#### **ORIGINAL PAPER**



# Resisting marginalization with culturally responsive mathematical modeling in elementary classrooms

Erin Turner<sup>1</sup> · Julia Aguirre<sup>2</sup> · Mary Alice Carlson<sup>3</sup> · Jennifer Suh<sup>4</sup> · Elizabeth Fulton<sup>3</sup>

Accepted: 23 December 2023 © FIZ Karlsruhe 2024

#### **Abstract**

Mathematical modeling (MM) - a cyclical process that involves using mathematics to make-sense of and analyze relevant, real-world situations - has the potential to advance equity and challenge spaces of marginalization in the elementary mathematics classroom. When informed by culturally responsive teaching practices, MM creates opportunities to center the knowledge and experiences that students from diverse racial, cultural, and linguistic backgrounds bring to the classroom as valuable resources to support learning and inform action. It can disrupt power and status hierarchies in the classroom that contribute to structural and ideological marginalization. This paper describes ways teachers connected their teaching of MM with key components of a culturally responsive mathematics teaching framework. Analysis synthesizes data from an innovative, research-based professional development for elementary teachers to support teacher learning of equity-centered, culturally responsive MM instruction. Data sources include end of year teacher interviews, and professional development discussions from 19 teachers at four geographically, racially, and culturally diverse sites. Findings focus on how teachers connected their teaching of MM with key dimensions of culturally responsive mathematics teaching, and affordances and challenges related to resisting ideological and structural forms of marginalization.

**Keywords** Mathematical modeling · Culturally responsive mathematics teaching · Equity · Professional development · Elementary education

Erin Turner eturner@arizona.edu

Julia Aguirre jaguirre@uw.edu

Mary Alice Carlson mary.carlson5@montana.edu

Jennifer Suh jsuh4@gmu.edu

Elizabeth Fulton elizabeth.fulton@montana.edu

Published online: 15 January 2024

- <sup>1</sup> University of Arizona, Tucson, AZ, USA
- <sup>2</sup> University of Washington Tacoma, Tacoma, WA, USA
- Montana State University, Bozeman, MT, USA
- George Mason University, Fairfax, VA, USA

#### 1 Introduction

Mathematical Modeling (MM) is a cyclical process that uses mathematics to make sense of and analyze relevant, realworld situations (Kaiser et al., 2011; Kaiser, 2017). Teaching MM involves supporting the development of modeling competencies (Maaß, 2006), including posing problems; making assumptions and defining variables; and creating, validating, and sharing models (COMAP & SIAM, 2016). While MM has long been a focus in secondary and university mathematics (Borromeo Ferri, 2021; Stender & Kaiser, 2015), recent research has shown that young children have the mathematical competencies and real-world understandings to explore modeling tasks (Carlson et al., 2018; English, 2009; English & Watters, 2005). In fact, modeling can support students with diverse backgrounds and a broad range of prior mathematics experiences to be confident and successful in mathematics (Lesh & Doerr, 2003).

Building on this work, we view MM as a *lever for equity* and a way to *resist marginalization* in the elementary classroom (Aguirre et al., 2022). In our work, equity means: All



students in light of their humanity—personal experiences, backgrounds, histories, languages, physical and emotional well-being- must have the opportunity and support to learn rich mathematics that fosters meaning-making, empowers decision-making, critiques, challenges, and transforms inequities/injustices (Aguirre et al., 2013). Our perspective on equity attends to ways that students are marginalized in mathematics education, via systemic structures that limit access to resources including rigorous, meaningful curriculum, and via ideologies about what it means to do mathematics and who can be competent in mathematics that shape student identities (Chen & Horn, 2022). While mathematics education has the potential to marginalize any student, it is critical to acknowledge that in the United States, "members of some groups have historically been and continue to be targeted for marginalization more so than others due to the structures—such as racism—that organize our society" (Chen & Horn, p. 806).

Scholars who research MM have suggested its potential for advancing equity, and therein, resisting marginalization, especially when it is taught with culturally responsive and/or emancipatory aims (Anhalt et al., 2018; Barbosa, 2006). First, when modeling activities connect in meaningful ways to students' lived, cultural experiences, this resists standardized curriculum structures that tend to exclude students' diverse identities, perspectives, and voices from the problems that are posed and the solutions that are generated (Brown, 2008). Second, modeling activities are challenging, and often include opportunities for deep mathematical thinking and critical analysis of situations in students' lives. When all students have opportunities to engage in MM, this resists deficit-based ideologies about students' mathematical competence that have historically restricted access to challenging, meaningful mathematics, particularly for students from immigrant or other minoritized backgrounds (Boaler & Staples, 2008; Sengupta-Irving, 2021). Third, compared to typical problem solving-based mathematics instruction, MM expands what it means to do mathematics, what knowledge and experiences are relevant to posing and solving problems, and what strategies and solutions are viable. This allows for a broader range of students to be successful (Lesh & Doerr; 2003). Our study seeks to understand the potential of MM, taught through the lens of culturally responsive mathematics teaching (CRMT), to address spaces of marginalization in the elementary mathematics classroom.

## 2 Teaching culturally responsive MM to resist marginalization

Culturally responsive mathematics teaching (CRMT) is a holistic and complex instructional practice that includes explicit attention to cultural/community funds of knowledge and lived experiences (Caswell et al., 2011; González et al., 2005; Hunter & Miller, 2022). Culturally responsive mathematics teachers hold high expectations for all students, and build authentic partnerships with students, families, communities, and sovereign Indigenous nations to support learning mathematics (Averill et al., 2009; Gay, 2000; Nicol et al., 2013). This is "significant work" for teachers of mathematics, and particularly challenging for teachers that may not share cultural identities with their students (Nicol, 2013). Research suggests that identifying cultural activities that reflect mathematics concepts - including traditional storytelling, games, songs, and tools - as resources for learning can support teachers in this practice (Averill et al., 2009; Hunter et al., 2018). For example, in work with teachers serving Indigenous students in Canada (e.g., Inuit People, Métis People), Nicol and colleagues (2013) found that learning about students' cultural practices shifted teachers' views from cultural and mathematical deficiency to viewing students' cultural knowledge as a resource for mathematics learning. In other words, teachers' ideologies about mathematical competence and what it means to do mathematics shifted towards ideologies that resist rather than reinforce marginalization.

Few studies on teaching MM have explicitly connected to culturally responsive teaching, and most that do tend to focus on secondary contexts (ages 12-18). A common theme across these studies is the potential of modeling tasks to honor students' knowledge of modeling contexts and their mathematical ideas. Anhalt et al. (2018) describes a secondary MM task related to home fence design and mathematical functions. Modelers explored the purpose of fences in their own and adjacent neighborhoods, including design assumptions (e.g. aesthetics, keeping children or pets safe; discourage trespassing). They recommended new fence designs to a builder, using functions to model the shape and size of each design. Cirillo et al. (2016) offered a middle school MM task focused on access to healthy food. Students analyzed fast food menu options and pricing, including unit rates, and explored how their analysis could help change pricing options or inform decisions related to healthy food. Brady et al. (2023) studied the enactment of a middle school task about designing a homeless shelter. Students were highly invested in the task, as it addressed an important community need, and "humanized the design process, empathizing with the inhabitants and showing concern for how the interior space would be experienced" (p. 5). In another study,



Barbosa (2006) described how middle school students used modeling to mathematically analyze and critique a government plan for distributing seeds to farmers in their community and to propose a more just distribution method. These examples illustrate how culturally responsive approaches to MM create opportunities for students to use mathematical analysis to understand relevant situations in their lives and communities. In this way, MM with a culturally responsive approach has the potential to resist structures such as standardized curricula that marginalize students' lived experiences in mathematics classrooms.

At the elementary level (ages 5-11), research has documented the potential of teacher-researcher collaborations to design MM tasks that reflect culturally responsive approaches (Turner et al., 2022, 2023). In the United States, Suh et al. (2023) described how teachers and researchers worked together to design a MM task about food insecurity, highlighting the dilemmas they faced as they tried to ensure authentic connections to a local context, and meaningful opportunities to use mathematics to analyze and take action. In related work, Tate et al. (2022) studied how a team of teachers collaboratively planned a MM task focused on mathematizing racial representation in library book collections. They found that teachers benefited from opportunities to reflect together on how to build background knowledge and navigate conversations about race and identity, particularly with young children. In Australia, English (2009) described how grade 3 through 5 teachers collaborated with researchers to design MM tasks connected to community contexts (i.e., creek pollution), and to anticipate student strategies and models. In South Africa, Paolucci and Wessels (2017) found that teachers were able to identify meaningful and relevant real-world contexts for MM, but struggled to pose modeling questions that reflected appropriately demanding mathematics content for younger students, noting that additional supports were needed. These studies reflect the potential of culturally responsive approaches to MM to resist marginalization by centering local contexts and student experiences, as well as some of the challenges that arise in task design. However, to better understand the potential of MM to resist marginalization, the research base needs a more holistic approach that attends not only to task design, but to the opportunities and challenges that arise when tasks informed by culturally responsive approaches play out in elementary classrooms.

## 3 A theoretical framework for culturally responsive mathematics teaching

To inform this holistic approach, we draw on Zavala and Aguirre's (2023) comprehensive framework for culturally responsive mathematics teaching (CRMT). Although it is not specific to MM, their framework attends to a range of instruction components including task context, mathematical rigor, and student experiences, and names observable actions indicative of a culturally responsive approach to teaching mathematics. The CRMT framework consists of three main strands: Knowledges and Identities; Rigor and Support; and Power and Participation (See Fig. 1). Each strand consists of multiple dimensions. Zavala and Aguirre's use of the term "strand" is intentional, as they expect "various threads of the CRMT" (p. 25) to be present and interwoven in culturally responsive teaching. In this section, we describe how the CRMT dimensions reflect ways to challenge structural and ideological forms of marginalization in mathematics classrooms.

The *Knowledges and Identities* strand elevates student cultural and community knowledge and experiences, affirms positive mathematical identities, and honors student thinking and ideas (Aguirre et al., 2013; Carpenter et al., 2014; Civil, 2007). By engaging students in meaningful and culturally relevant mathematical tasks, teachers position the knowledge and experiences that students from diverse racial, cultural, linguistic, and mathematical backgrounds bring to the classroom as resources for learning. This actively resists marginalization via ideologies that position specific students as academically inferior based on the color of their skin, the languages they speak or the neighborhood they live in (Adiredja & Louie, 2020).

The *Rigor and Support* strand emphasizes sustained opportunities for students to engage with high cognitive demand mathematics tasks that strengthen their analytical and inquiry skills (Smith & Stein, 1998). Students may need multiple and varied supports (i.e., social scaffolds, analytic scaffolds, see Anhalt, 2014) to access tasks and sustain their engagement. Furthermore, affirming multilingualism acknowledges that children who speak more than one language need to be centered as valuable contributors to the mathematical learning space. These dimensions represent ways to resist structural marginalization which systematically denies specific students access to high value resources such as rich mathematical tasks because of ideologies about academic readiness or dominant language acquisition (Chen & Horn, 2023).

The *Power and Participation* strand emphasizes distributing intellectual authority among students and teachers, disrupting stereotypes and status hierarchies that shape social relationships and classroom interactions, and engaging



#### **Knowledges & Identities**

### Centering Cultural and Community Funds of Knowledge

Helping students connect mathematics with relevant/ authentic issues or situations in their lives

#### (Re) Humanizing Mathematics

Supporting creativity and broadening what counts as mathematical knowledge, and affirming positive math identities for all students.

### Honoring Student Thinking and Ideas

Making opportunities to elicit, express, and build on student mathematical thinking in multiple ways (e.g. gestures, pictures, words, symbols)

#### **Rigor & Support**

#### Sustaining High Cognitive Demand

Enable all my students to closely explore and analyze math concept(s), procedure(s), and problem solving/reasoning strategies

#### Scaffolding Up

Maintaining high rigor with high support for all students.

#### **Affirming Multilingualism**

Making space for multilingual learners (MLL) to be central participants in mathematics activities

#### **Power & Participation**

#### **Distributing Intellectual Authority**

Distributing mathematics authority and make space for multiple forms of knowledge and communication

#### **Disrupting Status and Power**

Disrupt status differences, entrenched stereotypes, and inequitable power relationships present in all mathematics classrooms

#### **Analyzing and Taking Action**

Supporting student use of mathematics to analyze, critique, and address power relationships and injustice in their lives

Fig. 1 Culturally responsive mathematics teaching framework (Zavala & Aguirre, 2023)

critical consciousness through mathematical analysis and taking action (Featherstone et al., 2011; Gutstein, 2006; Ladson-Billings, 1995). The emphasis on disrupting traditional status and power hierarchies reflects ways to resist ideological marginalization based on narratives that devalue students' intellectual contributions, and limit the identities and roles available to them (Chen & Horn, 2022).

Although research has established critical components of CRMT as means to resist structural and ideological marginalization, less is known about the ways teachers understand and take up culturally responsive mathematics teaching practices during modeling lessons. We address this need in our study via the following research questions:

- How do teachers connect their teaching of modeling with key dimensions of CRMT?
- What affordances and challenges do teachers encounter related to resisting marginalization?

#### 4 Methods

#### 4.1 Context and participants

This study is part of a broader research and professional development program focused on culturally responsive MM in the elementary grades. Teachers participated in a year-long, hybrid professional development program that included monthly in-person sessions and asynchronous activities to deepen learning. In person sessions, facilitated by the authors, introduced frameworks for CRMT and included time to explore modeling tasks and routines, collaboratively plan activities, and reflect on classroom enactments. Asynchronous materials included readings, videos of modeling lessons, and reflection prompts.

This study focused on 19 teachers of kindergarten through 5th grade (ages 5 through 11) who participated in our professional development program at one of four research sites in different regions of the United States (southwest, midatlantic, northwest, and mountain west). 16 of the teachers worked in schools that served racially and linguistically diverse students from underserved communities. Classrooms included migrant and refugee students from diverse countries of origin, and significant numbers of multilingual students. Four teachers taught in predominantly white schools with a small but growing population of multilingual students.

#### 4.2 Professional development model

We grounded our professional development in two perspectives: Anhalt, et al.'s (2018) modeling cycle (Fig. 2), which drew on similar representations including Blum and Leiß



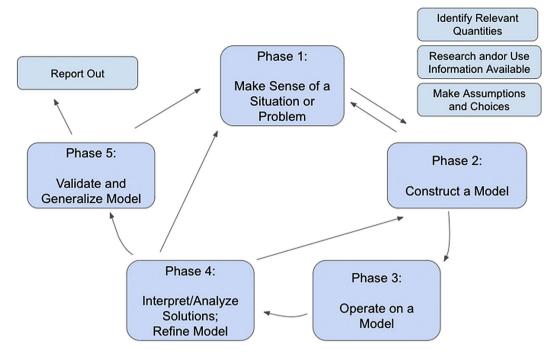


Fig. 2 The modeling cycle used in our professional development model

(2005) and the Common Core State Standards for School Mathematics (CCSSI, 2010), and Zavala and Aguirre's (2023) CRMT framework (Fig. 1).

In drawing on these frameworks, our aim was to help teachers support students' development of modeling competencies (Kaiser, 2007; Maaß 2006) and harness the power of MM to affirm student identities, cultivate problem-posing, engage all students in challenging and meaningful mathematics, and use mathematics to disrupt, rather than reify, longstanding inequitable power differences in and out of classrooms.

#### 4.3 Data sources

Our primary data source was transcripts from end-of-year interviews with teacher participants. Each teacher participated in an hour-long individual interview with project researchers after completing professional development activities and teaching several modeling lessons including: snack-sharing tasks, involving distributing with partitive and quotative division concepts; making tasks, with multiplicative thinking leading to proportional reasoning; and community-based tasks, involving collecting data and using statistical reasoning models to make decisions. Interview topics included teachers' perceptions of MM and connections to the CRMT framework, experiences implementing modeling lessons, student learning, and supports and challenges for teaching modeling. Secondary data sources used to triangulate findings from teacher interviews were detailed

field notes and transcripts of selected discussions during three professional development sessions at each site. Specifically, we focused on teachers' reflections on how MM activities enacted in their classrooms supported components of CRMT as well as the challenges that arose.

#### 4.4 Data analysis

We uploaded transcripts of teacher interviews and selected professional development discussions to ATLAS TI, a qualitative analysis program. Initially, we coded deductively using our theoretical framework to define categories and codes and identify segments where teachers' talk reflected connections to specific dimensions of the CRMT framework (Saldaña, 2021). Given that the framework includes three strands which are explicitly intended to be woven together to support culturally responsive mathematics teaching, when teachers described the ways that MM lessons connected to more than one framework dimension. we applied multiple codes to these segments. For example, several teachers described the range of mathematical representations that students, including multilingual learners, used when modeling. Students' varied approaches and representations gave teachers opportunities to broaden what counts as mathematical knowledge (RH) and affirm the contributions of multilingual learners (AM). In a second round of coding, we applied subcodes that focused both on supportive connections (i.e., how teaching MM supported dimensions of CRMT), and challenges (i.e., tensions



related to specific CRMT dimensions in teaching MM). For example, within the centering cultural and community funds of knowledge code, a supportive connection included modeling tasks that invited family and community member involvement. A challenge was ensuring task contexts were relevant and meaningful to students, while avoiding stereotypes. Figure 3 displays coding categories and definitions.

Two researchers coded each data source, and then reconciled differences. For research question 1, we used the interview coding to summarize connections and challenges that received substantive attention (i.e., repeated mention and/or elaborated examples) for each individual teacher. Next, we compiled the teacher-level summaries to identify patterns both within and across our four research sites. We then analyzed the coding of professional development discussions by site, to triangulate patterns across data sources. For research question 2, we used the patterns of supportive connections

and challenges identified in our first two rounds of coding to guide further analysis. We reviewed coded excerpts for each pattern and created analysis memos by site, focused on how connections between teaching MM and dimensions of CRMT might support or challenge efforts to resist marginalization in the mathematics classroom. We then looked across memos to establish themes.

#### 5 Findings

In the first findings section, we summarize patterns, across teachers, of supportive and challenging connections between teaching modeling and dimensions of the CRMT framework (Research Question 1). In the second section, we elaborate three key themes related to the potential of teaching MM to resist marginalization using illustrative excerpts

	Codes	Definition	Sub codes		
Knowledge and Identities	Centering Cultural and Community Funds of	Connect to relevant and authentic situations in students' lives; Honor students' funds of	Supports		
	Knowledge (FoK)	knowledge	Challenges		
	Rehumanizing Mathematics	Foster creativity; Broaden what counts as mathematical knowledge; Affirm positive math	Supports		
	(RH)	identities for all students	Challenges		
	Honoring Student Thinking and Ideas	Elicit and build on student mathematical ideas in diverse ways; Honor diverse math ideas and	Supports		
	(ST)	diverse representations	Challenges		
Rigor and Support	Sustaining High Cognitive Demand	Create opportunities to analyze math concepts and procedures, and engage in problem	Supports		
	(CD)	solving/reasoning	Challenges		
	Scaffolding Up (SU)	Maintain high rigor for all students; Scaffold access and understanding	Supports		
	(30)	access and understanding			
	Affirming Multilingualism	Make space for multilingual learners to be central participants in math activities	Supports		
	(AM)	participants in main activities	Challenges		
Power and Participation	Distributing Intellectual Authority	Distribute intellectual authority so that students make decisions, define questions, and determine	Supports		
	(IA)	validity of solutions in diverse ways	Challenges		
	Disrupting Status and Power	Disrupt status differences, entrenched stereotypes, and inequitable power relationships in mathematics	Supports		
	(DS)	classrooms	Challenges		
	Analyzing and Taking Action	Create opportunities to use mathematics to analyze, critique and address injustices	Supports		
	(TA)	anaryze, critique and address injustices	Challenges		

Fig. 3 Codebook excerpt



from interviews and professional development discussions (Research Question 2). The first two themes are organized around salient strands of the CRMT framework, but also reflect the ways multiple dimensions of the framework are present and interact in MM lessons that have the potential to resist marginalization. The third theme describes tensions related to resisting marginalization that arose as teachers worked within and across framework strands.

### 5.1 Summary of supports and challenges across teachers

As they reflected on modeling activities in their classrooms, teachers described numerous ways that activities supported specific CRMT dimensions (Fig. 4). The most frequent connection, noted by 17 of 19 teachers, was that teaching MM expanded opportunities to elicit and build on students' diverse mathematical ideas. Teachers emphasized that tasks opened space for diverse strategies and representations, allowing a broad range of students to contribute. Other common connections included that modeling (a) facilitated connections to students' funds of knowledge (n = 14 teachers), (b) allowed teachers to distribute intellectual authority to students (n = 15 teachers), and (c) disrupted status relationships that positioned certain students as more mathematically capable (n=12 teachers). While these connections received comparatively less emphasis than connections to student thinking, they were still discussed substantively by the majority of teachers across all four sites. Approximately half of the teachers described ways that teaching MM rehumanized mathematics by promoting creativity and positive mathematical identities, and sustained a high cognitive demand for all students via teacher scaffolds and supports. Across all four sites, fewer teachers described ways that teaching MM supported opportunities to affirm multilingualism, or to engage students in taking action. Analysis of professional development sessions revealed similar patterns. Teachers emphasized that MM built on students' diverse ideas and, in doing so, distributed intellectual authority to students. Teachers placed comparatively more emphasis on how MM engaged all students in high cognitive demand tasks. We view this as unsurprising, as professional development conversations often centered on the mathematics students used and the competencies demonstrated in student work.

Teachers also described challenges related to specific CRMT dimensions. The most pervasive challenge, noted by 15 of 19 teachers, was related to scaffolding student engagement without narrowing the task or lowering the cognitive demand. Interestingly, some of the same teachers who noted the affordances of MM lessons for scaffolding access to rigorous content also described scaffolding challenges. A similar pattern of supportive connections coupled with challenges was evident in other dimensions including honoring student thinking (challenges noted by 8 teachers) and distributing intellectual authority (9 teachers). In other words, some teachers simultaneously emphasized the potential of teaching MM to advance a specific dimension of CRMT, and acknowledged challenges they encountered. Teachers referred to two areas, affirming multilingualism and analyzing and taking action, primarily in terms of challenges, describing them as spaces for additional professional learning. This pattern was mirrored in professional development sessions, where teachers mentioned challenges related to scaffolding and cognitive demand, honoring student thinking, and distributing intellectual authority most often.

Next, to better understand the affordances and challenges of modeling for advancing CRMT, and thereby resisting marginalization, we elaborate three key themes in teachers' reflections. The first theme centers on ways that teaching MM honored diverse thinking and cultural experiences of students, including multilingual learners, and rehumanized mathematics by encouraging risk-taking and positive mathematical identities. The second theme focuses on how modeling restructured power and participation in mathematics classrooms by distributing intellectual authority to students and challenging existing status differences. The final theme focuses on the complex tensions related to openness, access, and disrupting status that MM introduced, and the ways teachers grappled with seemingly conflicting pedagogical and equity-oriented goals during lessons. In each theme, to acknowledge the ways that teachers connected to multiple

	Knowledge & Identities			Rigor & Support			Power & Participation		
Teachers	FoK	RH	ST	CD	SU	AM	IA	DS	TA
#/% described supportive connection	14 (74%)	9 (47%)	17 (89%)	11 (58%)	11 (58%)	3 (16%)	15 (79%)	12 (63%)	2 (11%)
#/% described challenge	3 (16%)	0 (0%)	8 (42%)	5 (26%)	15 (79%)	7 (37%)	9 (47%)	4 (21%)	5 (26%)

Fig. 4 Summary of supportive connections and challenges between teaching MM and specific CRMT dimensions across teachers

dimensions of the CRMT framework as they reflected on the potential of MM to resist marginalization, we use the abbreviated dimension codes (e.g., ST for student thinking, and AM for affirming multilingualism) to mark these connections.

### 5.2 Themes related to knowledge and identities and rigor and support

## 5.2.1 Resisting marginalization by broadening what counts as relevant knowledge and supporting diverse ways to communicate ideas

Teachers from across all four sites noted that teaching MM allowed them to embrace students' diverse ideas and experiences because tasks were open-ended and facilitated a range of strategies and representations (ST). Ms. I (grade 3, southwest), noted in her interview:

There's a saying in Spanish, "cada cabeza es un mundo," "every head is a different world," so if you apply that to all the math modeling, every student had a different approach and a different way of thinking of how they could solve it. So there was no standard, this is how you're going to do it. Some kids drew pictures, some kids did count by's, some kids did sticks, other kids painstakingly used methods that I knew would not be successful, but that's what worked for them, and you just let them go.

For her, the open and contextualized nature of modeling was particularly important for multilingual students learning English (the language of instruction) (AM, ST), explaining, "I have so many children that don't speak English....when you give them a math modeling project like snack sharing, they can draw pictures, they can make themselves heard through other ways."

Other teachers echoed this idea, noting that the meaningful, familiar contexts of modeling tasks broadened what counted as relevant knowledge. Modeling tasks created space for students to share experiences outside of school and knowledge from families and communities (FoK). This affirmed students' identities and resisted the ways out-of-school experiences are often marginalized in mathematics curricula. Ms. L (grade 4, northwest) explained in an interview:

I think it levels the playing field.... every kid is able to put in their unique ideas and ways of explaining and modeling forward. And that's very equitable... So all their funds of knowledge, all the things that they grow up with in our [community] around them and love to

bring into the classroom as well. So [in modeling] they began to ask questions about the things that they see and think critically about the world around them.

Across all four sites, teachers frequently returned to the ways the openness and relevance of modeling tasks supported students from diverse backgrounds to share their ideas and experiences. This resisted potential ideological marginalization that occurs when decontextualized tasks and narrow notions of what "counts" as a mathematical contribution exclude multilingual learners and other students from historically marginalized backgrounds from meaningful mathematics learning experiences that connect to their diverse knowledge and identities.

### 5.2.2 Resisting marginalization by encouraging risk-taking and positive mathematical identities

Some teachers argued that MM encouraged risk taking (RH), which helped them support students' sustained engagement in challenging tasks (CD). In one professional development conversation between three teachers at our northwest site (Ms. R and Ms. T, grade 1, and Ms. L, grade 4), teachers noted that the open-endedness of modeling tasks (as compared to closed problems with a single solution) helped them to scaffold students to revise their strategy or solution (CD, SU). Two teachers described using specific asset-based language to help students embrace the challenges associated with revising models.

Ms. T: So I think by using language like that (pointing to the paper) and observing what they're doing, saying things like "I love how you are showing your work, or how you are thinking critically about this problem", that might help them.

Ms. L: It's hard when they don't get the right answer.

Ms. T: Yeah, but when we use that language more and that's what they are hearing, rather than "that's not correct, [but] how can we fix it"?

Ms. R: By using that positive language with them... and using those words like "assumptions", "What did you know? How did this help? What is your experience?" It helps them come in. They feel braver taking those steps and those risks.

Ms. L: Yeah, well they're being praised for not the right answer but for taking those risks.



Teachers at other sites emphasized similar ideas. In an interview, Ms. C (grade 1, mountain west) noted that "students that maybe don't feel as confident [have] a chance to feel confidence in their thinking and especially in their mathematical thinking, because it [modeling] is so much more flexible" (RH). These reflections are important, because they suggest that MM fostered productive dispositions towards mathematics (i.e., taking risks) and positive mathematical identities (i.e., bravery, confidence). In other words, teaching MM has the potential to resist the ideological marginalization based on limited, and often deficit-based beliefs about who can do mathematics grounded in race, class, and gender-based stereotypes and what it means to be competent in mathematics (performing quick, accurate calculations) that students from underrepresented backgrounds often experience.

### 5.2.3 Resisting marginalization by expanding teachers' opportunities to learn about student strengths

Teachers at all sites explained that because modeling lessons supported diverse strategies and connections (ST), and encouraged students to take risks (RH), they learned more about students' strengths. For example, Ms. C (grade 1, mountain west) explained in her interview that as students worked on a task that involved rating and ranking options for a classroom sensory space, they demonstrated strengths related to fact fluency that she was unable to previously see (ST).

The sensory room [modeling lesson], that's where I really started to see their mathematical thinking come together like when they were adding all of their rubrics up and figuring that out, like the way they were using their fluency.... that I don't normally see when you're doing a fluency task. Like they were starting to pull out the five and add the fives and things like that. That was just really helpful for me.

Ms. T (grade 1, northwest) echoed this idea as she reflected in her interview on what she learned about students' strengths in MM lessons:

I learned that they are a lot more capable, than, sometimes we realized. I mean I hold all my kids to high standards, and I believe that they all have the abilities to grow and access things, but I think sometimes with the traditional structure or the curriculum that we have, it doesn't allow for them to show their strength.

Collectively, teachers found that the openness of MM activities, and the ways that they encouraged risk-taking

enhanced their opportunities to recognize student strengths (ST) and affirm positive mathematical identities (RH). This resisted the ideological marginalization that students from underrepresented backgrounds may experience when narrow curricular tasks do not allow students to showcase their diverse strengths and innovative mathematics ideas.

### 5.3 Themes related to power and participation dimensions

# 5.3.1 Resisting marginalization by redistributing intellectual authority so that diverse groups of students have power and agency

Teachers across our four sites described how MM redistributed authority (IA) in their classrooms, often contrasting modeling with "typical" instruction that focuses on reproducing teacher strategies. As Ms. W (grade 4, mid-atlantic) noted in her interview:

Math modeling really puts the power of how to solve these problems into the kids' hands rather than my own. I think, with normal, like direct instruction, I'm telling them how problems are solved. But with this, they're to really take that and tell us how problems are solved which is very powerful.

She discussed how "eye-opening" it was to see the mathematical connections students made and described her efforts to step back and "let them be problem solvers" as an important growth area in her teaching.

Ms. D (grade 4, mid-atlantic), also noted that MM shifted authority over strategies from the teacher to the students (IA). She explained, "All the knowledge is not coming from the teacher, right? Like they can explore this and work together to figure it out and have their own strategies that aren't necessarily the same as my strategies." Other teachers highlighted how MM decentered the teacher as the authority over correct answers, and instead empowered students to validate solutions. For example, in her interview Ms. M, (grade 2, northwest) noted:

[MM] also improved equity because it took the power and the knowledge off of the teacher and put it on the kids, which is an issue in all subjects, but I think, especially in math. When kids are looking for a right answer the teacher is like the holder of the knowledge and so when you do these modeling tasks that takes power and gives it back to the class.

Across these remarks, teachers emphasized that MM centered students as intellectual authorities in the classroom.



Teachers' comments demonstrate the potential MM holds to resist the ideological marginalization students experience in classrooms where authority rests with the teacher and students are positioned as receivers, rather than producers, of mathematical knowledge.

### 5.3.2 Resisting marginalization by disrupting status hierarchies in classrooms

Teachers also noted that as authority shifted away from the teacher, a more diverse group of students had opportunities to share their ideas (IA, DS). More students sharing substantive ideas disrupted existing power structures between and among students. For example, Ms. C (grade 1, mountain west), reflected in an interview that modeling disrupted narratives around students who receive extra support in mathematics.

I was constantly shocked at the ability that the kids had. Students that might be even receiving extra support in the area of math outside of the classroom were coming in with amazing ideas and still able to complete the project or using either different materials, really checking their work, if I scaffold it with them, if they used a partner to help them. Every kid was able to feel accomplished in completing the [MM] task. So that kind of disrupted that idea of like, "I'm a good math student," or "I'm bad at math."

Teachers at other sites echoed this sentiment. During a professional development session at our southwest site, Ms. N, a mathematics coach who visited multiple classrooms each day, recounted the ways MM gave her opportunities to help students focus on understanding others' thinking irrespective of language status (AM) and challenged perceptions surrounding who can and cannot do math (DS).

Sometimes when I go into classrooms and I don't know who the non-English speaking students are, sometimes I'll ask a student to clarify or to show me and then a bunch of other students will jump in and say, "He can't, he doesn't know English. But I see that as a "he can't do the math" or "he can't explain it because of the language." And I try to reframe that as "how can we find out their thinking, even though we don't know their language?" And that way we can build those connections and not create an environment of because they are not speaking English they can't do this task or can't engage with us.

At all four sites, teachers found that MM created opportunities to change power and participation structures in their classrooms. MM helped teachers relinquish authority and gave students power to recognize peers' contributions as valuable. Valuing a broader range of ideas disrupted entrenched status ideologies related to who can do mathematics (DS), and created opportunities to position historically marginalized students as competent, contributing problem solvers.

### 5.4 Tensions across rigor and support and power and participation dimensions

## 5.4.1 MM activities may marginalize some students, including emerging bilingual learners, unless sufficient instructional supports are in place

Across sites, teachers grappled with how to best support emerging bilingual learners to fully participate in MM (AM, SU). They recognized that the language demands of modeling tasks (e.g. writing and speaking) and the importance of students driving modeling decisions may seem overwhelming to some students learning in another language. Responsively supporting emerging bilingual students was both a priority for teachers, and a challenge that needed additional attention.

Ms. J (kindergarten, southwest), explained in her interview that some emerging bilingual students may not fully participate given the language demands of MM.

One of my biggest challenges is [supporting] the bilingual kids because so much is spoken and written and when you don't have access to that it's like you're trying to show them something or you're trying to have them become part of something that is kind of over their heads, and I don't know how to make it more concrete.

Teachers at other sites echoed this idea, noting that "making space for multilingual learners to be central participants in mathematic[al] modeling" (Ms. F, grade 4, mountain west) was a priority in their future instruction (AM, IA).

Teachers also voiced this challenge in professional development sessions. In the following discussion at the southwest site, Ms. I (grade 3) described plans for a MM task that involved projecting the amount of tissue paper needed to create flowers for mother's day bouquets. She wanted the task to be open enough to allow students to make decisions (CD), but was concerned about providing enough support for emerging bilingual students (AM, SU). Her colleagues, Ms. S (grade 4), Ms. N (coach), and Ms. J (kindergarten), brainstormed possible instructional options.



Ms. I: We are going to make paper flowers for mother's day.... How much choice am I going to give the kids?...I wanted to have all the tissue paper pre-cut and maybe have them decide how many sheets they want in their flower?... I don't know if my kids are there. 50% of my class is IEP [Individualized Education Plan] or ELL [English language learners]... So knowing student ability, I want them to have success.....

Ms. S: Since they are cut into 4, and each kid needs 5, and if they are in groups of 4, hopefully they would notice that each kid needs 1 and then that is 1 extra, to divide into 4.

Ms. N: I thought that relationship might come up in a table. Five sheets, let 4 kids make a flower.

Ms. I: I don't know that they are there, but we could try. I just don't want it to flop.

Ms. J: What about blocks? [suggest using blocks to build a model of paper needed]

Ms. I: Maybe if I have a template, with squares divided into 4... I think if I give them something like that it would encourage them...That is why I do a lot of modeling with pictures and visuals, because the kids that don't speak English need to be able to connect... But how do I do that?

While brainstorming ideas, colleagues offered strategies to prevent the marginalization or exclusion of emerging bilingual students from participation in rigorous modeling tasks. These included using manipulatives to physically represent how many paper sheets are needed and graphic organizers such as tables to organize information. Ms. I emphasized that using visuals would help emerging bilingual children "connect" to the problem but remained uncertain about how to scaffold effectively. Teachers' comments reflect their developing ideas about the generative potential of MM activities to facilitate emerging bilingual students' participation and learning, alongside their concerns that additional scaffolds may be needed.

### 5.4.2 Collaborative group work may marginalize students unless structures are in place to resist status issues

Another challenge voiced by a few teachers focused on how to support students to make decisions and build models in collaborative groups. The project emphasized structuring groups based on multiple strengths (heterogeneous groups) rather than fixed ability groups (homogeneous groups) to support the high cognitive demand of modeling tasks (CD), and to disrupt status issues (DS). While teachers noted that MM supported broad participation and access, they also expressed concerns about how to effectively structure groups that supported all students without reinforcing status issues (DS).

Ms. W (grade 3, mid-atlantic) noted in her interview that some students needed more support with MM tasks and did not actively participate when in heterogeneous groups (SU, DS): "students that needed more support kind of took a back seat and didn't take as much action." This challenge also arose in professional development sessions. At the northwest site, teachers watched a video of groups of students presenting solutions to a MM task focused on projecting cafeteria waste over time, and Ms. L (grade 4) posed a "massive question."

Ms. L: If you have heterogeneous groupings, how do you differentiate? If you put who's very low, higher, mid, and you put them together, they do different things.

Facilitator: These tasks draw out different strengths of the kids. When you put the kids together and don't label them as high, medium, low, their strengths are different. Some have reasoning, some have more computation, some are creative thinkers.....

Ms. L: In this context, is it worth having kids working at a similar level? If those three girls [in the video] had another kid in the group who was really high performing in math and had multiplication tables memorized... that student would want to [build a model] for a year.

This exchange reflects the challenge of creating groups that support all students' engagement and learning. The teacher wondered whether ability grouping would better support varied learning needs, particularly for students with "low" status labels (SU, DS). The facilitator reframed status labels frequently used in schools such as high, medium, low into varied strengths that could be successfully combined to make progress on complex tasks. However, Ms. L's comments reflect a concern that students' access might be compromised if students with different mathematical strengths work together. In a final interview, Ms. L continued to grapple with this concern.

One of the ways I see differentiation happening is if they're in homogeneous groups because so much of this math [modeling] is choice, right? And, if they're



working in a group, they have to come to decisions together as a group. But, if you're in a heterogeneous grouping where one kid is still working on base 10 very fundamental, then you have somebody else in the group who is maybe already adding and subtracting and carrying and borrowing and maybe another kid who can do division or understands division, fundamentally I don't see that being a successful group....I mean the kids who are higher performing are going to take over, they're going to get bigger numbers, they're going to start writing and drawing [while] the other kids are not going to be able to access anything.

From Ms. L's perspective, ability grouping would enable all students to make progress on open-ended modeling tasks by supporting student choice and ownership over decisions, and sustaining high cognitive demand (CD, SU).

While this theme was less prevalent across sites, it highlights a common concern voiced by teachers related to disrupting status and scaffolding groups to work together. It reflects the complexity in resisting both ideological (i.e., beliefs that some students are more mathematically competent) and structural marginalization (i.e., homogeneous grouping practices) operating in classrooms to deny students, particularly those who may have less status in the mathematics classroom, from accessing challenging and meaningful tasks. This challenge also emphasizes teachers' need for practices and tools that maintain high cognitive demand for all students while they work in heterogeneous groups that are based on students' multiple strengths.

#### 6 Discussion

The themes reported in our findings reflected patterns in how teachers connected their MM teaching to key dimensions of CRMT (Zavala & Aguirre, 2023) (Research Question 1), and the affordances and challenges they encountered related to resisting structural and ideological marginalization in mathematics classes (Research Question 2) (Chen & Horn, 2022). Connecting to the Knowledge and Identities and Rigor and Support strands, teachers across all four sites explained that modeling lessons supported a diverse range of student ideas and representations (honoring student thinking and ideas), and encouraged connections to students' out of school experiences and funds of knowledge (centering cultural and community funds of knowledge). This openness and relevance, coupled with specific teacher moves that affirmed students' ideas and supported engagement (scaffolding up) and encouraged students to be brave and take risks (rehumanizing mathematics), expanded teachers' opportunities to learn about students' strengths. Teachers also found that MM lessons helped them attend to issues of Power and Participation. Teachers stepped back so that MM tasks were driven by student ideas(distributing intellectual authority). Decentering their own authority disrupted practices teachers accepted as "normal" in mathematics lessons (i.e., direct instruction, teacher validates answers), thereby expanding the roles and identities available to students. This allowed a broader range of competencies to become part of teaching and learning, which has the potential to disrupt marginalizing ideologies about what it means to be good at mathematics (disrupting status and power). In this way, teachers found "the hidden figures: the students who have been invisibilized by mechanisms of marginalization" (Chen & Horn, 2022, p 891).

Our findings also showed that the openness of MM simultaneously invited access, decision-making and connection, and created challenges that connect to the strands of Rigor and Support and Power and Participation. The challenges teachers voiced reflected efforts to prevent marginalization and maximize engagement, but teachers needed additional strategies to sustain cognitive demand and disrupt status. Teachers grappled with group work structures that supported students' access and progress in MM (scaffolding up and sustaining cognitive demand). They recognized that group work dynamics could reinforce status hierarchies in the classroom (disrupting status), suggesting teachers need additional tools to navigate this challenge. Teachers emphasized a need to learn to support emerging bilingual students with the language demands inherent in modeling tasks that emphasize collaboration (sustaining cognitive demand, scaffolding up). They also wanted to better support bilingual students as central, rather than peripheral, participants (affirming multilingualism).

MM was new to all of the teachers in our study, and teachers entered our work with varying levels of familiarity with culturally responsive mathematics teaching. We find it promising that at the end of the year-long program teachers not only deepened their understanding of teaching MM, but also expressed supportive connections between their work in modeling and the broader, equity-oriented goals of culturally responsive mathematics teaching (Caswell et al., 2011; Gay, 2000). For some teachers, these connections required taking risks as they tried new practices during MM lessons. Through this risk taking, teachers discovered ways that MM helped them affirm positive mathematical identities and distribute intellectual authority by giving more students more power and agency. This is important because it suggests that elementary teachers can learn to teach MM, and that teachers can use MM as an equity lever to actively resist marginality in the mathematics classroom.

Our study underscores the potential of culturally responsive MM instruction to address different spaces of



marginalization in mathematics education-including who is marginalized, how they are marginalized and what they are marginalized from. More specifically, teachers focused on who as they described the power of MM to affirm student identities, particularly by disrupting entrenched stereotypes (i.e. based on race, gender, social class, language) of who is good at learning mathematics (Featherstone et al., 2011). Teachers emphasized how as they explained the potential of open, culturally responsive modeling lessons to invite and elevate students' voices (Barbosa, 2006; Brown, 2008), and therein counter typical curricular practices that exclude students' diverse identities, experiences, and voices from mathematical problems (Nicol, 2013). Teachers focused on what they noted as efforts to ensure all students, including emerging bilinguals and students who have experienced less success in prior mathematics classrooms, are engaged in challenging, meaningful mathematics. In this way, our study contributes new understandings about the ways teaching MM, with culturally responsive aims, can resist spaces of marginalization in mathematics classrooms.

#### 7 Implications

Our study contributes important insights related to how elementary teachers connected their MM teaching with key components of CRMT. Our analysis focused on teachers' understandings and reflections on their teaching practices, and not on how they enacted these understandings in classroom instruction. Future research should explore how teachers' ideas about the potential of MM to broaden access to challenging mathematics, connect to students' ideas and experiences, distribute intellectual authority and disrupt status are evidenced in teaching practices and classroom interactions. In particular, future research should investigate how teachers respond to the challenges and tensions that arise during instruction, and to how teachers' instructional practices for culturally responsive MM develop over time.

Some teachers in our study experienced tensions related to the potential of MM tasks to reproduce status inequities in their classrooms. These challenges imply important implications for designing professional development programs and instructional tools to support teachers' work. For example, our findings suggest that elementary teachers benefit from opportunities to reflect on instructional challenges and brainstorm possible responses with colleagues and facilitators. Professional development should also include tools and instructional resources (e.g., scaffolding strategies, question prompts) that target the challenges teachers expressed, including how to support equitable participation with emerging bilingual students, and ways to structure small group work to minimize status issues. Finally,

mathematics teacher educators who facilitate teacher learning of culturally responsive MM need to consider their work in the context of institutional structures and norms, and teachers' current beliefs and mathematics teaching practices. Understanding how teachers' developing practices for culturally responsive MM are consistent with or diverge from the norms and practices where they work will help facilitators anticipate potential points of tension and co-construct strategies to successfully navigate tensions to challenge spaces of marginalization.

**Acknowledgements** The work here is supported by NSF DRL 2008997, 2010202, 2010269, and 2010178.

#### References

- Adiredja, A. P., & Louie, N. (2020). Untangling the web of deficit discourses in mathematics education. For the Learning of Mathematics, 40(1), 42–46.
- Aguirre, J., Mayfield-Ingram, K., & Martin, D. (2013). The impact of identity in K-8 Mathematics Learning and Teaching: Rethinking equity-based practices. National Council of Teachers of Mathematics.
- Aguirre, J. M., Suh, J. M., Tate, H., Carlson, M. A., Fulton, E. A., & Turner, E. E. (2022). Leveraging Equity and Civic Empathy through Community-Based Mathematical Modeling. North American Chapter of the International Group for the Psychology of Mathematics Education, 349–358. Retrieved from https://par.nsf.gov/biblio/10404296.
- Anhalt, C. O. (2014). Scaffolding in Mathematical modeling for ELLs. The Common Core State standards in Mathematics for English Language learners: Grades K-8 (pp. 111–126). Teaching English to Speakers of Other Languages (TESOL) International Association Publications.
- Anhalt, C. O., Staats, S., Cortez, R., & Civil, M. (2018). Mathematical modeling and culturally relevant pedagogy. *Cognition, Metacog*nition, and Culture in STEM Education: Learning, Teaching and Assessment, 307–330.
- Averill, R., Anderson, D., Easton, H., Maro, T., Smith, P., D., & Hynds, A. (2009). Culturally responsive teaching of mathematics: Three models from linked studies. *Journal for Research in Mathematics Education*, 40(2), 157–186.
- Barbosa, J. C. (2006). Mathematical modelling in classroom: A sociocritical and discursive perspective. *ZDM-The International Journal on Mathematics Education*, *38*(3), 293–301. https://doi.org/10.1007/BF02652812.
- Blum, W., & Leiß, D. (2005). Filling up—The problem of independence-preserving teacher interventions in lessons with demanding modelling tasks. In M. Bosch (Ed.), *Proceedings of 5th Congress European Research Mathematics Education*. Gerona: Universitat Ramon Llull.
- Boaler, J., & Staples, M. (2008). Creating Mathematical futures through an Equitable Teaching Approach: The case of Railside School. *Teachers College Record*, 110, 3, 608–645.
- Borromeo Ferri, R. (2021). Mandatory Mathematical Modelling in School: What do we want the teachers to know? In F. K. S. Leung, G. A. Stillman, G. Kaiser, & K. L. Wong (Eds.), Mathematical Modelling Education in East and West. International perspectives on the teaching and learning of Mathematical Modelling. Springer.



- Brady, C., Jung, H., Alejandro, J., Coleman-King, C., & de Araujo, Z. (2023). Engineering connections in culturally-responsive mathematical modeling problems Presentation at the Annual Conference of the American Society of Engineering Education, Baltimore. MD.
- Brown, K. (2008). Employing mathematical modeling to respond to indigenous students' needs for contextualized mathematics problems. In M. Goos, R. Brown, & K. Makar (Eds.) *Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 93–99).
- Carlson, M. A., Wickstrom, M. H., Burroughs, E. A., & Fulton, E. W. (2018). A case for modeling in the elementary school classroom. In C. R. Hirsch, A. Roth, & McDuffie (Eds.), Annual perspectives in mathematics education: Mathematical modeling and modeling mathematics (pp. 121–130). National Council of Teachers of Mathematics.
- Carpenter, T., Fennema, E., Franke, M., Levi, L., & Empson, S. (2014). Children's Mathematics: Cognitively Guided Instruction, 2nd Edition. Portsmouth, NH.
- Caswell, B., Esmonde, I., & Takeuchi, M. (2011). Towards culturally relevant and responsive teaching of mathematics. In C. Rolheiser, M. Evans, & M. Gambhir (Eds.), *Inquiry into practice: Reaching* every student through inclusive curriculum practices (pp. 64–71). Ontario Institute for Studies in Education.
- Chen, G. A., & Horn, I. S. (2022). A call for critical bifocality: Research on marginalization in Mathematics Education. *Review of Educational Research*, 92(5), 786–828.
- Cirillo, M., Bartell, T., & Wager, A. (2016). Teaching mathematics for social justice through mathematical modeling. In C. Hirsrch, A. Roth, & McDuffie (Eds.), *Mathematical modeling and modeling Mathematics* (pp. 87–96). APME.
- Civil, M. (2007). Building on community knowledge: An avenue to equity in mathematics education. *Improving access to mathematics: Diversity and equity in the classroom*, 105–117.
- Common Core State Standards Initiative (2010). Common core state standards for mathematics. Washington, DC: National Governors Association Center for Best Practices and Council of Chief State School Officers. Retrieved from https://www.thecorestandards.org/wp-content/uploads/Math Standards1.pdf.
- Consortium for Mathematics and its Applications (COMAP) and Society for Industrial and Applied Mathematics (SIAM) (2016). Guidelines for assessment & instruction in mathematical modelling education. Authors. https://www.comap.com/Free/GAIMME/.
- English, L. D. (2009). Promoting interdisciplinarity through mathematical modelling. ZDM-The International Journal on Mathematics Education, 41(1), 161–181.
- English, L. D., & Watters, J. J. (2005). Mathematical modeling in the early school years. *Mathematics Education Research Journal*, 16, 58–79.
- Featherstone, H., Crespo, S., Jil, L., Oslund, J., Parks, A., & Wood, M. (2011). Smarter together! Collaboration and equity in the Elementary Math Classroom. National Council of Teachers of Mathematics.
- Gay, G. (2000). Culturally responsive teaching: Theory, research, and practice. Teachers College Press.
- González, N., Moll, L. C., & Amanti, C. (2005). Funds of knowledge: Theorizing practices in households, communities, and classrooms. Routledge.
- Gutstein, E. (2006). Reading and writing the world with mathematics: Toward a pedagogy for social justice. Taylor & Francis.
- Hunter, J., & Miller, J. (2022). The use of cultural contexts for patterning tasks: Supporting young diverse students to identify structures and generalise. ZDM-Mathematics Education, 54, 1349–1362.
- Hunter, R., Hunter, J., Anthony, G., & McChesney, K. (2018). Developing Mathematical Inquiry communities: Enacting culturally

- responsive, culturally sustaining, ambitious Mathematics Teaching. SET: Research Information for Teachers, 2, 25–32.
- Kaiser, G. (2007). Modelling and modelling competencies in school. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modelling: Education, engineering and economics*(pp. 110–119). Horwood.
- Kaiser, G. (2017). The teaching and learning of mathematical modelling. In J. Cai (Ed.), Compendium for Research in Mathematics Education (pp. 267–291). National Council of Teachers of Mathematics.
- Kaiser, G., Schwarz, B., & Buchholtz, N. (2011). Authentic modelling problems in Mathematics Education. In G. Kaiser, W. Blum, B. Ferri, & R. Stillman, G. (Eds.), Trends in Teaching and Learning of Mathematical Modelling. International perspectives on the teaching and learning of Mathematical Modelling (Vol. 1). Springer. https://doi.org/10.1007/978-94-007-0910-2 57.
- Ladson-Billings, G. (1995). Toward a theory of culturally relevant pedagogy. American Educational Research Journal, 32(3), 465–491.
- Lesh, R., & Doerr, H. M. (Eds.). (2003). (Eds.). Beyond constructivism: Models and modelling perspectives on mathematics problem solving, learning, and teaching. Lawrence Erlbaum.
- Maaß, K. (2006). What are modelling competencies? *ZDM-The International Journal on Mathematics Education*, 38(2), 113–142.
- Nicol, C., Archibald, J. A., & Baker, J. (2013). Designing a model of culturally responsive mathematics education: Place, relationships and storywork. *Mathematics Education Research Journal*, 25, 73–89.
- Paolucci, C., & Wessels, H. (2017). An examination of preservice teachers' capacity to create mathematical modeling problems for children. *Journal of Teacher Education*, 68(3), 330–344.
- Saldaña, J. (2021). The Coding Manual for qualitative researchers.SAGE Publications Limited.
- Sengupta-Irving, T. (2021). Positioning and positioning apart: Mathematics learning as becoming undesirable. *Anthropology and Education*, 52, 2, 187–208. https://doi.org/10.1111/aeq.12378.
- Smith, M. S., & Stein, M. K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School*, *3*, 344–350.
- Stender, P., & Kaiser, G. (2015). Scaffolding in complex modelling situations. *ZDM- Mathematics Education*, 47(7), 1255–1267.
- Suh, J. M., Tate, H., Rossbach, M., Green, S., Matson, K., Aguirre, J. M., Seshaiyer, P., & Steen, S. (2023). Dilemmas and Design principles in planning for justice-oriented community-based Mathematical modeling lessons. *Mathematics Teacher Educator*, 11(3), 210–230.
- Tate, H., Proffitt, T., Christensen, A., Hunter, C., Stratton, D., Fleshman, E., Aguirre, J., & Suh, J. (2022). Mathematizing representation in children's libraries: An anti-racist Math Unit in Elementary grades Teaching for Excellence and Equity in Mathematics—Special Issue: Anti-racism in Mathematics Education. TODOS:Mathematics for all, 13(1), 23–40.
- Turner, E., Bennett., A., Granillo, M., Ponnuru, N., McDuffie, R., Foote, A., Aguirre, M., J., & McVicar, E. (2022). Authenticity of elementary teacher designed and implemented mathematical modeling tasks. *Mathematical Thinking and Learning*. https://doi.org/10.1080/10986065.2022.2028225.
- Turner, E., Suh, J., Tate, H., Sotelo, D., Carlson, M. A., Aguirre, J., & Fulton, E. A. (2023). Cultivating equity and empathy in community-focused elementary math modeling. In M. Strutchens, G. Krause, D. White & J. Bay-Williams (Eds.), Antiracist mathematics education: Stories of acknowledgement, action and accountability. TODOS: Mathematics Education for All.
- Zavala, M. R., & Aguirre, J. M. (2023). Cultivating mathematical hearts: Culturally responsive math teaching in elementary classrooms. Corwin.

**Publisher's Note** Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Springer Nature or its licensor (e.g. a society or other partner) holds exclusive rights to this article under a publishing agreement with the author(s) or other rightsholder(s); author self-archiving of the accepted manuscript version of this article is solely governed by the terms of such publishing agreement and applicable law.

