Backward Transfer and Students' Performance on Linear Functions Problems

Sara Gartland, Charles Hohensee, & Crystal Collier
University of Delaware

This study examines the phenomenon of backward transfer in the context of high school students learning and reasoning about linear and quadratic functions. Using quantitative methods, this study provides statistical evidence that it is possible to produce intended productive backward transfer effects on students' prior ways of reasoning about linear functions with quadratic functions instruction that emphasizes quantitative and covariational reasoning. Using qualitative methods, this study characterizes the quality of the backward transfer effects on students' quantitative and covariational reasoning. The significance of these results is that if intended productive backward transfer is possible, then it represents a new way for mathematics education to be improved.

Backward transfer, defined as when learning about new concepts influences how students reason about previously learned about concepts (Hohensee, 2021), is a byproduct of learning. Research has shown that sometimes this byproduct is *unintended* (e.g., Hohensee et al., 2021), but also that it can be *intended* (e.g., Hohensee, 2014). Additionally, evidence has shown that sometimes backward transfer is *unproductive* and sometimes *productive* (i.e., when new learning undermines or enhances students' ways of reasoning about previously learned about concepts, respectively). For instance, Lima and Tall (2014) found students who tried to misapply the newly learned quadratic equation to a previously learned linear functions problems (unproductive), whereas Moore (2021) found a student with new understanding of rise over run after they engaged in new learning about graphing on unconventional axes (productive). In the backward transfer study reported here, we were focused on productive and intended backward transfer.

Much of our backward transfer research has been situated in the context of linear and quadratic functions (e.g., Hohensee et al., 2021). In our other studies, linear functions was the topic that was previously learned about and quadratic functions the topic for new learning. Therefore, we identified students who had already learned about linear functions and had yet to learn about quadratic functions and examined their ways of reasoning about linear functions before and after they participated in a unit of new instruction on quadratic functions. The goal was to identify changes in linear function reasoning that the instruction may have produced.

In the backward transfer research described above, we focused exclusively on qualitative analyses of data to identify and characterize backward transfer effects. For example, we qualitatively compared students' *covariational reasoning* (Carlson et al., 2002) about linear functions before and after they participated in quadratic functions instruction to characterize changes in those students' covariational reasoning (Hohensee et al., 2021; Hohensee et al., 2023). We also qualitatively compared students' *action* versus *process* conceptions of linear functions (Briedenbach et al., 1999) before and after they participated in quadratic functions instruction to characterize changes in those students' conceptions of linear functions (Hohensee et al., 2022). In all three studies, qualitative analyses revealed changes in the quality of students' prior ways of reasoning about linear functions.

An important new aspect of the research being reported here is that we made a shift from purely qualitative analyses of student reasoning toward mixed-methods (i.e., qualitative and quantitative). The goal of this mixed-methods analysis of new data was to examine whether quadratic functions instruction that was associated with qualitative changes in students' prior ways of reasoning about linear functions was also associated with statistically significant changes in student performance on linear functions problems. Our rationale for conducting this backward transfer research is that it could lead to a better understanding of how to produce more intended productive backward transfer and less unintended unproductive backward transfer, which would benefit the field of mathematics education.

Theoretical Foundation

The theoretical foundation on which this study was constructed is composed of theory on the *transfer of learning*, and foundational ideas about reasoning about functions.

Transfer of Learning

The transfer of learning has been a long sought-after educational outcome (e.g., Woodward & Thorndike, 1901). Many studies have been conducted on transfer and, for much of that research, the traditional transfer orientation has guided inquiry. According to the tradition orientation, transfer is conceptualized as when knowledge acquired in one context is correctly applied to a novel context (Singley & Anderson, 1989). However, a number of researchers have for various reasons called the traditional orientation to transfer into question, including because it is a relatively blunt instrument that is not sensitive to the many nuanced ways that knowledge acquired in one context could play a role in a novel context (Lobato, 2012).

In response to questions raised about the traditional orientation to transfer, several progressive orientations were recently developed (i.e., in the last 20 years). The progressive orientation most relevant to our study was the *actor-oriented transfer* (AOT) perspective (Lobato, 2012). According to this orientation, transfer is conceptualized as an influence on reasoning rather than as an application of knowledge. Specifically, transfer is defined as "the influence of a learner's prior activities on her activity in novel situations" (p. 233). Among other things, this orientation takes a much more nuanced view of the transfer phenomenon.

Interestingly, it was during transfer research from an AOT perspective that we first became aware of what we began calling *backward transfer* (Hohensee, 2014). In particular, when considering the nuanced ways that prior knowledge was influencing students reasoning about novel contexts, we noticed that the prior knowledge itself was sometimes being influenced. This led us to first consider the influence that reasoning in a new context (e.g., such as when learning about a new concept) can have on prior knowledge (e.g., such as on one's ways of reasoning about previously learned about concepts) as itself a focus of study. We think of backward transfer as the backward version of AOT because we too are interested in more than the application of knowledge. We are interested in influences that one kind of knowledge can have other reasoning.

Reasoning About Functions

Two foundational ideas related to reasoning about functions that figured prominently in our study were *quantitative reasoning* and *covariational reasoning* about functions.

Quantitative reasoning about functions. Smith and Thompson (2007) defined *quantities* as "attributes of objects or phenomena that are measurable" (p. 101) and *quantitative reasoning* as "conceptualiz[ing], reason[ing] about, and operat[ing] on quantities and relationships in sensible problem situations" (p. 95). Quantitative reasoning is important for reasoning about functions and was foundational for our study because, as Ellis (2011) states, "adopting a quantitative reasoning approach can support students' meaningful engagement with algebra in general and with functions in particular" (p. 218).

Covariational reasoning about functions. Confrey and Smith (1991) defined *covariation* as "one quantity changes in a predictable or recognizable pattern, the other also changes, typically in a differing pattern . . . how x_1 changes to x_2 and how y_1 changes to y_2" (p. 57). Carlson et al. (2002), in turn, defined *covariational reasoning* as "the cognitive activities involved in coordinating two varying quantities while attending to the ways in which they change in relation to each other" (p. 354). Covariational reasoning was foundational for our study because how quantities change in relation to each other is at the heart of what makes linear functions linear and quadratic functions quadratic (Confrey & Smith, 1994).

Research Question

We can now state the research question that guided our study: *How do students' performance on a linear functions assessment change from pre- to post-test if, between the two tests, students participate in a quadratic functions unit that emphasizes quantitative and covariational reasoning?*

Methods

Data collection occurred during a 2.5-week program focused on quadratic functions, and that emphasized quantitative and covariational reasoning. It took place at a Mid-Atlantic university during Summer 2022. Twenty-five students in grades 9-10, who had all recently completed a version of Algebra I during the school year, volunteered to participate in the summer program. Students participated in audio/video-recorded instructional sessions for 2.5 weeks. The students also took pre- and post-tests before and after the 2.5-week instructional unit.

The pre- and post-test both consisted of three linear functions problems and one quadratic functions problem. Each linear functions problem targeted a different linear function representation (i.e., graph, table, pictorial). The quadratic functions problem involved a table representation of a quadratic function because the summer program focused on data tables.

Three teachers (the first and second author and a high school mathematics teacher) provided instruction during two 1-hour instructional sessions per day during the summer session. Each instructional session focused on one activity involving a computer animation of a moving character on SimCalc Mathworlds software that allowed students to reason quantitatively and covariationally with various features of distance-time quadratic functions. Most activities required students to record coordinated time and distance pairs in data tables, and the tables served as a structure for supporting and organizing their quantitative and covariational reasoning.

We developed a three-point rubric for each test part to assess student performance on the pre- and post-tests (i.e., each problem was part out of 3) and met to discuss each part of the rubric and to practice grading. Next, we randomly distributed students among the three authors of this proposal for first round grading. Then, we each did second round grading of 20% of the test parts to check reliability. We reached 85% agreement or greater on all test parts and resolved all disagreements. To test for statistically significant differences, we ran a paired sample t-test on our graded pre- and post-tests. Our ongoing qualitative analysis involves coding for changes in students' reasoning using a priori codes about quantitative and covariational reasoning from our prior research, as well as new codes that emerge from the data.

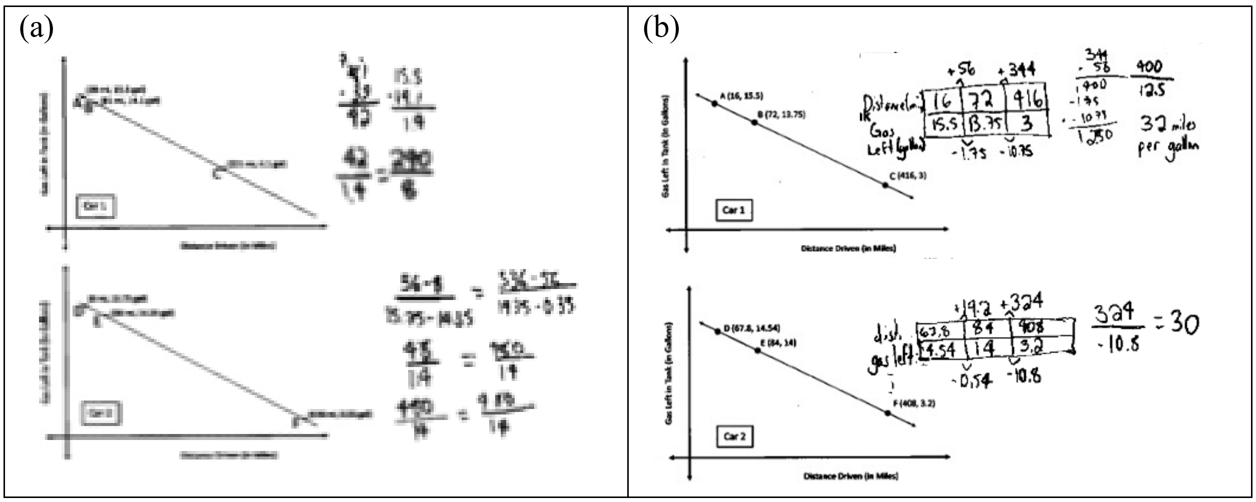
Results

On average, students showed a statistically significant increase from pre- to post-test (t = 2.334, df = 24, p = 0.028). Because the pre- and post-tests included three linear functions problems, this result suggests that learning about quadratic functions during a program focused on quantitative and covariational reasoning had a productive backward transfer influence on students' ways of reasoning about linear functions. Additionally, because the pre- and post-tests included one quadratic functions problem, this result suggests that students also made progress toward achieving the learning goals for the quadratic functions program.

To supplement the quantitative result above, we also include emerging results from ongoing qualitative analysis of student work on the pre- and post-tests. According to our analysis, students post-test reasoning includes more quantitative and covariational reasoning than their pre-test reasoning. The case of Samad (a pseudonym) illustrates that the quality of these changes. On the pre-test, Samad began using the slope formula but did not find rates. Moreover, Samad did not attend to whether changes in quantities were positive or negative (see Figure 1a). On the post-test, Samad more closely tracked the changes in both variables by creating data tables, he attended more to whether the changes in quantities were positive or negative, and he found rates (see Figure 1b). We conceptualized these changes in reasoning as examples of intended productive backward transfer (i.e., with our instruction we intended to influence productive changes). Other students in our data set exhibited similar changes in reasoning.

Figure 1

Sample of Samad's Pre-Test Responses (1a) and Post-Test Responses (1b).



Discussion

This study is the first we know of to provide statistically significant evidence of intended productive changes in students' performance on linear functions problems after students have participated in an instructional unit on quadratic functions. This result is significant for the field because producing intended backward transfer represents be a new way that mathematics education could be improved. Moreover, because backward transfer effects are by-products of new instruction, they are produced without dedicating additional instructional time.

Now that this study has provided quantitative evidence that intended productive backward transfer effects are possible, more research is needed to understand the underlying mechanisms, to develop activities that reliably produce these effects, to examine the production of intended productive backward transfer effects in other content domains, and so on. Although an abundance of research on transfer already exists, our study provides motivation for much more mathematics education research on backward transfer as well.

Attendee Engagement

During our presentation, we will engage attendees in examining with a partner specific pre- and post-test responses to consider how student responses to linear functions problems in our data changed from pre- to post-test i. We plan on doing this twice (5 minutes each). Each time, we will end the attendee engagement time by presenting our interpretations of the changes and how those changes were associated with the quadratic functions instruction that students participated in. Our learning goal for these attendee engagement activities will be to help attendees develop a better understanding and more awareness of productive backward transfer.

References

Breidenbach, D., Dubinsky, E., Hawks, J., & Nichols, D. (1992). Development of the process conception of function. *Educational Studies in Mathematics*, *23*(3), 247–285. https://doi.org/10.1007/BF02309532

Carlson, M., Jacobs, S., Coe, E., Larsen, S., & Hsu, E. (2002*).* Applying covariational reasoning while modeling dynamic events: A framework and a study. *Journal for Research in Mathematics Education, 33*(5), 352–378. https://doi.org/10.2307/4149958

Confrey, J., & Smith, E. (1991). A framework for functions: Prototypes, multiple representations, and transformations. In R. Underhill & C. Brown (Eds.), *Proceedings of the Thirteenth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 57–63). Blacksburg, VA: Virginia Polytechnic Institute & State University.

Confrey, J., & Smith, E. (1994). Exponential functions, rates of change, and the multiplicative unit. *Educational Studies in Mathematics*, *26*(2/3), 135–164. https://doi.org/10.1007/BF01273661

Ellis, A. B. (2011). Algebra in the middle school: Developing functional relationships through quantitative reasoning. In J. Cai & E. Knuth (Eds.), *Early algebraization: A global dialogue from multiple perspectives advances in mathematics education* (pp. 215–235). New York, NY: Springer.

Hohensee, C. (2014). Backward transfer: An investigation of the influence of quadratic functions instruction on students' prior ways of reasoning about linear functions. *Mathematical Thinking and Learning, 16*(2), 135–174. https://doi.org/10.1080/10986065.2014.889503

Hohensee, C. (2021). A case for theory development about backward transfer. In C. Hohensee & J. Lobato (Eds*.), Transfer of learning: Progressive perspectives for mathematics education and related fields* (pp. 81–102). Dordrecht, The Netherlands: Springer. https://doi.org/10.1007/978-3-030-65632-4_4

Hohensee, C., Gartland, S., Willoughby, L., & Melville, M. (2021). Backward transfer influences from quadratic functions instruction on students' prior ways of covariational reasoning about linear functions. *Journal of Mathematical Behavior*, *61*. https://doi.org/10.1016/j.jmathb.2020.100834

Hohensee, C., Willoughby, L., & Gartland, S. (2022). Backward transfer effects on ways of reasoning about linear functions with instruction on quadratic functions. *Mathematical Thinking and Learning*. https://doi.org/10.1080/10986065.2022.2037043

Hohensee, C., Melville, M., Collier, C. L., & Ma, Y. (2022). *Backward transfer influences from quadratic functions instruction on levels of linear function reasoning abilities.* Manuscript under review.

Lima, R. N., & Tall, D. O. (2008). Procedural embodiment and magic in linear equations. *Educational Studies in Mathematics*, *67*(1), 3–18. https://doi.org/10.1007/s10649-007-9086-0

Lobato, J. (2012). The actor-oriented transfer perspective and its contributions to educational research and practice. *Educational Psychologist, 47*(3), 232–247. https://doi.org/10.1080/00461520.2012.693353

Moore, K. C. (2021). Graphical shape thinking and transfer. In C. Hohensee, J. Lobato (Eds.), *Transfer of learning, research in mathematics education* (pp. 145–171). https://doi.org/10.1007/978-3-030-65632-4_7

Singley, M. K. & Anderson, J. R. (1989). *The transfer of cognitive skill*. Cambridge, MA; Harvard University Press.

Smith, J., & Thompson, P. W. (2007). Quantitative reasoning and the development of algebraic reasoning. In J. J. Kaput, D. W. Carraher & M. L. Blanton (Eds.), *Algebra in the early grades* (pp. 95–132). New York: Erlbaum.

Woodworth, R. S., & Thorndike, E. L. (1901). The influence of improvement in one mental function upon the efficiency of other functions. (I). *Psychological Review, 8*(3), 247–261. https://doi.org/10.1037/h0074898