UNPACKING EQUITY-ORIENTED TEACHING DILEMMAS IN ELEMENTARY MATHEMATICAL MODELING LESSONS

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Our paper details the ways teachers understand and navigate equity-oriented dilemmas (Berlak & Berlak, 1981) when teaching mathematical modeling and how mathematics teacher educators can support teachers' learning of culturally responsive mathematics teaching. Using Zavala and Aguirre's (in press) framework for culturally responsive mathematics teaching, we explored the ways teachers describe and frame their choices when faced with dilemmas. Findings revealed that teachers identified dilemmas with Rigor and Support most often, followed by Knowledges and Identities. Dilemmas related to Power and Participation occurred far less frequently. Implications for teacher professional development are discussed.

Keywords: Equity, Inclusion and Diversity; Modeling, Culturally Relevant Pedagogy; Professional Development

Teaching mathematical modeling involves supporting students to pose problems; make assumptions and define variables; and create, validate, and share usable models (COMAP & SIAM, 2016). Mathematical modeling has the potential to advance equity because it gives students opportunities to draw on their own knowledge bases as they answer meaningful questions (Aguirre et al., 2019; Anhalt et al, 2018; Carlson et al., 2016; Cirillo et al, 2016; English & Watters, 2004; Suh et al., 2018; Turner et al., 2022), immerses students in rigorous content (Fulton, 2018), broadens participation, and disrupts traditional classroom power structures (Anhalt, 2014; Featherstone et al, 2012; Lesh & Doerr, 2003). Teaching modeling is pedagogically demanding (Carlson, 2021), as teachers must anticipate, elicit, and interpret students' ideas about problem contexts and mathematical approaches, and then respond in-the-moment (Jacobs, Lamb, & Philipp, 2010). Research has demonstrated that teaching mathematical modeling and developing equity focused pedagogies is challenging, but learnable (Anhalt et al, 2018). Meeting these challenges requires support and more research is needed to understand how teachers navigate the demands of equity-focused mathematical modeling.

Theoretical Perspectives

We focus on dilemmas that arise when culturally responsive pedagogies are foregrounded in teaching mathematical modeling. The language of *dilemmas* is useful in understanding the complexities of instructional decision-making (e.g. Berlak & Berlak, 1981; Lampert, 1985). Berlak and Berlak (1981) use dilemmas to describe "both the forces which shape teachers' actions…and the capacity of teachers not only to select from alternatives, both to act to create alternatives" (p. 124). Instead of focusing on a "right" or "best" course of action, we focus on understanding dilemmas from teachers' perspectives and on supporting teachers to develop and select from a range of alternatives.

Teaching mathematical modeling is challenging because modeling problems are more open and less predictable than tasks in most lessons (Cai et al., 2014). They require teachers to know about the contexts that motivate modeling problems, potential mathematical solutions, and ways to maintain rigor and support for students as they develop, refine, and communicate their models (Carlson, 2021). Historically, mathematical modeling has been enacted at the secondary and undergraduate level but a growing number of researchers have found that elementary students can successfully engage in mathematical modeling (Albarrucín, 2021; Turner, et al., 2021; English 2012; English & Watters, 2006). We posit that mathematical modeling is also a lever for equity. Modeling empowers teachers to elicit and build on the knowledge and cultural resources that students bring to the classroom and empowers students to draw on their identities and experiences to inform mathematical work and take action (Aguirre, et al., 2019; Turner and Bustillos, 2017). In addition, modeling elicits diverse student contributions and gives teachers opportunities to assign competence and "recognize and reward a broader range of mathematical abilities than those traditionally emphasized" (Lesh & Doerr, 2003, p. 23).

Our perspective on equity-focused pedagogies is informed by Zavala and Aguirre's (in press) three-strand framework for culturally responsive mathematics teaching. The first strand, *knowledge and identities,* focuses on how teachers draw on students' funds of knowledge, build on student thinking, and support positive mathematics identities. The second strand, *rigor and support,* attends to how teachers maintain high cognitive demand, support students by scaffolding, and affirm multilingualism. The third strand, *power and participation*, focuses on distributing intellectual authority among students, disrupting status differences and stereotypes, and supporting students to take action. Explicitly and intentionally foregrounding culturally responsive pedagogies turns the potential mathematical modeling holds as a lever for equity into a reality for students. However, there is a gap in the extant literature related to how teachers learn to advance equity through mathematical modeling. The purpose of our study was to explore how teachers understand and navigate equity-oriented dilemmas when teaching modeling lessons.

Methods

Our study focused on the following research questions: (1) What dilemmas do teachers encounter as they teach culturally responsive mathematical modeling? (2) When faced with dilemmas, how do teachers frame their choices?

Context and Participants

This study is part of a collaborative, multi-year, funded project on mathematical modeling in the elementary grades. The project involves four universities and four geographically, racially, and culturally diverse school districts. Teachers engage in seven workshops spanning across the school year with in-person sessions and asynchronous assignments between workshops. There they learn about elementary mathematical modeling and ways to engage in culturally responsive mathematics teaching practices. In between workshops, teachers also implement at least three modeling lessons in their classrooms. This study draws on data from post-lesson enactment interviews with 15 teachers new to modeling lessons at four different sites from across grades K-5. Modeling lessons focused on deciding how a classroom should share snacks fairly. Students created models to answer questions such as "How many snacks do we need?" or "Do we have enough?" or "How long will these snacks last?"

Data and Analysis

Our analysis focused on dilemmas related to the three strands of the culturally responsive mathematics teaching framework described above. Our primary data source was transcriptions of post-lesson teacher interviews. Using an inductive, open-coding process (Saldaña, 2021) we first

coded interview transcripts to identify teacher described dilemmas (i.e., moments when teachers recounted uncertainty about what to do in a lesson, or reflected on decisions made). Next, we identified dilemmas that related to one or more strands of our equity framework and generated a codebook (Table 1). A second round of analysis focused themes within each strand of equity-oriented dilemmas and on how teachers understood their choices in commonly occurring dilemmas. We wrote memos to describe patterns and themes within each strand and created heat maps to analyze the relative frequency of codes within each strand.

Table 1: Codebook for Equity-Oriented Dilemmas in Elementary Mathematical Modeling

Knowledge and Identities	
Cultural/ Community Funds of Knowledge	How to elicit or build on students' funds of knowledge, including connecting students' out-of-school experiences with ideas central to the mathematical demands of the modeling task.
Rehumanizing	Affirming mathematical identities, supporting creativity, broadening what counts as knowledge, and helping students feel confident taking intellectual risks.
Student Thinking and Ideas	Eliciting to and responding to student ideas, getting many student ideas on the table and deciding how to respond to unanticipated/unreasonable ideas.
Rigor and Support	
Cognitive Demand	How to support students to engage in the rigor of the task, especially whether the task seems too easy or too hard.
Scaffolding Up	Actions and interventions teachers take, or could take, to maintain high rigor with high support for students.
Affirming Multilingualism	Giving multilingual learners access to the task, and to centering multilingual learners' perspectives during modeling.
Power and Participation	
Distributing Intellectual Authority	Transferring authority from the teacher to the students, especially deciding what quantities/strategies to use or whether a model is correct.
Disrupting Status and Power	Attending to the ways students are positioning each other or to the ways teachers position students.
Taking Action	The ways the lesson is supporting students to use mathematics to analyze, critique and address power relationships and injustice in their lives.

Findings

Looking across the three categories, Knowledges and Identities, Rigor and Support, and Power and Participation, teachers described dilemmas related to Rigor and Support most often (n = 52), followed by Knowledges and Identities (n = 40). Dilemmas related to Power and Participation occurred far less frequently (n=17). The figure below shows the relative frequency of the dilemmas teachers described within each category. Darker colors indicate more dilemmas in a particular culturally responsive mathematics teaching dimension. Teachers identified dilemmas related to *student thinking and ideas*, *scaffolding up*, *cognitive demand,* and *distributing intellectual authority* most often. In the section that follows, we share examples of

dilemmas from these categories, following each with a discussion of how teachers framed their options.

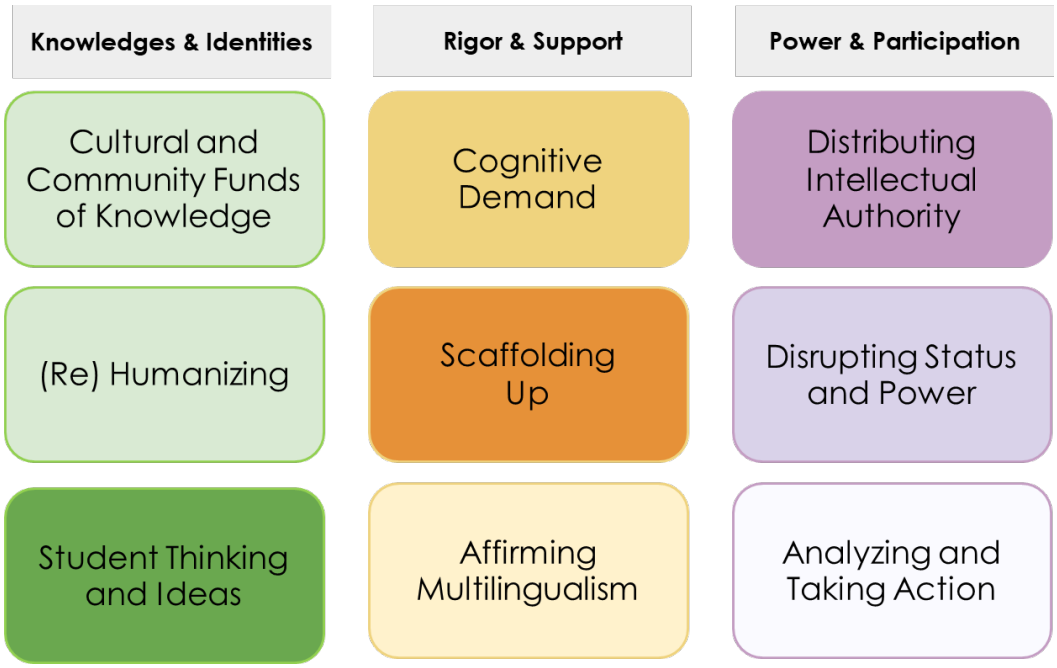

Figure 1: Culturally Responsive Math Teaching Framework Shaded to Indicate the Relative Frequency of Codes

Knowledge and Identities: Student Thinking and Ideas

In the Knowledge and Identities category, dilemmas coded as *student thinking and ideas* occurred most often. These dilemmas centered on eliciting and responding to students' ideas about the modeling context, the quantities and assumptions relevant to the context, and the mathematical strategies they might want to use. The most prominent pattern was that teachers wondered about what to do when students did not generate the ideas, strategies, or approaches teachers anticipated, or when students struggled to come up with an approach at all. For example, M. R., a kindergarten teacher, recounted challenges helping students share and discuss options around what to do with leftover snacks.

> And then the other ones, I guess, didn't know what to do with the [extra snacks]. So then one time, I chimed in and said, "I'll eat them," and so they put me down because they didn't know what to do with the extra. I was hoping that they would say split it, but I couldn't pull that out of them at all. I guess my struggle is trying to get like, I know that they can do it, but I think they need a lot of help getting there.

Mrs. R. anticipated students would want to split any extra snacks and when they struggled to come up with options for the leftovers, she offered to eat the extras. Underlying her dilemma about what to do when students are hesitant to share their thinking were questions around how much to guide and support students in generating ideas to begin with. It seemed Mrs. R.'s belief that students *could* generate ideas was in tension with what she saw as her best option for moving the lesson forward; namely offering an option for the leftovers herself.

Rigor and Support: Scaffolding Up

Dilemmas coded as *scaffolding up* were related to tensions teachers experienced around actions they took, or could take, in order to maintain high rigor and high support for students. Because modeling problems are open and foreground student decision-making, teachers wondered when and how to make use of examples, model mathematical strategies, or guide students' work and they often struggled to decide when and how to provide scaffolds that give students access while maintaining rigor and support. For example, Ms. S. reflected on her efforts to provide appropriate support to her fourth-grade class.

> I didn't have them write [the number of fourth grade classes] down because this is the first time we did it, and I didn't want to overwhelm them with too many numbers. So we put it on the board. There were 26 kids, and me, so we had 27. [We had] about 530 pretzels. We figured that out using multiplication because there were 24 servings and 22 pretzels in a serving. That was the problem we had together. 22 x 24 is a little too advanced for them to do without the manipulatives, and then I just kind of let them go. And that was, I think, the mistake. Not talking about it first, like just a little bit at least.

Initially, Ms. S. decided to ease the demands of the task by limiting the number of quantities the students worked with at one time. However, directing students to decide whether or not they had enough pretzels for their class and rather than the entire fourth grade funneled students to use a single approach: find the number of pretzels needed per class, and then find the number of pretzels needed per grade. As her reflection indicates, Ms. S. also felt students needed additional support finding the number of pretzels in a container but chose to "let them go" - a decision she recounted as a "mistake." Ms. S.'s dilemma highlights teachers' challenges to use teaching strategies that maintain high rigor *with* high support. At times, teachers responded by offering supports that reduced demand and constrained students' choices. At other times, and in an effort to maintain rigor, they withheld support but let students become sidetracked by large numbers and challenging computations.

Power and Participation: Disrupting Intellectual Authority

Teachers discussed dilemmas related to *power and participation* less frequently than the other two categories. When they did discuss power and participation, dilemmas were most often related to *distributing intellectual authority.* Many teachers described these dilemmas as a tension between intervening with explicit guidance and stepping back so that students could drive the discussion or take ownership of ideas. Teachers used terms like "stepping back" or "standing back" to signal a shift in power structures from the teacher as the driver of mathematical ideas to the students. For teachers, shifting from the role of explicit instructor to listener and facilitator was significant. Mrs. B, a fourth-grade teacher, described her dilemma as follows.

> So really, for me, just like letting go of what I thought was maybe going to be the incorrect way. Because honestly, when they started wanting to add all those 16s together, I was like, oh my gosh. My brain was like. "No, no, no, no. Let's not do that. Don't do that." But the math that came out of that was so beneficial that I wouldn't do. I would have let them keep going. Now, if I were ever to do that again, I would have said, "Yeah. Absolutely. Try that." Right? So that was challenging just to be able to slow down and to just praise what they were doing correctly and listening. And a lot of it was not me talking, but just like letting them listen or let go or letting them share out.

Mrs. B's description of her internal struggle to step back and let students pursue what appeared to be an incorrect strategy brings the dramatic shift from resisting to affirming student ideas into focus. At the same time, it highlights her opportunity to benefit from letting students develop and explore their own strategies, and the significance of the shift she was learning to make.

Discussion

Teaching culturally responsive mathematical modeling involves ambitious and challenging teaching practices. It requires teachers to learn new ways of working with student ideas and new ways of providing scaffolds and support. Like Mrs. B, many teachers also confront a mismatch between the roles they and their students play in traditional mathematics lessons and the new roles and responsibilities they have to adopt. Investigating dilemmas teachers new to modeling encounter gives the field insights into the nature of the demands teachers are facing, and of the ways teachers understand their options. Such insights can help professional development designers create resources to broaden teachers' perspectives on what aspects of their practice need to be problematized and give teachers expanded options for navigating dilemmas.

Given our explicit focus on culturally responsive mathematical modeling, we believe it is also important to attend to the dilemma categories that did not come up in teacher interviews. Although our current analysis does not focus on why particular dilemmas occurred more often than others, we believe the task context and teachers' prior experiences played an important role. We note that the modeling context was snack sharing and was situated within the teachers' classrooms. Snack sharing is a common classroom routine and is familiar to most teachers and students. Mathematical opportunities embedded in snack sharing are clear and connections to K-5 mathematics content are readily available. However, the *connecting to students' cultural and community funds of knowledge* and *analyzing and taking action* dimensions of the culturally responsive mathematics teaching framework involve explicit connections to contexts and practices outside the classroom. Because snack sharing does not require teachers to make substantive connections to students' out of school experiences, they may have limited opportunities to connect to and affirm students' cultural and community knowledge bases. Likewise, snack sharing may give classes opportunities to discuss, define, and mathematize fairness for the purpose of their model, but it does not come with ready opportunities to analyze, address, and critique power relationships and injustice.

Second, teachers in our project may have had more opportunities to develop language and practice around *student thinking and ideas* and *scaffolding up* outside of our professional development. Working with student ideas and providing scaffolds and supports are common practices in elementary school teaching (e.g., Cohen, 2004; van de Pol, Volman, & Beishuizen, 2010). Thus, teachers may be more adept at noticing and reflecting on their own practices in these areas. We posit that *(re)humanizing* mathematics, *disrupting status and power*, and *analyzing and taking action* introduced new ways to frame mathematics teaching, so teachers may need additional support to learn to notice their own teaching habits in these areas, as well as more opportunities to learn new practices.

Conclusion

Unpacking and understanding both the named and unnamed dilemmas that arise as teachers learn to implement culturally responsive mathematical modeling lessons has implications for the design and implementation of teacher professional development. In our own project, we have

used our growing understanding dilemmas to design "teacher moves tables" that suggest strategies aligned with specific dimensions of our culturally responsive mathematics teaching framework, developed annotations that foreground teacher decisions to accompany lesson planning tools, and worked to provide time during lesson debriefs and teacher professional development sessions to discuss culturally responsive teaching dilemmas. We are also beginning to investigate the varied potential of different kinds of modeling tasks to address dimensions of culturally responsive mathematics teaching, especially *power and participation,* and the ways modeling tasks give teachers opportunities to notice and reflect on their pedagogical choices.

Although our work focuses on culturally responsive mathematical modeling, we believe a focus on teaching dilemmas could benefit a range of professional development initiatives. The ways teachers understand and frame their choices provides a bridge between the ideals espoused and modeled during professional development and the ways teachers negotiate those ideals during instruction. Explicitly foregrounding equity through tools like Zavala and Aguirre's (in press) culturally responsive mathematics teaching framework grounds teacher learning in the practices critical to realizing the equitable and just learning experiences each and every child deserves.

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