; or with probability 0.19, only 3 of the paths will be operational. Thus, when we use this network, we may be able to use only a subset of the paths, and we do not know in advance which ones to use. If we simply send uncoded data, we cannot control the information received[2]. Instead, we want to control what information will be received if some paths are operational.

This problem can be addressed using symmetric multilevel codes that we propose to deploy over space (across paths). Multilevel codes (see our review in Section II) allow a graceful performance degradation: if less than the expected amount of blockages occur, we can take advantage of this to improve the information rate; and if more than the expected amount of blockages occur, although the information rate will decrease, it will not decrease to zero. A remaining challenge is the operational complexity of such codes, which increases with the number of paths utilized (see Section II).

A second challenge of this work is that, there is no clear notion of "optimality": multiple rate-outage probability trade-off curves can be attractive, and a different one can be "dominant" depending on the optimization criterion. For example, if we use a single erasure code, we can achieve the highest average rate (averaged over all network realizations), yet at the cost of high outage (many of our networks may be non-operational). Even if smoother trade-offs can be achieved, none of them clearly "dominates". Thus, instead of adopting a single opti-

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[1]The blockage probabilities are proportional to the density and the velocity of blockers, as well as the physical distances between nodes.

[2]If we send 6 independent information streams, one through each path, we would have no control on which information stream would be received.

mization criterion and claiming optimality for it, we provide coding schemes and argue why they are suitable proactive mechanisms for link blockages in mmWave networks.

Contributions and Paper Organization. In Section II, we provide an overview on the 1-2-1 model for mmWave networks, erasure codes, and symmetric multilevel codes.

In Section III, we propose a proactive transmission mechanism for arbitrary mmWave networks by deploying symmetric multilevel codes over space, i.e., we encode the source sequences in packets and send these packets over multiple network paths. We propose an optimization formulation that seeks to suitably balance the average information rate with a graceful performance degradation. We also present a low-complexity design that approximates well the aforementioned trade-off.

In Section IV, we numerically assess the performance of the proposed coding schemes and compare them with some baseline methods. Our evaluations show that: (i) the proposed schemes provide a more graceful performance degradation compared to fixed-resilience coding schemes, such as erasure correcting codes; and (ii) our complexity reduction technique gives a comparable performance in terms of information rate, while significantly reducing the code complexity. Finally, in Section V we conclude the paper.

II. Coding Framework

Notation. $[a:b]$ is the set of integers from a to $b > a$, and $|\cdot|$ denotes the cardinality for sets; for a vector v, we denote with $\|v\|$ the ℓ_2-norm of v.

We consider the so-called 1-2-1 network model, which was introduced in [8] to capture the fact that mmWave communications require beamforming with narrow beams to compensate for the high path loss. The 1-2-1 model 'strips' the physical layer component and focuses on capturing the *directivity* key characteristic of mmWave communication. In this sense, the 1-2-1 network model is simple, yet informative.

We focus on an N-relay 1-2-1 network where N relays (operating either in full-duplex or half-duplex mode) assist the communication between the source node (node 0) and the destination node (node $N+1$). At any particular time, a node can transmit to at most one node and it can receive from at most one node. Two nodes communicate by steering their beams towards each other. This activates a link that connects them (termed as a 1-2-1 link in [8]). Even though in this paper we assume that every relay has a single transmit and receive beam, our results naturally extend to scenarios in which the relays have multiple transmit and receive beams.

In [8], the unicast capacity of an N-relay Gaussian 1-2-1 network was approximated to within an additive gap that only depends on N. A linear program was proposed in [8] to compute the approximate capacity and the optimal beam schedule in polynomial time, both for full-duplex and half-duplex modes of operation. Recently, in [9] we showed that these results hold even if the network links experience blockage. These results show that we can efficiently find an optimal set of paths (i.e., a set of paths that achieve the approximate capacity when they are operated) and their schedule. Thus, in this paper we assume that such paths and schedules are given.

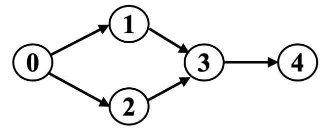


Fig. 1: An example network with $N = 3$ relay nodes.

In mmWave networks, links are susceptible to blockage (failure) and the blockage probabilities can be estimated through existing models [4]–[7]. Particularly, the blocker arrival process can be modeled as a Poisson point process to derive the blockage rate of the line-of-sight links. The blockage rate $\alpha_{j,i}$ of the link from node $i \in [0:N]$ to node $j \in [1:N+1]$ can be written as $\alpha_{j,i} = \lambda_{j,i} d_{j,i}$. Here, $d_{j,i}$ is the distance between nodes i and j; and the parameter $\lambda_{j,i}$ is proportional to the blocker density and velocity as well as to the heights of the blockers, the receiver, and the transmitter [4].

Since we are interested in low-delay communication, we consider a permanent (as compared to the timescale of communication) blockage model, where the link from node $i \in [0:N]$ to node $j \in [1:N+1]$ is blocked with probability $q_{j,i}$ and it is not blocked with probability $(1 - q_{j,i})$. That means that, if a link of capacity $\ell_{j,i}$ is blocked, its capacity is assumed to be zero and any packet transmitted through this link is blocked. If the link is unblocked, it successfully transmits packets at rate $\ell_{j,i}$. This model is different from the erasure channel model where a packet is blocked with probability $q_{j,i}$ at every channel use. In an erasure channel, even if a packet is blocked at one channel use, it can be successfully transmitted through the same link in another channel use. On the contrary, in the permanent blockage model, if a link is blocked it is assumed to be blocked during the timescale of communication. Thus, any transmitted packet through that link is blocked. In the following example, we show that an optimal schedule for erasure channels is not necessarily optimal for the permanent blockage model.

Example 1. Let $\ell_{j,i}$ denote the capacity of the link from node $i \in [0:N]$ to node $j \in [1:N+1]$. Consider the network in Fig. 1 for $\ell_{2,0} = 4$, $\ell_{3,2} = 12$, $\ell_{1,0} = \ell_{3,1} = 3$, $\ell_{4,3} = 6$ and the link blockage probabilities are zero except for $q_{3,2} = 2/3$. There are two paths connecting the source (node 0) to the destination (node 4): $p_1 : 0 \to 1 \to 3 \to 4$ and $p_2 : 0 \to 2 \to 3 \to 4$. In Fig. 1, in the erasure channel model, we can simply replace the link capacities $\ell_{j,i}$'s with the average link capacities $(1 - q_{j,i})\ell_{j,i}$. An optimal schedule transmits through p_2 because it has a higher rate than p_1. However, in the permanent blockage model that we consider, two scenarios can happen: (1) the link with capacity $\ell_{3,2}$ is blocked and hence, p_2 is blocked with probability $2/3$; or (2) none of the links is blocked with probability $1/3$. The rates of p_1 and p_2 are similar but there is a high probability that p_2 is blocked. Thus, an optimal schedule transmits through p_1.

A. Erasure Correcting Codes

A classical approach for resilience against link blockages is to use an erasure correcting code [10]–[17], which is a forward error correction code under the assumption of bit (or packet) erasures. An erasure code (n, k) transforms k information packets into n packets in a way that the original message can be recovered from any k packets (out of n packets). Thus, the information rate is equal to k/n. An erasure code supports a given number of blockages, i.e., it has a single threshold of blockage: we enter "outage" if the number of blockages is larger than the design (i.e., less than k packets are received resulting in a zero information rate), and we succeed (at least k packets are received resulting in a k/n information rate) if it is lower. Thus, erasure codes do not exhibit a graceful performance degradation. Moreover, even if we succeed, having fewer blockages than anticipated does not increase the information rate. We next define the average rate and the outage probability of an erasure code.

Definition 1: The *average* information rate of an erasure code (n, k) is defined as,

$$R_{\mathrm{E,k}} = \frac{k}{n}\left(1 - P_{\mathrm{out}}\right), \tag{1}$$

where P_{out} is the outage probability defined as,

$$P_{\mathrm{out}} = P(X < k), \tag{2}$$

where the random variable X denotes the total number of packets received by the destination.

B. Multilevel Diversity Coding

We here explore opportunistic resilience code designs that exhibit a graceful performance degradation. Particularly, we discuss multilevel diversity coding (MDC), which is a classical coding scheme where i.i.d. source sequences are encoded such that different reliability requirements can be guaranteed to different source sequences. Multilevel codes can be designed in two different ways: symmetric and asymmetric multilevel codes. In this paper, we focus on symmetric multilevel codes.

In the symmetric MDC [18], [19], there are H i.i.d. source sequences that are encoded into H descriptions using H encoders. These H descriptions are sent to H decoders, each through a different channel. The source sequences have certain levels of importance, ordered from 1 (the most important) to H (the least important). Each decoder has access to a subset of the descriptions. The encoders aim to produce the descriptions such that each decoder with h available descriptions can reconstruct the h most important source sequences. For the symmetric problem, superposition coding is information-theoretic optimal [18], [19]. That is, each source sequence is compressed separately, and descriptions are created by concatenating the compressed source sequences appropriately. The next example illustrates this point and showcases how multilevel codes can be helpful in mmWave networks.

Example 2. Consider the network with $N = 6$ relays in Fig. 2. There exist $H = 6$ edge-disjoint paths connecting the source (node 0) to the destination (node 7). We let $U_i, i \in [1:6]$, be the i.i.d. source sequences which are ordered with decreasing importance. The source sequences are encoded by $H = 6$ encoders and the descriptions are denoted by $E_i, i \in [1:6]$. Each description is sent through a different path. In Fig. 3, we show the setting for a 6-level symmetric multilevel code for the network in Fig. 2. The goal of the symmetric multilevel code design in this network is to reconstruct $U_i, i \in [1:h]$, if *any* h paths succeed (or equivalently, if any $H - h$ paths fail).

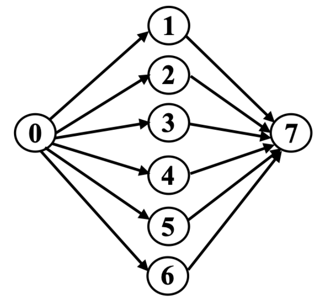


Fig. 2: An example network with $N = 6$ relays.

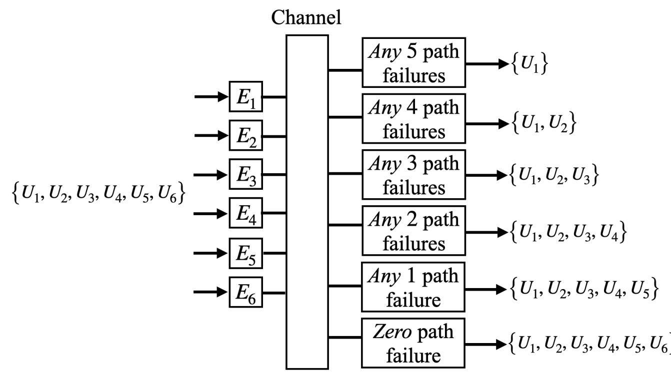


Fig. 3: 6-level symmetric multilevel code setting.

Remark 1: Multilevel codes can be deployed over space or time (or a combination of both). While deploying over space, we encode the source sequences and send the packets over *multiple* paths. While deploying over time, we encode the source sequences and send the packets over a *single* path but at different time intervals. As an example, for 6 source sequences $U_i, i \in [1:6]$, we can encode them by creating 6 encoded packets, and send them over 6 paths as in Fig. 2. While deploying over time, we can encode these source sequences by creating 6 packets in the same way as in deploying over space, and then send the packets over a single path at 6 different time intervals. In the rest of the paper, we deploy multilevel codes over space by leveraging the multipath environment of mmWave networks. If a mmWave network does not support a multipath environment or if we would like to design the code according to time correlations of blockages, our proposed design can be deployed over time or over a combination of space and time. Our results naturally extend to these scenarios.

III. Symmetric Multilevel Code Designs

In this section, we discuss how symmetric multilevel codes can be deployed over mmWave networks with arbitrary topol-

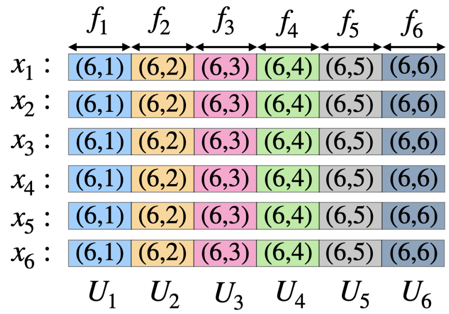


Fig. 4: Symmetric multilevel code for the network in Fig. 2.

ogy. We let H denote the number of edge-disjoint paths in the network (connecing the source to the destination) and $p_{[1:H]}$ denote the corresponding set of edge-disjoint paths. We denote the i.i.d. source sequences by $U_i,\ i \in [1:H]$, which are ordered with decreasing importance. Since superposition coding is optimal for the symmetric problem [18], [19], we encode each U_i with a different rate erasure code. In particular, we create *combined* packets $x_i, i \in [1:H]$, where each packet x_i is sent through path $p_i \in p_{[1:H]}$. Each packet x_i consists of H components, and each component is generated based on a different erasure code; we use erasure codes $(H,1),\ (H,2),\ldots,(H,H)$ to create the combined packets. Thus, if the components of a combined packet x_i are denoted by $x_{i,j}$ for $j \in [1:H]$, each component $x_{i,j}$ is generated based on an erasure code (H,j). We allocate a packet fraction to each erasure code, such that f_j denotes the fraction of a combined packet that is allocated to the erasure code (H,j) for $j \in [1:H]$. Under this design, if i number of path blockages occur for $0 \leq i < H$, the total rate achieved by a symmetric multilevel code is equal to $\sum_{j=1}^{H-i}(j/H)f_j$. This leads to the following average rate[3].

Definition 2: The *average* information rate of a symmetric multilevel code with H i.i.d. source sequences is

$$R_{\mathrm{M}} = \sum_{j\in\delta}\left(\frac{j}{H}P(X\geq j)f_j\right), \tag{3}$$

where the random variable X denotes the total number of packets received by the destination, and $\delta = [1:H]$.

Remark 2: The average information rate R_{M} in (3) is a weighted summation of average rates of erasure codes as defined in Definition 1. The weights are the fractions $f_j, j \in \delta$. Fig. 4 provides an illustration of the proposed symmetric multilevel code design for the network in Fig. 2. For instance, we encode U_1 with a $(6,1)$ erasure code which can be decoded if at least 1 path succeeds (or equivalently, at most 5 paths fail).

Our proposed multilevel code design (and its average information rate R_{M}) depends on the fraction f_j's where $j \in \delta$. In the rest of this section, we propose an algorithm to select them. Our objective is to allocate the packet fractions such that: (i) the average rate of a symmetric multilevel code is maximized; and (ii) there is a graceful performance degradation. Thus, we propose to solve the following optimization problem,

$$\begin{aligned}&\max_{f}\ \textstyle\sum_{j\in\delta}\left(\frac{j}{H}P(X\geq j)f_j\right) - \mu\|f\|^2\\ &\text{subject to C1}:\ \textstyle\sum_{j\in\delta} f_j = 1,\\ &\text{and}\qquad\quad \text{C2}:\ \ f \geq 0,\end{aligned} \tag{4}$$

where f denotes the vector of fractions $f_j, j \in \delta$, and μ is a nonnegative trade-off parameter given as input to the optimization problem. In particular, μ determines the relative importance of the ℓ_2-norm of the packet fractions versus the average rate. As the value of μ decreases, the solution of (4) focuses more on maximizing the average rate of the multilevel code rather than minimizing the ℓ_2-norm penalty. In the extreme case $\mu = 0$, the objective function in (4) reduces to (3); due to the constraint C1 in (4), the optimal solution will then select the packet fraction $f_{j^\star} = 1$ where $j^\star = \arg\max_{j\in\delta}(j/H)\,P(X\geq j)$, and all of the remaining packet fractions will be zero. This is equivalent to using a *single* erasure code $(H, j^\star)$. Even though this approach maximizes the average rate, as we discussed in Section II, it does not provide a graceful performance degradation. As the value of μ increases, the optimal solution starts to emphasize more on the penalty term, and allocates nonzero values to a higher number of packet fractions to decrease the ℓ_2-norm. When μ is sufficiently large, an optimal solution allocates an equal packet fraction to erasure codes. This leads to a more graceful performance degradation while achieving a lower average rate. Thus, there is no unique optimal selection for the fractions as there is an inherent trade-off between maximizing the average rate and achieving a graceful performance degradation. The parameter μ is tuned to have an attractive trade-off. In the rest of the paper, we refer to this heuristic as *MC-Optimized.*

Remark 3: In (4), we consider average rates of erasure codes which equally emphasize the rate and reliability (having a small outage probability). However, certain applications can focus on achieving high rates rather than providing high reliability, and vice versa. In this case, the objective function in (4) can be modified to maximize the *weighted* average rate which multiplies the rate and the probability terms by certain coefficients that can be chosen by a specific application.

So far, we selected the packet fractions for $|\delta| = H$ erasure codes. However, H can be exponential in the number of relays N, which increases the code complexity. Thus, we propose to reduce the complexity by selecting only m (e.g., $m = \lceil\log(H)\rceil$ so that the complexity is polynomial in N) erasure codes with the highest average rates, and only combine these selected codes in the symmetric multilevel code. The optimization problem in (4) allocates the packet fractions of these m erasure codes, and δ in (4) denotes the set of indices of the m erasure codes that are selected. This reduces the complexity of solving the optimization problem which depends on $|\delta| = m$. In what follows, we will refer to this heuristic as *MC-RC*. The average information rate of MC-RC, denoted by R_{RC}, is defined as in (3).

[3]We assume that each packet is transmitted during one transmission time interval denoted by t_d (e.g., $t_d = 250\ \mu$s [20]). Thus, the transmission duration of H packets is equal to t_d.

As discussed, $(H, j^\star)$ (the erasure code with the highest average rate) is selected by the solution of the problem in (4) when $\mu = 0$. As defined in (1), $R_{\mathrm{E,j}^\star}$ is the average rate of $(H, j^\star)$. The parameter μ in (4) allows a graceful performance degradation for MC-RC at the cost of achieving a lower average rate than $R_{\mathrm{E,j}^\star}$. Proposition 1 evaluates at most how much we lose in terms of average rate by deploying MC-RC.

Proposition 1: Consider an N-relay 1-2-1 network with an arbitrary topology, and let H denote the number of edge-disjoint paths in the network. Let $(H, j^\star)$ represent the erasure code that has the highest average rate $R_{\mathrm{E,j}^\star}$, and R_{RC} be the average information rate of MC-RC. Then,

$$R_{\mathrm{E,j}^\star} - R_{\mathrm{RC}} \leq \frac{1}{H} \min\{\varepsilon, j^\star\}, \tag{5}$$

where $\varepsilon = \sum_{i=1}^{H} \varepsilon_i$ with ε_i being the probability that the ith packet is received by the destination, i.e., the probability that the ith path is not blocked.

Proof: The proof follows from: (1) $R_{\mathrm{E,j}^\star}$ in Definition 1; (2) the fact that $R_{\mathrm{RC}} \geq 0$; (3) the Markov's inequality $P(X \geq j^\star) \leq \min\left\{1, \frac{\mathbb{E}[X]}{j^\star}\right\}$; and (4) noting that X is a Poisson binomial random variable for which $\mathbb{E}[X] = \varepsilon$. ■

IV. NUMERICAL EVALUATIONS

In this section, we numerically assess the performance of our proposed coding schemes MC-Optimized and MC-RC. We consider the network in Fig. 2. Our coding scheme can be applied to networks with arbitrary topologies by selecting edge-disjoint paths among all paths, thus the network in Fig. 2 can be considered as a snapshot of a larger network with an arbitrary topology and 6 edge-disjoint paths. We assume that all paths have the same blockage probability[4] equal to $1/3$. In Fig. 5a, we show the probability of any k paths to fail for $k \in [0\!:\!6]$, e.g., the probability of any 2 paths to fail is 0.33.

We compare the performance of our proposed coding schemes with the following baseline methods[5].

1) Erasure Code (EC). This method uses a single erasure code, namely the one with the highest average rate. As discussed in Section III, this is the solution of the optimization problem in (4) with $\mu = 0$.

2) Erasure Code-Reduced Outage (EC-RO). The baseline method EC described above aims to maximize the average rate by using a single erasure code; however, this can lead to a high outage probability. The method EC-RO uses a single erasure code as well, but it selects the code such that the outage probability in (2) is smaller than a given threshold γ.

3) Uniform allocation (MC-Uniform). This method uses a symmetric multilevel code while allocating an equal packet fraction to the erasure codes, i.e., $f_j = 1/H$ for $j \in [1\!:\!H]$. As discussed in Section III, a uniform allocation is the solution of the optimization problem in (4) for a sufficiently large μ. Thus, this method focuses more on ensuring a graceful performance degradation rather than on maximizing the average rate.

4) Proportional allocation (MC-Proportional). This method uses a symmetric multilevel code by assigning packet fractions proportional to the average rates of the erasure codes. That is, it allocates the packet fraction f_j for an erasure code (H, j) proportional to $(j/H)\, P(X \geq j)$ for $j \in [1 : H]$ while satisfying the constraints of the optimization problem in (4),

$$f_j = \frac{(j/H)\, P(X \geq j)}{\sum_{i=1}^{H} (i/H)\, P(X \geq i)}. \tag{6}$$

Compared to MC-Uniform, this method assigns higher packet fractions to codes that have higher average rates. It is different from our proposed schemes in Section III because it assigns fixed packet fractions as in (6) instead of optimizing over them. However, allocating the packet fractions as in (6) is still a feasible solution for our proposed optimization problem in (4).

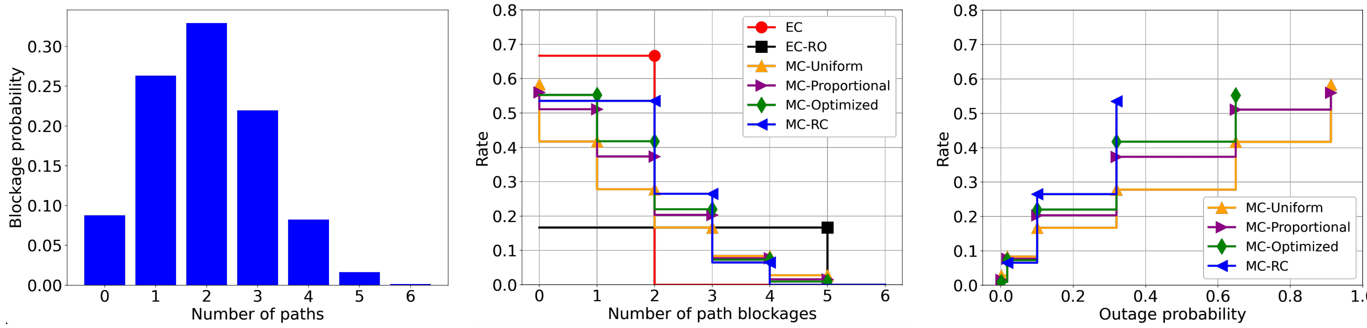
In Fig. 5b, as the number of path blockages increases from 0 to 6, we show the information rate achieved by each coding scheme. For each method, the markers in Fig. 5b are placed at the corner points of erasure codes that are combined by that method[6]. As expected, the information rate achieved by each method decreases as the number of path blockages increases. The method EC uses a single erasure code with the highest average rate, an erasure code $(6, 4)$ in this example. It can achieve rate $4/6$ whenever the number of path blockages is less than or equal to 2. However, its outage probability $P_{\mathrm{out}} = 0.32$ is considerably large. For the method EC-RO, we set $\gamma = 0.01$. Thus, EC-RO selects a single erasure code for which $P_{\mathrm{out}} < 0.01$. In this example, an erasure code $(6, 1)$ is selected with $P_{\mathrm{out}} = 0.001$, and it achieves rate $1/6$ whenever the number of path blockages is less than or equal to 5. From Fig. 5b, we note that the erasure code methods EC and EC-RO do not exhibit a graceful performance degradation.

Different from EC and EC-RO, all multilevel codes offer a graceful performance degradation. As shown in Fig. 5b, the performance of MC-Uniform is underwhelming compared to other methods because it allocates the same packet fraction to all erasure codes, while the other methods are allocating higher fractions to erasure codes that have higher average rates. Overall, MC-Optimized has the best performance because it maximizes the average rate while allowing a graceful performance degradation. The parameter μ is tuned to balance this trade-off and selected as 0.6. MC-Optimized outperforms MC-Proportional because MC-Proportional chooses fix packet fractions as shown in (6), and it even uses erasure codes that have a low average rate. Differently, MC-Optimized optimizes the packet fractions through μ to allocate higher fractions to codes that have higher average rates, while allocating zero fraction to codes that have low average rates. Moreover, our complexity reduction method MC-RC only uses $m = 3$ erasure codes while still giving a comparable performance with respect to MC-Optimized that uses 5 erasure codes, and it outperforms the baseline methods that use 6 erasure codes. We note that MC-RC sometimes outperforms MC-Optimized because μ is

[4] We refer the reader to Remark 4 that motivates these assumptions.

[5] For all discussed methods, the transmission duration of 6 packets is equal to $t_d = 250$ μs [20].

[6] The corner point of an erasure code indicates the maximum number of path blockages for which the code can provide a nonzero information rate.



(a) (b) (c)

Fig. 5: Performance of the proposed coding schemes and alternative schemes.

tuned separately for the two schemes to achieve an attractive trade-off ($\mu = 0.6$ for MC-Optimized and $\mu = 0.3$ for MC-RC). Although MC-RC achieves zero information rate when more than 4 paths fail, the probability of this event is significantly small as shown in Fig. 5a.

We also present information rate-outage probability curves for the multilevel codes[7] in Fig. 5c. For example, the outage probability corresponding to rate 0.58 of MC-Uniform is 0.91; that is, the probability that MC-Uniform does not achieve rate 0.58 is 0.91. From Fig. 5c, we observe similar behaviors to those in Fig. 5b. For instance, as shown in Fig. 5c, our proposed schemes can achieve similar rates to those achieved by the baseline methods but with smaller outage probabilities, e.g., MC-RC achieves rate 0.53 with outage probability 0.32.

Remark 4: We ran extensive numerical evaluations also for a larger number of edge-disjoint paths (e.g., 12 and 20), and when the link blockage probabilities are not necessarily the same (e.g., drawn from a Gaussian distribution). We observed similar results to those observed for 6 edge-disjoint paths with equal blockage probabilities (described above). Thus, we have not provided them because of space considerations.

V. Conclusions

In this paper, we provided first steps towards developing *low-complexity proactive* transmission mechanisms for mmWave networks that are resilient to blockages. We built on the multipath environment and on the accurate models for link blockage probabilities in mmWave networks. Particularly, we deployed symmetric multilevel codes to achieve an attractive trade-off between the average rate and a graceful performance degradation. Our evaluations show that our coding schemes provide a graceful performance degradation compared to alternative schemes while significantly reducing the code complexity. Thus, our work shows that multilevel codes are promising and worth further exploration in mmWave networks.

References

[1] G. Chittimoju and U. D. Yalavarthi, "A comprehensive review on millimeter waves applications and antennas," *Journal of Physics: Conference Series*, vol. 1804, no. 1, p. 012205, 2021.

[2] X. Wang, L. Kong, F. Kong, F. Qiu, M. Xia, S. Arnon, and G. Chen, "Millimeter wave communication: A comprehensive survey," *IEEE Commun. Surv. Tutor.*, vol. 20, no. 3, pp. 1616–1653, 2018.

[3] A. N. Uwaechia and N. M. Mahyuddin, "A comprehensive survey on millimeter wave communications for fifth-generation wireless networks: Feasibility and challenges," *IEEE Access*, vol. 8, pp. 62 367–62 414, 2020.

[4] I. K. Jain, R. Kumar, and S. Panwar, "Driven by capacity or blockage? A millimeter wave blockage analysis," in *2018 30th International Teletraffic Congress*, vol. 01, 2018, pp. 153–159.

[5] A. Samuylov, M. Gapeyenko, D. Moltchanov, M. Gerasimenko, S. Singh, N. Himayat, S. Andreev, and Y. Koucheryavy, "Characterizing spatial correlation of blockage statistics in urban mmWave systems," in *IEEE Globecom Workshops*, 2016, pp. 1–7.

[6] Y. Wang, K. Venugopal, A. F. Molisch, and R. W. Heath, "Blockage and coverage analysis with mmWave cross street bss near urban intersections," in *IEEE Int. Conf. Commun.*, 2017, pp. 1–6.

[7] V. Raghavan, L. Akhoondzadeh-Asl, V. Podshivalov, J. Hulten, M. A. Tassoudji, O. H. Koymen, A. Sampath, and J. Li, "Statistical blockage modeling and robustness of beamforming in millimeter-wave systems," *IEEE Trans. Microw. Theory Tech.*, vol. 67, no. 7, pp. 3010–3024, 2019.

[8] Y. H. Ezzeldin, M. Cardone, C. Fragouli, and G. Caire, "Gaussian 1-2-1 networks: Capacity results for mmWave communications," *IEEE Trans. Inf. Theory*, vol. 67, no. 2, pp. 961–990, 2021.

[9] M. G. Dogan, M. Cardone, and C. Fragouli, "Proactive resilience in 1-2-1 networks," in *2022 IEEE International Symposium on Information Theory (ISIT)*, 2022, pp. 3220–3225.

[10] I. S. Reed and G. Solomon, "Polynomial codes over certain finite fields," *J. Soc. Ind. Appl. Math.*, vol. 8, no. 2, pp. 300–304, 1960.

[11] L. Rizzo, "Effective erasure codes for reliable computer communication protocols," *SIGCOMM Comput. Commun. Rev.*, vol. 27, no. 2, p. 24–36, 1997.

[12] W. C. Huffman and V. Pless, *Fundamentals of Error-Correcting Codes*, 1st ed. Cambridge University Press, 2003.

[13] A. G. Dimakis, P. B. Godfrey, Y. Wu, M. J. Wainwright, and K. Ramchandran, "Network coding for distributed storage systems," *IEEE Transactions on Information Theory*, vol. 56, no. 9, pp. 4539–4551, 2010.

[14] A. G. Dimakis, K. Ramchandran, Y. Wu, and C. Suh, "A survey on network codes for distributed storage," *Proceedings of the IEEE*, vol. 99, no. 3, pp. 476–489, 2011.

[15] J. van Lint, *Introduction to Coding Theory*, 3rd ed., ser. Graduate Texts in Mathematics. Springer Berlin Heidelberg, 2012.

[16] R. Ziemer and W. Tranter, *Principles of Communications: Systems, Modulation, and Noise*, 7th ed. Wiley, 2014.

[17] B. S. Babu, M. N. Krishnan, M. Vajha, V. Ramkumar, B. Sasidharan, and P. V. Kumar, "Erasure coding for distributed storage: an overview," *Sci. China Inf. Sci.*, vol. 61, no. 10, pp. 100 301:1–100 301:45, 2018.

[18] J. Roche, R. Yeung, and K. P. Hau, "Symmetrical multilevel diversity coding," *IEEE Trans. Inf. Theory*, vol. 43, no. 3, pp. 1059–1064, 1997.

[19] R. Yeung and Z. Zhang, "On symmetrical multilevel diversity coding," *IEEE Trans. Inf. Theory*, vol. 45, no. 2, pp. 609–621, 1999.

[20] F. Rinaldi, A. Raschellà, and S. Pizzi, "5g nr system design: A concise survey of key features and capabilities," *Wireless Networks*, vol. 27, no. 8, p. 5173–5188, 2021.

[7] In Fig. 5c, we did not provide the curves for EC and EC-RO since they (as also highlighted in Fig. 5b) do not offer a graceful performance degradation.