Sources of uncertainty in drag modeling and its effect on predicted orbit state covariance

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Abstract

Realism of the predicted or forecasted orbital state covariance are crucial for several aspects of space traffic management and space domain awareness including sensor tasking, probability of collision estimation, track association, and maneuver detection. In this paper, we focus on reliably modeling the uncertainties in atmospheric drag parameters for forecast purposes and investigating the relationship between the dynamical uncertainties and orbital state uncertainty. Two sources of space weather uncertainty - thermosphere driver uncertainty and thermosphere model uncertainty - are considered in this work. First, we present a new probabilistic modeling approach for the solar driver inputs of thermosphere density models and evaluate the performance against operational forecast models. Next, we evaluate methods for propagating the driver uncertainty through the stochastic HASDM-ML model for thermosphere mass density. Finally, we perform orbital case studies to highlight the relative contribution of the two sources of uncertainty on predicted orbital state covariance.

1 Introduction

Forecasting of atmospheric or orbital drag is the largest source of uncertainty and a challenge for the owner/operator (O/O) communities critically affecting probability of collision (PoC) calculation and decision making. The acceleration due to the atmospheric drag force, a_{drag},

$$a_{drag} = -\frac{1}{2}\frac{\rho\, C_D\, A}{m} v_{rel}^2 \quad with \quad B = \frac{C_D A}{m} \tag{1}$$

is coupled to variations in neutral density, caused by the direct heating of the thermosphere through solar EUV absorption and indirect heating through high latitude heating caused by geomagnetic storm. Other terms in the drag model are the drag coefficient, C_D, cross sectional area, *A*, object's mass, *m*, the velocity relative to the atmosphere, v_{rel}. The typically used ballistic coefficient, *B*, allows for satellite specific parameters to be grouped into a single term. Here, we focus on the uncertainties in ρ caused primarily by inaccurate forecasts of the density model drivers (solar and geomagnetic), henceforth referred to as driver uncertainty, and the inaccurate parameterization of the energy deposition and physical processes driving the density changes, henceforth referred to as model uncertainty.

We propose to establish a new framework (seen in Figure 1) that couples the sources of uncertainty (driver and model) with the predicted orbital state. Current operations do not robustly and rigorously account for these uncertainty sources and therefore result in unrealistic covariances. We present progress towards development of this framework.

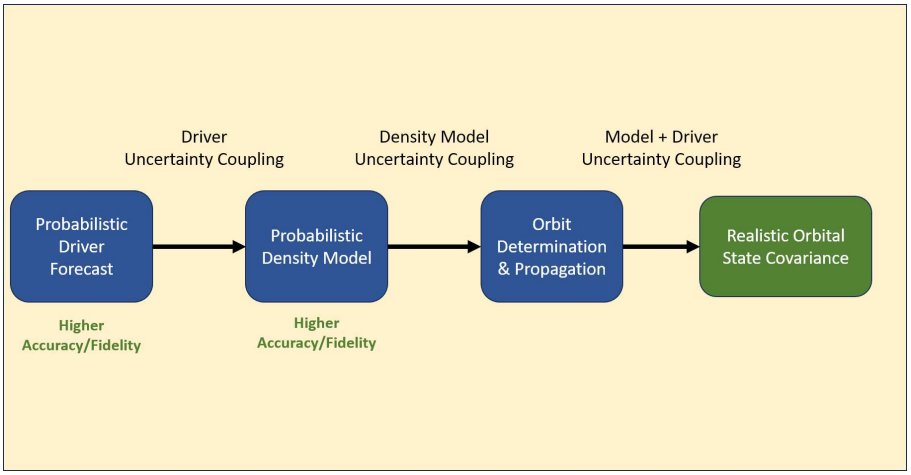

Figure 1. The proposed framework links both sources of uncertainty together to provide improved overall uncertainty estimates for use in orbital state determination.

2 Probabilistic thermospheric density modeling

We started developing the framework with significant efforts to advance the field and develop first-of-its-kind probabilistic density models. Most existing models of thermosphere density can be classified as either empirical or physics-based. Significant efforts over the last two decades have reduced the mean global error of empirical models (e.g. JB2008 and HASDM) to sub-10% level during solar active conditions, however the errors during geomagnetically active or storm conditions can be upwards of 25%. Physics-based models can model the storm conditions with higher fidelity and potentially more accuracy but are computationally expensive and can be biased. None of the existing models are truly probabilistic (cannot provide a probabilistic output given a deterministic input).

To date, we have developed four probabilistic density models: HASDM-ML, CHAMP-ML, MSIS-UQ, and TIE-GCM ROPE. The High Accuracy Satellite Drag Model - Machine Learned (HASDM-ML) was trained on 20 years of HASDM validation outputs [1, 2, 3]. The full three-dimensional density grids were reduced to 10 spatiotemporal coefficients through principal component analysis (PCA). The original implementation of HASDM-ML used Monte-Carlo dropout to achieve stochasticity. The model was trained with the negative logarithm of predictive density (NLPD) loss function to achieve meaningful predicted distributions of density. This was later improved with a direct prediction of the probability distribution (DPPD), improving the computational efficiency of the model [4]. An example of the probabilistic density output provided by HASDM-ML in response to a geomagnetic storm is seen in Figure 2.

CHAllenging Mini-satellite Payload - Machine Learned (CHAMP-ML) used in-situ density estimates from the CHAMP satellite to predict local density distributions [4]. This approach provided millions of samples for training and evaluation, but it had added complexity since location was an added input. CHAMP-ML also utilized NLPD for training and a DPPD approach, and it has a high spatiotemporal resolution because of the data it was trained on.

Mass Spectrometer and Incoherent Scatter radar with Uncertainty Quantification (MSIS-UQ) is different from the previous two models, because it is an exospheric temperature model [6]. It was trained on exospheric temperature estimates from a multi-satellite dataset [7] that – when combined with MSIS 2.0 [8] – predicts density with high accuracy and resolution. MSIS-UQ predicts local exospheric temperature distributions (through a combination of NLPD in training and DPPD), then samples are fed into MSIS 2.0 to get a corresponding density distribution.

Thermosphere-Ionosphere Electrodynamics General Circulation Model Reduced Order Probabilistic Em-

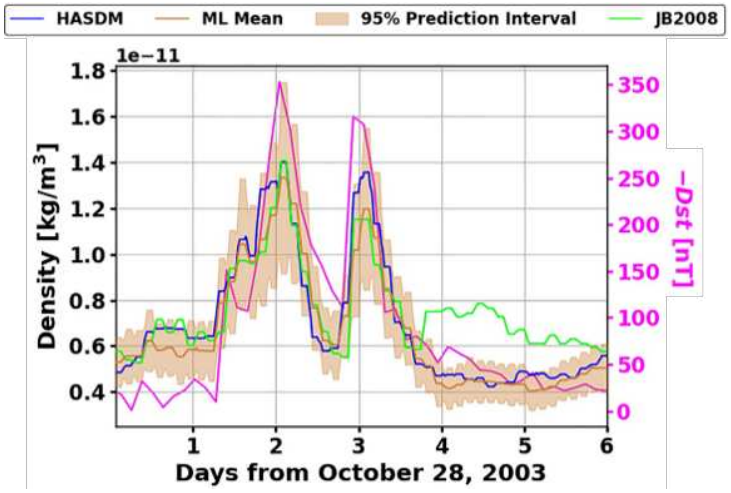

Figure 2. HASDM, HASDM-ML mean, and JB2008 orbit-averaged density for CHAMP's orbit for the 2003 Halloween geomagnetic storm. The shaded region represents the 95% prediction interval for HASDM-ML and storm-time disturbance index is shown on the right axis [5].

ulator (TIE-GCM ROPE) operates on a reduced representation of TIE-GCM mass density, similar to HASDM-ML [9]. Since TIE-GCM is a dynamic physics-based model, we opted to use Long Short-Term Memory neural networks (LSTMs) to make density forecasts. The dynamic nature of these models caused previous UQ approaches to become ineffective for the task at hand. We instead chose to use an ensemble approach to get density distributions from the individual global density models.

3 Probabilistic solar driver modeling

The JB2008 model is used to provide a density nowcast to HASDM-ML, which is corrected by a set of calibration satellites. The JB2008 empirical thermosphere density model relies on in part, a set of solar and geomagnetic drivers $F_{10.7}$, $S_{10.7}$, $M_{10.7}$, and $Y_{10.7}$, which map energy from solar irradiance to major thermosphere layers and are highly correlated with thermosphere heating [10]. All solar drivers are scaled to solar flux units (SFU), where 1 SFU = $10^{-22}\frac{W}{Hzm^2}$.

The current operational method for forecasting solar drivers uses a linear auto-regressive algorithm, known as "TS_FCAST" in the interactive data language (IDL). These forecasts are short-term (6-days) and are used by HASDM and HASDM-ML to provide deterministic and probabilistic density forecasts, respectively [11]. Forecasting of $F_{10.7}$, has been investigated thoroughly with both deterministic and probabilistic approaches. Work by [12] showed promising results using machine learning (ML) approaches, specifically neural network ensembles to provide short term probabilistic forecasts. Long short term memory (LSTM) models with a variety of architectures, lookbacks, and weight initializations provide a diverse set of solar driver predictive models, which are combined to produce a probabilistic forecast. The ML ensemble methods outperformed the current operational methods and provide uncertainty estimates, seen in Figure 3.

The other three solar drivers have only been measured since 1997 [10] and have not yet had probabilistic methods applied. A similar neural network ensemble method can enhance forecasts for all solar drivers required by JB2008, including $S_{10.7}$, $M_{10.7}$, and $Y_{10.7}$. This ensemble approach involves applying ensemble methods to provide multivariate forecasts of the other solar drivers, resulting in improved performance, and provides well calibrated uncertainty estimates for all drivers. Using such ML methods to forecast solar drivers, probabilistic inputs (and their uncertainty estimates) can be coupled with probabilistic thermosphere density models to provide more robust and reliable uncertainty estimates

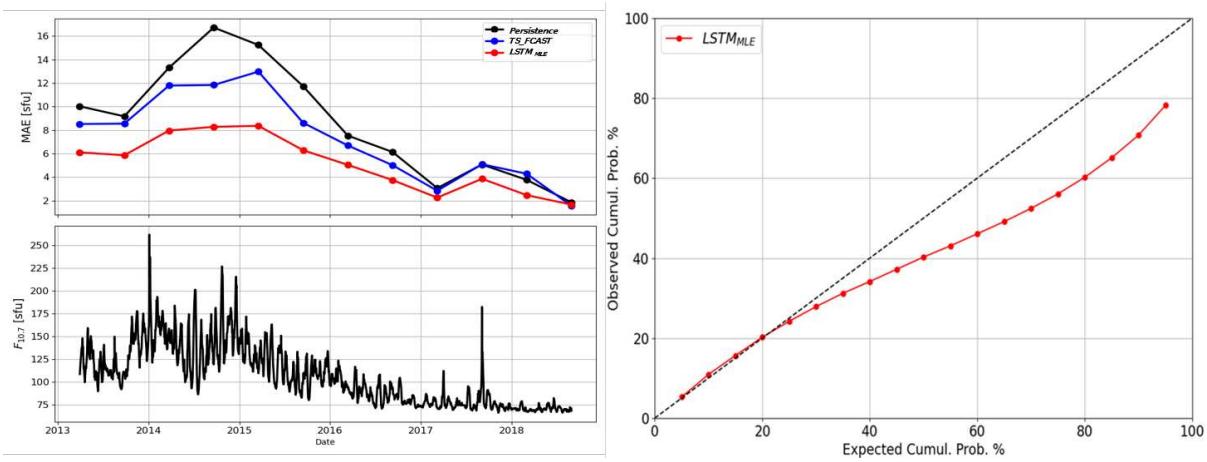

Figure 3. **Left:** Probabilistic $F_{10.7}$ LSTM ensemble model by [12] outperforms both the operational method and a persistence baseline, especially at high solar activity levels. **Right:** A test set calibration curve for probabilistic forecast indicates a well calibrated probabilistic model, with the dotted line at 45^o indicating perfect model calibration.

for density used in orbit propagation and orbital state uncertainty.

4 Coupling of density model uncertainty with orbit propagation

The next step for the framework requires probabilistic density models to be paired with orbital propagation techniques. The work by [13] investigated such evolution of orbit error distribution in the presence of atmospheric density uncertainties, which are modeled using probabilistic machine learning techniques. This work only considered the uncertainty associated with density models and did not consider the driver uncertainties. The authors developed several modified Monte Carlo (MC) methods to perform orbit uncertainty propagation and developed other methods which were computationally more efficient than traditional MC. The authors also introduced an ensemble approach which combines the epistemic uncertainties predicted by HASDM-ML, CHAMP-ML, and MSIS-UQ, to characterize the uncertainty in orbital states of a space object.

It was determined by [13], that the high sampling frequency in traditional MC, led to a partial "cancellation" in drag perturbations and led to unrealistically small errors. To combat this, the authors introduced a modified MC scheme, which relies on the sampling of so-called "bias" factor κ. The bias factor is desired to have a Gaussian distribution. If κ_i is a sampled value of the bias factor at any point during the Monte Carlo run, then the corresponding density sample is,

$$\rho_i = \mu_{\rho_i} + \kappa_i \sigma_{\rho_i} \tag{2}$$

where the mean density μ_{ρ_i} and the standard deviation σ_{ρ_i} are obtained from one of the probabilistic density models.

To determine the effectiveness of the modified MC approach, we simulate a 3-day orbit propagation (details of initial conditions are described in [13]) to examine along-track position error between the mean orbit and the MC runs. Figure 4 shows the comparison between a modified MC scheme and traditional MC. Figures 4(a) and 4(c) show density values for the first 3 hours of the propagation. The black curve is the mean density and the colored curves indicate five separate MC runs. Figures 4(b) and 4(d) show the normal PDF for the along-track error at the end of propagation. We see that the modified MC

approach yields more realistic density values as opposed to the near discontinuous changes seen from traditional MC. Due to the high computational expense of MC, coupled with the need for propagation thousands of catalog objects, [13] also investigated a consider covariance sigma point (CCSP) filter approach, which is based of the work by [14]. The authors found that CCSP could be used in place of MC, providing similar error propagation results, even at solar maximum/storm conditions; with the benefit that CCSP is much quicker than MC.

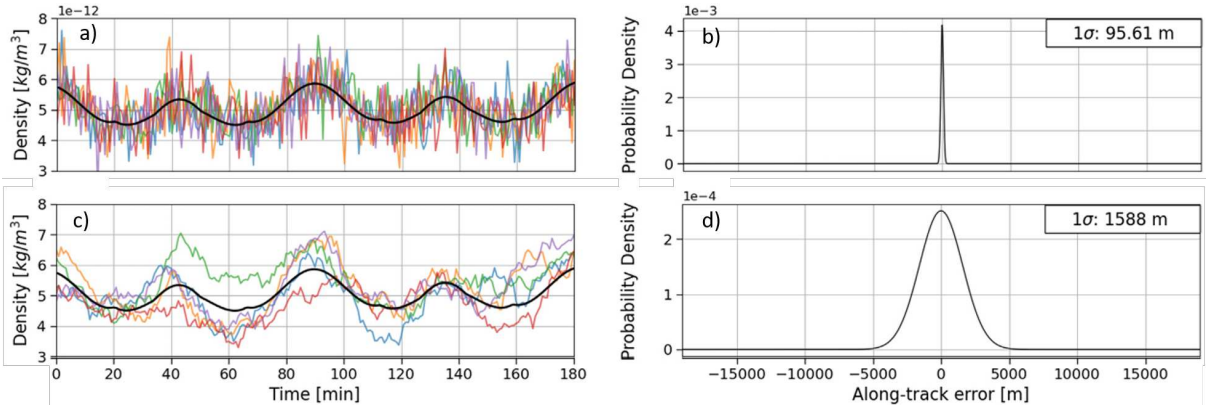

Figure 4. Comparison of traditional and chosen modified Monte Carlo technique. Traditional MC approach is shown in (a),(b) and modified MC is shown in (c),(d) [13].

There is no evidence that any single atmospheric density model is always more accurate than others under all space weather conditions [15]. Drawing conclusions about orbital state uncertainty from a single model is not recommended, so [13] considered the use of a multi-model ensemble approach to combine orbital state PDFs of HASDM-ML, CHAMP-ML, and MSIS-UQ. The ensemble approach can combine probabilistic density forecasts using any number of models, and produce a combined orbital error PDF,

$$p(\mathcal{X}; \mu_1, P_1, ..., \mu_i, P_i) = \frac{1}{N} \sum_{i=1}^{N} \mathcal{N}(\mathcal{X}; \mu_i, P_i) \tag{3}$$

where $\mathcal{N}(\mu_i, P_i)$ represents the multivariate normal distribution with mean μ_i and covariance P_i, which are associated with N individual probabilistic density models.

We investigate the evolution of orbital state uncertainty using the probabilistic ensemble approach (Eq. 3) for a geomagnetic storm during solar maximum (September 07, 2002). A 3-day orbit uncertainty propagation is performed (details regarding simulation configuration can be seen in [13]). It is seen in Figure 5 that models have varied fidelity and provide different forecasts for the storm case, and that a combination of several model types using the CCSP approach can provide an overall higher fidelity probabilistic forecast. It is clear that the individual models produce fairly different normal distributions, but can be combined to produce a more complex distribution, which cannot be produced by a single probabilistic density model.

We see that machine learning-based probabilistic density models can be used to investigate the effect of atmospheric density uncertainty on the evolution of orbit state uncertainty. The ensemble approach can be used for predicting orbit state PDF that combines the uncertainty predicted by various probabilistic density models. With a critical source of overall uncertainty linked to orbit propagation and orbit state uncertainty, we must now discuss a how uncertainty in drivers of density model can be evaluated and linked to uncertainty in probabilistic density models; which contribute to the overall uncertainty in the framework.

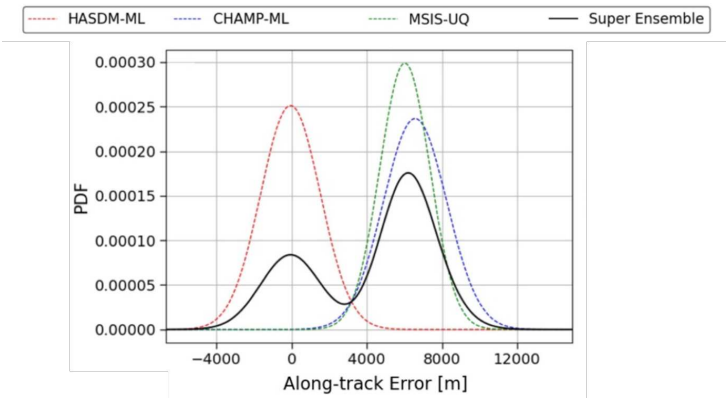

Figure 5. Orbit state PDF for the along-track direction using the ensemble approach. The black curve represents the combined result, known as "super ensemble". The red curve corresponds to HASDM-ML, the blue curve corresponds to CHAMP-ML, and the green curve corresponds to MSIS-UQ [13].

5 Coupling driver uncertainty with density modeling

In the previous section, we showed the effectiveness of coupling model uncertainty to orbital state uncertainty. However, probabilistic models like HASDM-ML do not adequately consider driver uncertainty. In this work, we seek to demonstrate the ability of a coupling method that will consider the effects of driver uncertainty on probabilistic density models. We show the technique by incorporating probabilistic forecasting of $F_{10.7}$ with HASDM-ML.

Building off of the previous work by [13] and due to the limitations of traditional MC, an alternate approach is considered. The Unscented Kalman Filter (UKF) [16] is useful for non-linear systems, as it does not make any linear approximations and is computationally less expensive than MC. The UKF approach uses the Unscented Transform (UT), which accurately captures the mean and covariance from a set of data points that behaves as a Gaussian distribution by selecting carefully sample points known as sigma points. This method helps in capturing the most relevant information needed with only a small number of sample points and can be used to realistically link driver uncertainty to probabilistic density models.

This coupling method uses the framework from [16] by selecting the sigma points and a set of weights from the mean and variance of $F_{10.7}$. Then, sigma points are propagated through HASDM-ML to obtain a density distribution, and a new mean and variance are calculated. A comparison between the MC approach and the UT approach is necessary. To observe the advantage of UT, we sampled 10000 cases for MC to obtain the mean and variance. From the UT approach, a set of 3 sigma points were used. We can see in Figure 6, that the effects of uncertainty in $F_{10.7}$ are captured effectively when using UT, and provide a similar distribution of the atmospheric density when compared to the MC case.

6 Model and driver uncertainty impacts on predicted orbital state covariance

We previously studied the impact of driver and model uncertainties on the future position of satellites in LEO. We considered a single satellite over a three-day window for four different space weather conditions: low, moderate, elevated, and high solar activity. For each condition, we used driver forecasts provided by Space Environment Technologies (SET). This source of uncertainty analysis does not consider probabilistic models or coupling presented in this work, but instead illustrates the need for improved probabilistic modeling considering both sources of uncertainty. Using historical error statistics and Monte Carlo approaches, we obtained probabilistic samples of solar drivers, keeping geomagnetic drivers deterministic. For driver uncertainty cases (DUQ), probabilistic solar drivers were passed to

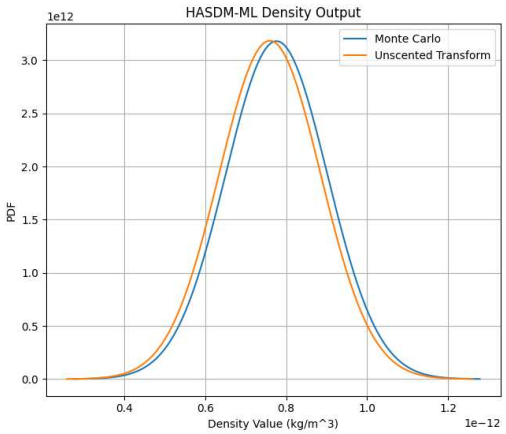

Figure 6. Comparison of the Monte Carlo approach and the Unscented Transform Technique from HASDM-ML Output. The blue curve displays the distribution of Monte Carlo, and the orange curve represents the distribution from the Unscented Transform.

HASDM-ML, CHAMP-ML, MSIS-UQ, and TIE-GCM ROPE, and the mean model densities were used. For model uncertainty cases (MUQ), all deterministic drivers were passed to the models, but the models' density distributions were sampled via MC to yield probabilistic density. Figure 7 shows the along-track standard deviation of the satellite position over the course of the three-day periods.

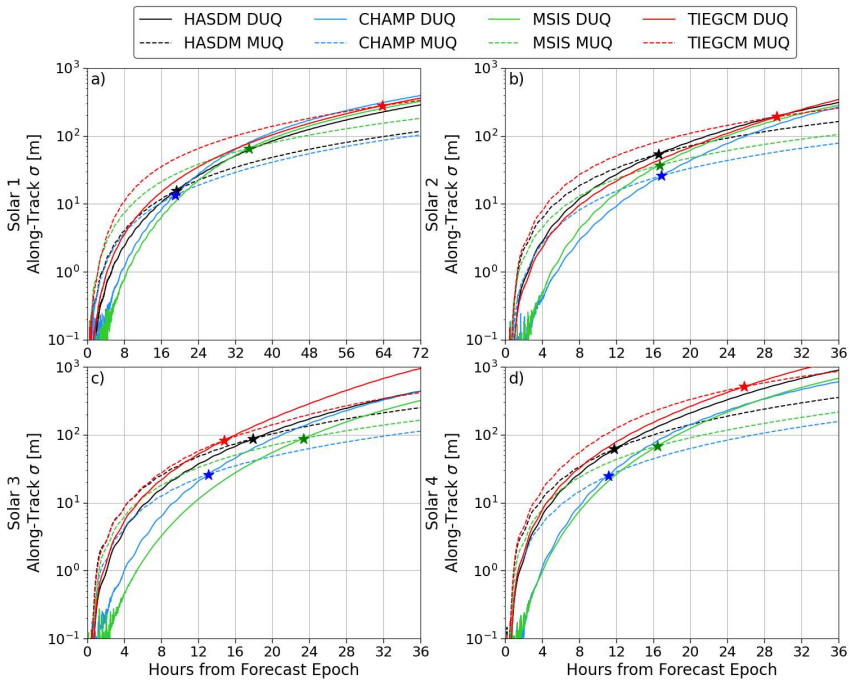

Figure 7. In-track position standard deviation as a function for time for the four solar activity conditions. The markers refer to the time where the dominant uncertainty takes over for a particular model [17].

In Figure 7, it is clear that over three days, driver uncertainty is the dominant source of uncertainty. Granted, this is impacted by the method for obtaining the uncertainty in the drivers themselves. In the first 8–30 hours, however, model uncertainty is dominant, and it is on the same order as driver uncertainty even after 36 hours. When using a deterministic density model, driver uncertainty can be considered but not model uncertainty, leaving out a crucial component and causes an underestimation in the overall uncertainty of a satellite's state.

7 Integration of driver and model uncertainties for orbit propagation

Considering the uncertainties in orbit state propagation, density modeling, and solar driver forecasting has led to significant improvements in understanding each aspect individually. Now, we aim to integrate these various methods into a cohesive approach. The coupling of density modeling with orbit propagation has already been established in previous work. With the introduction of the UT approach, we can further integrate probabilistic driver modeling with probabilistic density modeling and orbit propagation. This implementation will enable us to account for driver and model uncertainties and combine them with probabilistic density modeling when implementing orbit propagation to provide better predicted orbital state covariance.

8 Conclusions and future work

In this work, we presented a framework which is capable of coupling the two major sources of uncertainty in thermosphere density modeling, driver and modeling uncertainties. Typically, these two sources of uncertainty have been overly simplified or ignored. The framework presented allows for a consideration of both sources of uncertainty simultaneously for use in orbit propagation and orbital state uncertainty estimation, which could be provided to space traffic management operators.

We showed results and discussed each piece of the proposed framework. Four probabilistic density models, HASDM-ML, CHAMP-ML, MSIS-UQ, and TIE-GCM, have been developed to provide uncertainty estimates for modeled mass density. The inherent uncertainties in deterministic density models were evaluated. When using deterministic density models, driver uncertainty can be considered, but model uncertainty cannot. It is critical to consider both simultaneously, so as a first step, probabilistic density models were introduced.

Work was also done to develop probabilistic forecasting models for the inputs to HASDM-ML. The neural network model ensemble approach was created to provide short-term forecast for each solar driver and provide associated uncertainty estimates. The method provided first-time probabilistic forecasts of $S_{10.7}$, $M_{10.7}$, and $Y_{10.7}$. These solar drivers are sampled using MC or UT and provide inputs to HASDM-ML, coupling the driver uncertainty with a probabilistic thermosphere density model. Then, when probabilistic density models were considered, work was done to sample the probabilistic density using MC and CCSP approaches, which provided a capability for coupling probabilistic density forecasts with orbital state uncertainty estimation. This work resulted in the coupling of model uncertainty with orbital state uncertainty.

The effectiveness of the proposed framework has not yet been fully investigated, it is key to evaluate the full end to end density modeling capability. It is also critical to determine the affects of both sources of uncertainty on orbital state errors. A primary focus moving forward is to investigate the coupling of driver uncertainty and model uncertainty with estimated orbital state. We believe it is necessary to provide a realistic framework, so its effectiveness, robustness, and reliability needs to be further investigated.

9 Acknowledgements

This work was made possible by NASA Established Program to Stimulate Competitive Research Grant #80NSSC19M0054, the NASA SBIR contract #80NSSC20C0292, the NASA West Virginia Space Grant Consortium Training Grant #NNX15AI01H, NASA grant #80NSSC20K1362 to Virginia Tech under the Space

Weather Operations 2 Research Program with subcontracts to WVU and SET. We also acknowledge support under NSF ANSWERS award #2149747 (sub award to West Virginia University from Rutgers University), and NSF CAREER award #2140204.

This research is based upon work supported in part by the Office of the Director of National Intelligence (ODNI), Intelligence Advanced Research Projects Activity (IARPA), via 2023-23060200005. The views and conclusions contained herein are those of the authors and should not be interpreted as necessarily representing the official policies, either expressed or implied, of ODNI, IARPA, or the U.S. Government. The U.S. Government is authorized to reproduce and distribute reprints for governmental purposes notwithstanding any copyright annotation therein.

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