Characterizing Mathematics Teacher Learning Patterns Through Collegial Conversations in a Community of Practice

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Abstract

We examined secondary (6-12) mathematics teachers' participation in a professional development (PD) model where they collectively investigated video cases of students engaging with ambitious instructional materials. We leveraged frame analysis, frame processes, and the Teaching for Robust Understanding framework to characterize the learning of professional learning communities. We found that teacher learning was supported within collegial environments where teachers respectfully challenged or transformed ideas on how to solve problems of practice. Our findings highlight how engagement in a PD model supports teachers in establishing participation and reification patterns that encourage them to engage collegially, justify their positions, and align to ambitious teaching practices. These findings implicate a need for mathematics education leadership communities to take action to support collegial conversations in PD intentionally.

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Teachers have constrained opportunities to systematically develop and share ideas about their practice (Ball et al., 2014). Even when teachers investigate teaching practice together, the mathematics education leadership community is limited in capturing their ideas so that they can be used and improved upon by others at scale (Hiebert et al., 2002). The decentralized nature of public education, coupled with the reluctance or inability to share ambitious teaching ideas, is a persistent problem and has been posited as a primary obstacle to improving American education (Charalambous & Delaney, 2020; Dewey, 1929).

This problem is important as the mathematics education leadership field continues to develop standards, assessments, and instructional materials that move teachers past lecture-based, teacher-centered instruction towards engaging students regularly in activities involving conceptual thinking, complex problem-solving, and mathematical discussions (National Council of Teachers of Mathematics [NCTM], 2014, 2018, 2020; Porter et al., 2011; Stigler & Hiebert, 1997). With this ambitious vision of mathematics instruction, there is a strong need for mathematics education leadership to provide opportunities to ground the work of teacher learning in the classroom (Gallagher, 2016; Kazemi & Hubbard, 2008). Such opportunities must also empower teachers to leverage their experiences in developing shared professional knowledge about the teaching and learning of mathematics (Hiebert & Stigler, 2017).

Professional development (PD) can be key in supporting instructional shifts that deepen learning opportunities for students (Rosli & Aliwee, 2021; Sztajn et al., 2017). Mathematics education leadership can leverage PD as a natural mechanism to empower teacher learning and contribute to a knowledge base that supports ambitious instruction. Ambitious instruction establishes "learning environments from which students emerge as agentive, knowledgeable, and resourceful thinkers and problem solvers" (Schoenfeld, 2023, p. 165). As such, this work aims to provide insight into the creation of learning spaces that can help teachers create powerful and transformative mathematics classrooms. Our work is based within a research-practice partnership that integrates key elements of coherent instructional systems within a PD model for secondary teachers. An important element of such systems is the use of ambitious instructional resources developed to support powerful mathematics teaching. Another key element of our PD model is the collective investigation of video cases featuring students engaging with these instructional resources.

In this paper, we explore how evidence of teacher learning manifests in sustained PD sessions focused on implementing mathematics instructional resources effectively. We employ a theoretical perspective of a community of practice (CoP) while incorporating principles of effective PD to understand the collective learning that occurs as these professional communities engage in both congenial and collegial dialogue. Thus, the research question guiding our work is: *How does learning about mathematics teaching practices manifest within a CoP during a PD model focused on the collective investigation of video cases of students engaging with ambitious instructional materials?*

Theoretical Perspective and Background

We draw on sociocultural theory to study the ways in which a community of secondary mathematics teachers engages in PD focused on ambitious mathematics teaching practices. The following sections will review the literature on the theory of learning within a CoP, PD that supports such learning, and a research-based framework that details powerful mathematics teaching practices. Furthermore, we describe the nature of congenial and collegial conversations and their relationships to teacher learning in PD settings.

Socioculturalism and Learning Within a Community of Practice

Sociocultural theorists (Brown et al., 1989; Collins et al., 1988; Lave & Wenger, 1991) argue that learning is inseparable from the activity, context, and culture in which it takes place because learning occurs through social engagement (Cobb & Yackel, 1996). Socioculturalism regards learning as participation in cultural practices and social engagements that enable learners to participate in the activities of the expert (Cobb, 1994). Furthermore, this perspective views knowing as a way of speaking and acting within cultural practices (Goos et al., 1999). According to Forman (1996), in order to facilitate learning, it is necessary to have "access to meaningful practice in a community" (p. 117) rather than focusing on instructional resources or materials (e.g., textbooks) that individual learners may use to internalize knowledge. Broadly, Lave and Wenger (1991) depict learning from this perspective as the legitimate peripheral-to-full participation in a CoP.

Communities of practice are groups of people who mutually engage in an activity, are connected by a joint enterprise, and engage with a shared repertoire of resources (Wenger, 1998a, 1998b). A CoP consists of learners, such as newcomers and more-knowledgeable others, moving from peripheral-to-full participation (Kelly, 2006; Lave & Wenger, 1991). For example, Lave and Wenger describe clothing tailors as newcomers who may learn how to cut out cloth first before learning other steps, such as sewing by hand or using a sewing machine. As the newcomers participate in a CoP of clothing tailors by learning how to perform each step of tailoring, the peripheral participation of the newcomers moves to full participation by producing a garment. In the context of teaching communities, a new teacher can enter a department as an outsider and begin by observing the normal interactions and discourse within department meetings, possibly offering passive agreement to others' discussions. Over time, this type of peripheral participation can shift towards full participation as they learn the communication norms and can authentically contribute to discussions and possibly challenge others.

Within a CoP, evidence of learning occurs through patterns of participation and reification (Wenger, 1998a, 1998b). Wenger defines participation as the experiential process of taking part in a CoP. Reification gives form to that experience through "objects that congeal this experience into 'thingness'" (1998b, p. 58). A CoP is constantly evolving in its mutual engagement among members, and the evolution of such mutual engagement can form patterns of participation indicative of the community's collective learning process. In teaching, CoPs allow members to address challenges that arise in their instructional practice by affording space to create reflective professional narratives. Professional narratives highlight practice and professional knowledge and reveal insight into cultural values (Allard et al., 2007). Because collective participation in creating professional narratives occurs through dialogue, patterns of participation in a CoP are noted as patterns that emerge in that dialogue. Participants in a CoP can create new patterns by changing how they engage in conversations within that community from one of "respectful turn taking and individual turns of talk" (Bannister, 2015, p. 357) to ones in which participants press each other for justification and ask clarifying questions in order to co-construct understanding. These changes are reified by specific community actions, including when the participants focus their discussions on a particular shared repertoire, such as a framework for best teaching practices or powerful lessons, to enhance their understanding.

Professional Development and its Design Elements

In the context of teaching and teachers, CoPs, known as professional learning communities (PLCs), can be designed and enacted by teacher leaders as an effective PD form that provides opportunities for participants to collaborate and learn. From a sociocultural perspective, PLCs are CoPs because community members are (i) mutually engaged in a communal activity of learning about and reflecting on teaching, (ii) connected by a joint enterprise to improve teaching practice, and (iii) engaged with a shared repertoire of resources, such as regular instructional routines or a common curriculum (Wenger, 1998a, 1998b). Moreover, as a PD structure, PLCs can align closely to the five elements of effective PD identified by Garet et al. (2001): content focus, active learning, coherence, sustained duration, and collective participation.

Content-focused PD grounds participants in subject matter content and focuses on how students learn that particular content (Desimone, 2011; Desimone & Garet, 2015). Content-based PD is often situated in teachers' classrooms, allowing teachers to study students' work, try new curricula, or study a particular element of pedagogy or student learning (Darling-Hammond et al., 2017). Borko et al. (2008) studied a group of teachers in a learning community focused on video cases in which all seven sessions revolved around a different mathematical task. Participants focused on aspects of the teacher's role during the enactment of mathematics tasks as well as students' mathematical reasoning with the tasks. In this PD, focused on specific mathematical content (e.g., proportional reasoning or ratios), teachers diligently worked with teacher leaders to understand the videotaped students' solution strategies, even when they did not align with any of the proposed teacher strategies. Teachers expressed that the content topics covered were meaningful, motivating the participants to learn, improve their practice, and better serve their students. Also, they found that the teachers' conversations changed to focus more on mathematical content as the PD progressed. From a CoP perspective, teachers in this PLC were able to refine their understanding of the content or shared repertoire collaboratively.

Active learning in PD refers to “opportunities for teachers to observe, receive feedback, analyze student work, or make presentations, as opposed to passively listening to lectures” (Desimone & Garet, 2015, p. 253). Active learning experiences in PD move teacher leaders away from traditional lecture modalities and instead engage teachers directly in practice connected to their classrooms and students (Darling-Hammond et al., 2017). Active learning in PD often incorporates collaboration, coaching, feedback, and modeling. It can also include analysis of student artifacts and video clips from actual mathematics classrooms. For instance, Alles et al.’s (2018) study of PD incorporated active learning by engaging teachers and teacher leaders in a learning community who worked collectively to incorporate strategies discussed in the PD into teacher planning, videotaping teacher lessons, and analyzing these lessons as a community. They found that teachers engaged in this PD showed a significant positive change in their dialogue practices in their classrooms compared to teachers who participated in a one-time traditional PD program. Similarly, Borko et al. (2008) incorporated active learning in their PD study of mathematics teachers through a two-year-long program utilizing the Problem-Solving Cycle model, which analyzed video from teachers’ classrooms. The active learning in this context manifested in the PD’s focus on teacher planning, implementing, and analysis of their classroom lessons. They found that, over time, teachers’ conversations became “more focused, in-depth, and analytical” (p. 432). Patterns emerged about how teachers in both of these PLCs participated and reified concepts, specifically from changes in their engagement within the PD and their teaching practices.

Coherence describes the alignment of the PD content with other aspects of a teacher’s profession. Such PD grounds teacher learning in their classroom, school, and district contexts (Kazemi & Hubbard, 2008). Thus, coherent PD content addresses teachers’ curriculum, builds on prior teacher learning, and focuses on sustained and collaborative communication with other teachers in similar contexts. Coherent PD experiences should also be relevant to teachers' belief systems, school initiatives, and policies (Desimone, 2011; Garet et al., 2001), and support local teacher and school needs and interests (Bayar, 2014; Koellner et al., 2011). For instance, PD has been found to be more successful when coherently linked to classroom lessons. Smith et al. (2020) studied 24 teachers in a PLC from a single district that participated in PD centered on a model of professional learning in which teachers collaboratively planned and reflected on lessons they were concurrently teaching during a summer school session. The study found that the PLC members found the PD to be coherent and relevant; as a result, their practice had changed by incorporating ideas from the PD.

For PD to have sustained duration, the sessions must occur regularly over extended periods of time and remain focused on the same learning goal. Research shows that traditional, one-day PD sessions, even if there is a brief follow-up, often do not produce the intended outcomes. Ross and Bruce (2007) studied teacher learning between a group of teachers engaged in a one-day PD session with three short follow-up sessions and a control group who engaged in no PD. They found no significant difference between groups on all but one of the teacher efficacy variables and inferred the limited duration of the PD program as a way to explain this finding. Other researchers have found more sustained durations of PD to be more effective, yet the suggested duration has varied. Garet et al. (2001) suggest that teachers work together for at least one semester and have a minimum contact time of 20 hours. Yoon and colleagues (2007) found that effective PD programs averaged 49 hours of contact time. It is also important to note that more time does not guarantee more effective PD. "Time must be well organized, carefully structured, and purposefully directed" (Guskey, 2003, p. 749). For example, Santagata and

Bray's (2016) study focused on a learning community of teachers studying student mathematical errors, and illustrated how a sustained duration of PD can be designed and implemented effectively. In this PD, teachers met for two full days at the beginning of the PD and then monthly for the remaining six months of the school year. At each meeting, teachers jointly planned lessons and engaged in video analysis of teachers' enactment of lessons. Findings indicated that the sustained duration helped the teachers grow in their understanding of students' mathematical misconceptions and refine their practices.

Collective participation within PD refers to groups of teachers that share a common interest. PD should provide collective experiences for groups of teachers with similar needs and challenges (Desimone & Garet, 2015), such as teachers from the same grade, subject, or school. When such groups participate in PD activities together, they build an interactive learning community (Desimone, 2011) which can allow for more "collaboration, integration, and targeting of specific student needs" (Smith et al., 2020, p.81). For example, van Es and Sherin's (2008) study of PD with mathematics teachers illustrated the collective participation of teachers working towards the concept of noticing through mutual engagement in a video club. All participants in this study were mathematics teachers from the same district, taught similar grade levels, and were in the third year of implementing a new reform curriculum. Throughout the PD, each teacher shared video clips of their classroom activities (e.g., whole class discussion, small group work), and their peers analyzed and discussed the clips to learn to notice and interpret students' mathematical thinking. Through the teacher leaders' intentional design of this PD, teachers' patterns of participation changed, wherein participants increasingly attended to detailed noticing of students' mathematical thinking. From a CoP perspective, participation in this PD helped teachers reify the concept of professional noticing in mathematics classrooms.

As argued above, PLCs can be designed and enacted by teacher leaders as an effective form of PD that emphasizes collaborative learning and can often align closely with Garet et al.'s (2001) five elements of effective PD. Threaded through all the characteristics of effective PD, teachers consistently engage in dialogue about mathematics teaching and learning. However, how teachers engage in such dialogue is also an important component impacting the effectiveness of a PD endeavor.

Congenial and Collegial Conversations

Within PD sessions, members of a CoP participate through dialogue. That dialogue generally takes the form of congenial or collegial conversation. Congenial conversations focus on politeness and privacy and are generally agreeable (Evans, 20012). Within PD sessions, congeniality could be one teacher suggesting a particular teaching move and another teacher cordially agreeing with that suggestion, regardless of their true opinion. In contrast, collegial conversations focus on constructive disagreements, development, and performance around practice (Evans, 2012). True collegiality requires more than being cordial and caring; it means examining ideas and problems of practice safely, where teachers can speak their truth without fear of repercussion (Zepeda, 2020). Within PD sessions, collegiality could be teachers disagreeing with all or some parts of their and others' suggestions for practice, which offers opportunities for members of the teacher community to suggest and argue for something different. Collegial conversations do not always mean disagreement; a collegial conversation could be one in which a community member creates a new understanding based on a posited idea.

To create a culture of growth in a PLC, teacher leaders must encourage teacher conversation that embraces collegiality because doing so authentically respects both similarities and differences (Zepeda, 2020). Collegial conversations are a catalyst for PLCs to reify their patterns of participation because these conversations allow teachers to build on or challenge each other's understanding by respecting different perspectives. In other words, collegial conversations entail deep discourse that promotes learning in a PLC. In order for communities to shift from congenial to collegial conversations, it is necessary for there to be shared repertoires for eliciting different ideas and feedback from all teachers in a PLC (Nelson et al., 2010). Collegial conversations are sociocultural because such dialogue can manifest itself as community members engage with "evolving forms of mutual engagement," "understanding and tuning their enterprise," and "developing their repertoire, styles, and discourses" (Wenger, 1998b, p. 95). Borko (2004) argues that in order to create successful learning communities, we need to create norms of interaction that support teachers to take risks in their dialogue with each other. These norms allow teachers in a PLC to discuss and justify their true opinions without the fear of dissimilar or dissenting ideas (Zepeda, 2020). In fact, recent research has shown that collegial conversations within a PLC can help teachers reify their understanding of powerful mathematics classrooms (Leonard et al., 2022).

Both congeniality and collegiality are necessary to create an effective PLC and should be actively supported by teacher leaders during PD. Congenial conversations help establish a safe space where members feel supported and their views are honored. Moreover, establishing such comfort amongst members can motivate collegial conversations, enabling the PLC to create new ideas and disagree or dissent constructively. However, not all congenial conversations lead to collegiality since the nature of congenial conversations is to avoid conflict and keep the status quo (Nelson et al., 2010; Selkrig & Keamy, 2015). Thus, in order to move from congenial to collegial conversations, the members in the PLC need to value communicative virtues, including:

> tolerance, patience, respect for differences, a willingness to listen, the inclination to admit that one may be mistaken, the ability to reinterpret or translate one's own concerns . . . , the self-imposition of restraint in order that others may "have a turn" to speak, and the disposition to express oneself honestly and sincerely. (Burbules & Rice, 1991, p. 411)

Since collegial conversations involve disagreement or different opinions, as well as new meanings or honest opinions, these communicative virtues are essential to foster authentic collegiality.

There is a caveat to the dichotomy between congenial and collegial conversations: conversations may not solely fall into congeniality or collegiality. According to Burbules and Rice (1991), different conversation forms can be categorized along the following spectrum: full agreement and consensus; partial agreement with a common understanding of different opinions; disagreement with a partial understanding of differences; disagreement with little understanding but with a respect for differences; and full disagreement without a respect for differences. This spectrum shows the complexity of conversation forms and that the classification of conversations is not absolutely dependent on the dichotomy between congenial and collegial conversations. Therefore, we interpret dialogue within PLCs as existing within a spectrum of congenial and collegial conversation. The conversation types that promote shifts from congeniality to collegiality will be discussed later in the data analysis section.

The Teaching for Robust Understanding Framework

An important aspect of a PLC comprised of mathematics teachers is the development of a shared repertoire built around best practices for teaching and learning mathematics. The

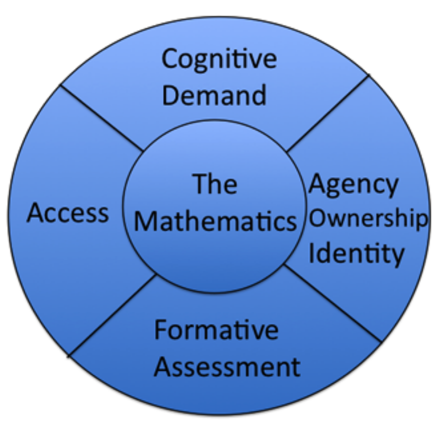
Teaching for Robust Understanding (TRU) framework creates an engaging and equitable educational experience for students and aligns the PLC's vision of ambitious instruction with what occurs in powerful classrooms (Schoenfeld, 2015). The TRU framework is informed by decades of research (see Schoenfeld, 2013 for some of the history of TRU) and details five interrelated dimensions (see Figure 1): The Mathematics; Cognitive Demand (CD); Equitable Access (EA); Agency, Ownership, and Identity (AOI); and Formative Assessment (FA). When established as the focal point of a PD program, the TRU framework supports teacher learning about classroom environments in which all students are supported in becoming independent mathematical thinkers (Schoenfeld & the TRU Project, 2016).

The Mathematics

Powerful mathematics classrooms are built on rich mathematical content with which students are able to engage in meaningful ways. Such content must focus on important mathematical ideas in a coherent manner (NCTM, 2000, 2014, 2018, 2020; National Governors Association [NGA], 2010; National Research Council, 2001), reflecting the deeply connected logical structure of mathematical concepts (Schmidt et al., 2005). Nearly as important as the

Figure 1

Teaching for Robust Understanding Framework (Schoenfeld, 2017)



content students encounter in their mathematics classrooms are the mathematical practices they use to engage with that content. When students use mathematical practices, such as making conjectures and constructing mathematical arguments to justify conclusions, they actively make connections to both their prior knowledge and other ideas in mathematics (Cuoco & McCallum, 2018; NGA, 2010). Understanding that grows from this connection-making is conceptual in nature (Hiebert, 2013; Rittle-Johnson et al., 2001), and is more easily applied in novel situations (Baroody et al., 2007; Brophy, 1999; Fries et al., 2021).

Cognitive Demand

The mathematical tasks with which students engage in classrooms set boundaries for how they are able to think about mathematical content, and the depth of disciplinary understanding they are able to achieve (Doyle, 1988). Tasks that are implemented with a consistently high level of CD afford students the opportunity to struggle productively, facilitating the development of conceptual understanding (DiNapoli & Morales, Jr., 2021; Hiebert & Grouws, 2007; Warshauer, 2015). Such tasks provide improved opportunities to learn (Jackson et al., 2013; Stein et al., 1996; Tekkumru-Kisa et al., 2020), are associated with higher student achievement (Boaler & Staples, 2008; Stigler & Hiebert, 2004), and challenge students to develop sophisticated solution strategies (Downton & Sullivan, 2017). When students struggle with high-level tasks, it is critical for teachers to provide support that does not lower the CD. This can take the form of supplying adequate time for the tasks, providing proper scaffolding, and modeling effective use of mathematical practices, such as using mathematical reasoning to support a claim (Smith & Stein, 2018). Research shows that these supportive learning environments can help students persevere in their in-the-moment problem solving and nurture their willingness to productively struggle over time (DiNapoli & Miller, 2022).

Equitable Access

Access to ambitious mathematical content and instruction is important for all students, and is essential to their academic and economic prospects (Moses & Cobb, 2001; NCTM, 2018, 2020). What have historically been characterized as differential outcomes in mathematical achievement associated with student gender, socioeconomic status, race, ethnicity, language, culture, and (dis)ability can more productively be framed as differential opportunities to learn (Flores, 2007; Hung et al., 2020; Milner, 2012). While many issues regarding inequitable opportunities to learn cannot be remedied at the classroom level (e.g., district-wide tracking policies), there are many ways teachers can work to provide all students with access to powerful mathematics. Teachers can choose tasks having multiple entry points and solution strategies, providing various ways students can meaningfully engage with content, thus positioning more students as capable doers of mathematics (Boaler, 2016; Hodge & Cobb, 2019; LaMar et al., 2020). Teachers can also limit their use of activities or participation structures that repeatedly privilege the same students, such as those that reward speed over depth of understanding.

Agency, Ownership, and Identity

Students' mathematical identities shape the ways in which they choose to participate in the classroom, and are therefore intimately connected to their learning (Boaler, 2000; Hand & Gresalfi, 2015; Lave & Wenger, 1991). These mathematical identities are shaped by a multitude of factors, such as students' racial, ethnic, and gender identities, family and community influences, and prior mathematical experiences (Levya, 2021; Martin, 2000, 2012). Within each classroom, students' mathematical identity development is also influenced by the shared understanding of what it means to be a competent doer of mathematics in that classroom (Boaler & Greeno, 2000; Cobb et al., 2009). When teachers are mindful of students' multiple identities and position them with agency as mathematical meaning-makers, they support students in constructing positive mathematical identities for themselves (Aguirre et al., 2013). Further, when students are expected to support their ideas with mathematical reasoning and are responsible for evaluating the validity of others' reasoning, they become "authors and producers of knowledge, with ownership over it, rather than mere consumers of it" (Engle & Conant, 2002, p. 404). Teachers can support such ownership by publicly attributing ownership of ideas to students, utilizing participation structures that encourage students to build off of these ideas (e.g., think-pair-share), and by establishing classroom norms wherein mathematical reasoning and argumentation are the standard for determining the validity of student solutions, rather than the teacher or a textbook.

Formative Assessment

Effective use of FA in the classroom has been linked to positive student learning outcomes and the development of metacognitive habits (Andersson & Palm, 2017; Black & Wiliam, 1998; Darling-Hammond et al., 2020). In contrast to summative assessment (e.g., quizzes, exams), FA is used to inform instruction rather than to evaluate student performance. FA can occur via formal classroom tasks or through in-the-moment student-teacher interactions. For example, teachers can enact pre-assessment and exit-ticket tasks to surface students' mathematical thinking. Also, teachers can ask students open-ended questions to gain insight into their thinking and understanding (Schildkamp et al., 2020), which they can then use to provide appropriate scaffolding or additional instruction. The use of FA can support students' development of a growth mindset by shifting focus away from extrinsic, performance-based motivation (Shepard, 2000), and can encourage metacognitive behaviors in students, such as self-reflection and goal-setting (Granberg et al., 2021). FA pedagogies allow teachers to solicit student thinking during a lesson and adjust instruction to “respond to those ideas, by building on productive beginnings or addressing emerging misunderstandings” (Schoenfeld, 2014, p. 408), to ultimately improve teaching and learning.

Related to the TRU dimensions are Formative Assessment Lessons (FALs; see Mathematics Assessment Resource Service [MARS], 2015a). In collaboration with others, Schoenfeld’s team developed FALs as instructional materials aligned to TRU. In particular, they designed FALs to be incorporated by teachers within their existing curriculum. These lessons involve tasks and activities that can foster robust, equitable learning environments where “all students are supported in becoming knowledgeable, flexible, and resourceful disciplinary thinkers" (Schoenfeld & the TRU Project, 2016, p. 3). In a study of the FALs’ implementation in Kentucky, in spite of a myriad of methods that teachers chose to implement the FALs, their use was responsible for an additional 4.6 months of growth over the course of the year (Herman et al., 2015).

While each of the TRU dimensions can be viewed as distinct facets of powerful mathematics classrooms, they are all deeply connected and enhance each other as learning unfolds in the classroom. For example, providing as-needed support to all students in a way that maintains CD is heavily reliant on the in-the-moment information gathered from FA. Schoenfeld (2017) explained that “these dimensions are arranged spatially in [Figure 1] to illustrate both the individual dimensions and their connections – everything is connected, but each dimension has its own integrity” (p. 419). In the context of this work, the TRU framework is the core of the shared repertoire of resources for this CoP, and teachers’ reification of the TRU framework includes developing an understanding of each distinct dimension as well as how they can be connected. Furthermore, the TRU framework offers a common language for dialogue within this

PD setting. The next section details our methodology used to answer our research question: *How does learning about mathematics teaching practices manifest within a CoP during a PD model focused on the collective investigation of video cases of students engaging with ambitious instructional materials?*

Methods

Participants and Context

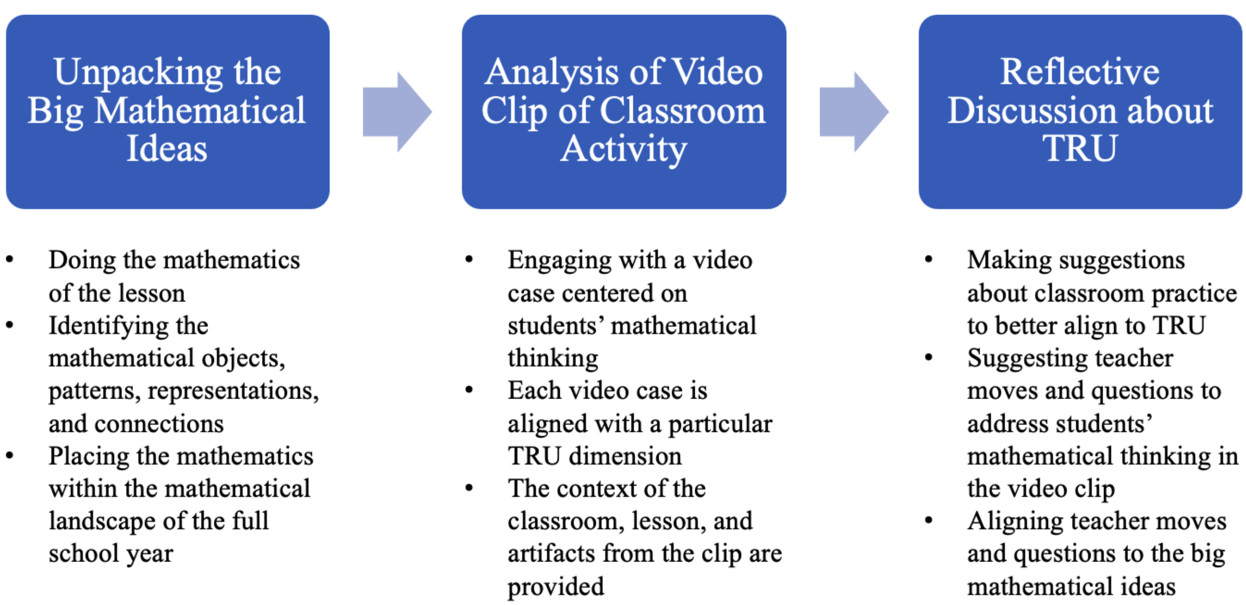
This paper focuses on one of many CoPs that were part of a larger project spanning multiple regions. The CoP studied in this work consisted of three PLCs in an urban Midwestern city. The entire CoP was composed of 30 members, with each PLC containing 10 secondary mathematics teachers. Moreover, each PLC had two of its members serve as participant-facilitators. We studied this CoP for two years as they engaged in a TRU-aligned mathematics PD model called Analyzing Instruction in Mathematics Using the TRU Framework (AIM-TRU). For context, most members of this CoP were from different middle schools and high schools in the region, most of which served low-income and racially diverse neighborhoods. The majority of CoP members were familiar with the TRU framework and had some experience teaching with FALs. Teachers in these PLCs had varying amounts of mathematics teaching experience, spanning 0-25 years with an average of approximately nine years. Across the two years of study, the PLCs met 24 times for 2.5-hour PD sessions conducted both in-person and via Zoom.

The AIM-TRU PD model engaged these secondary (6-12) mathematics teachers in a collaborative investigation of ambitious instructional materials to deepen instructional knowledge and support shifts in practice aligned to the TRU framework (Schoenfeld, 2015). This research team designed the model to align with Garet et al.'s (2001) five elements of effective PD: content focus, active learning, coherence, sustained duration, and collective participation.

This PD model allows teachers and teacher educators to generate collective professional knowledge for teaching and learning mathematics using the dimensions of ambitious instruction that are necessary and sufficient to produce equitable environments supporting deep mathematical learning opportunities for students (Schoenfeld & the TRU Project, 2016). We have also designed our PD model in accordance with Wenger's (1998a) theory that learning occurs within CoPs, and that teacher communities can serve as levers for equitable praxis and generative settings for robust teacher learning. To leverage mathematically rich student conversations for teacher learning, the AIM-TRU PD model focuses on the following components: (a) unpacking a lesson's big mathematical ideas, (b) making observations about video cases demonstrating students' mathematical thinking while engaging in TRU-aligned FALs, and (c) sets of video case reflective discussion questions based on the TRU framework (see Figure 2). Specifically, in component (c), PLC participants were prompted to (i) posit possible teacher moves or questions that would support students in the video case to engage with

Figure 2

Overview of AIM-TRU PD Model

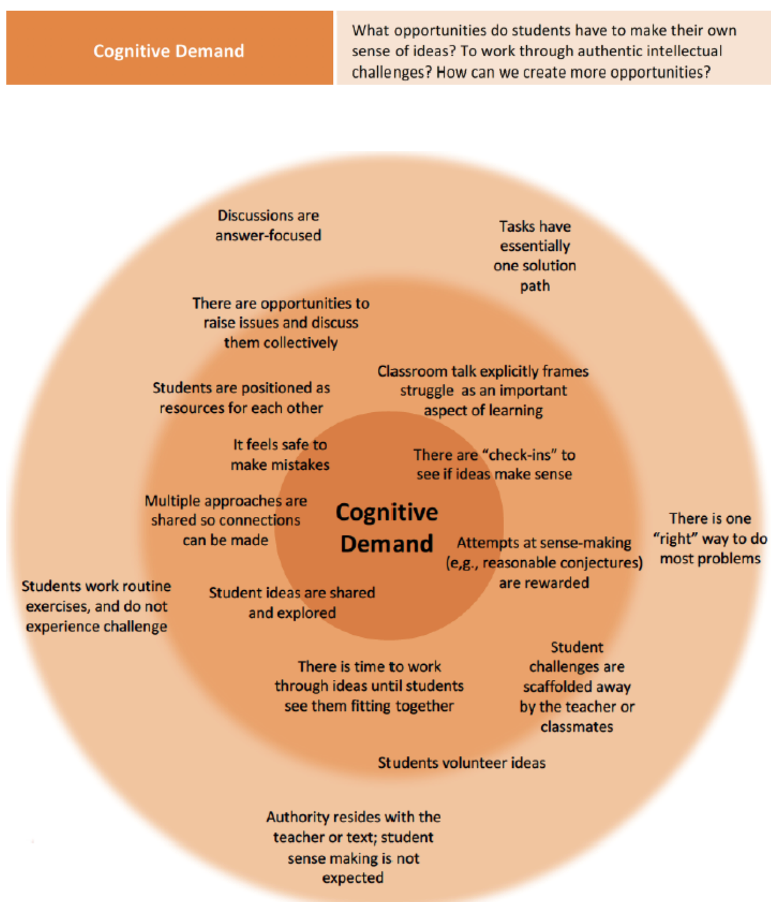


the mathematics based on a particular dimension of TRU, and (ii) to align possible teacher moves or questions to the big mathematical ideas of the lesson featured in the video case. The participant-facilitators followed a detailed protocol to enact the model, which helped ensure a natural and equitable conversation among participating teachers.

To further support participants in reflecting about the video cases relative to TRU, the AIM-TRU PD model incorporates TRU On-Target Tools (Schoenfeld et al., 2023) to situate each TRU dimension in the context of classroom activity, adapted with permission to fit our context (see Figure 3). The TRU On-Target Tools offer a visual representation of teacher moves

Figure 3

Example of a TRU On-Target Tool: Cognitive Demand



and their alignment to a particular TRU dimension. Schoenfeld and colleagues explained the On-Target Tools as follows:

> On the outer rings of the targets are descriptions of classroom attributes and activities that are commonly found in mathematics lessons, but that, with some adjustments, hold the potential to support more equitable and ambitious learning opportunities . . .. As you move toward the center of that target, the attributes listed describe increasingly powerful opportunities for student learning. (p. 2)

Participant-facilitators encouraged all participants to use the TRU On-Target Tools to help them engage in reflective discussion about TRU. Thus, the TRU On-Target Tools supported participants in positing productive teacher moves that aligned to TRU and to the big mathematical ideas of the lesson featured in the video case.

Data Collection

A researcher collected video and audio recordings of the 24 PLC meetings and artifacts created by the CoP. Artifacts included shared documents capturing participants' ideas generated both individually and collectively in small group discussions during each PLC meeting. We transcribed component (c) of the AIM-TRU PD model focused on PLC participants' reflective discussion of the video cases as they related to TRU and the big mathematical ideas. We chose to focus on these reflective discussions as a data reduction strategy (see Bannister, 2015) because, in our view, those conversations contained the most concentrated evidence of teacher learning relative to our theoretical framing about how CoPs learn. All of these transcriptions were cross-referenced with the related artifacts. Thus, the primary data sources were video recordings of PLC participants studying and discussing video clips of students engaged in rich mathematical activity.

Data Analysis

To make claims about how evidence of teacher learning manifests in this context, our analysis plan considered patterns of participation and reification within the PLCs. For transparency, see Appendix A for a detailed example of the coding involved in our analysis plan. For this stage of our analysis, we focused on teacher dialogue within a particular component of the PD model that occurred after the group watched and independently reflected on the video case. After individual reflection, teachers collectively engaged with reflective discussion questions about the video case based on the TRU framework, during which they had opportunities to (i) posit possible teacher moves or questions that would support students in the video case to engage with the mathematics based on a particular dimension of TRU, and (ii) align possible teacher moves or questions to the big mathematical ideas of the lesson featured in the video case. For this section of the transcript, we applied frame analysis, a method to study the ways teachers collectively shape and structure meanings through participation and reification in a CoP (Bannister, 2015, 2018). Frames are co-constructed objects among a community that represent existing meanings in the group at any given time. Frames have been used as ways to classify and organize teacher conversations in the short term (Horn & Kane, 2015) and to demonstrate growth in a CoP over time (Bannister, 2015).

The first level of analysis was to code the transcript by core framing types: diagnostic, prognostic, or motivational (Bannister, 2015, 2018; Benford & Snow, 2000). We viewed these three frame types as different ways teachers could participate in PLC. In particular, when discussing the video case of classroom activity, a teacher could state their observation about a problem of practice (a diagnostic frame, e.g., "During group work, the students aren't listening to each other."). If the teacher provided a diagnosis, they might additionally suggest an in-the-moment teaching move that could resolve that problem of practice (a prognostic frame, e.g., “The teacher could ask one student to explain in their own words what their classmate said.”). Finally, if the teacher both diagnosed and prognosed a particular problem of practice, they may also provide a rationale for a particular suggestion (a motivational frame, e.g., “Encouraging students to explain what their classmate said could help them build on each other's ideas and develop agency.”). We viewed motivational frames as the most powerful type of participation within the CoP because they imply agency and motive of the community members to address the joint challenge that arose about mathematics teaching practice. Furthermore, motivational frames imply collegiality because community members are justifying a point of view that may be in contrast to earlier ideas. In previous work applying frame analysis (e.g., Bannister, 2015), researchers have used the content of the frame types to understand patterns of participation over time. In our work, we instead looked to provide additional descriptors for the frames to create a more fine-grained classification system.

Iterating on frame analysis, we recognized the need to further classify prognostic and motivational frames by their framing process to better capture the complexities of the discourse of the PLC, particularly the spectrum of conversations (Burbules & Rice, 1991) that could occur relative to congeniality and collegiality. We focused on prognostic and motivational frames because these were talk turns that contained suggested teaching moves and justifications, respectively, which aligned to our previous data reduction strategy of focusing solely on component (c) of the AIM-TRU PD model. Benford & Snow (2000) described frame processes as the several factors associated with the development of any diagnostic, prognostic, or motivational frame. Their review of the literature established several frame processes that help describe how frames are constructed in a CoP, and suggested alignment to a spectrum of conversation. The frame processes included: articulating, punctuating, bridging, amplifying, extending, transforming, countering, and disputing (see Table 1). Other than articulating, each of these frame processes implies that the central idea communicated in the frame is connected to a previous frame or frames, constructed by either building off of or contradicting others' ideas. By coding each prognostic and motivational frame according to its frame processes and by noting the transcript lines of any connected frames, we were able to capture a fuller picture of how PLC participants co-constructed ideas through dialogue (see Appendix A for a coding example). Our synthesis of frame processes and the collegiality literature revealed evidence of the alignment of certain frame processes with congenial and collegial conversations (see Table 1). We acknowledge that conversations are not binarily congenial or collegial; however, in this work, we simplified our categorization of such conversation to help us develop the general story of

Table 1

Frame Alignment Processes

Category	Frame Processes	Definition
Congenial	Articulating	Expressing experiences, observations, and/or interpretations of implementing instructional materials
	Punctuating	Highlighting some issues, events, or beliefs as being more important than others
	Bridging	Connecting two or more unconnected frames
	Amplifying	Clarifying a previous frame
	Extending	Building on a previous frame
Collegial	Transforming	Generating new meanings or understandings based on previous frames
	Countering	Opposing or disagreeing with previous frames
	Disputing	Disagreeing with a portion of a previous frame, not the frame entirely

teachers' learning patterns as they participated in their PLCs. We viewed transforming, countering, and disputing frame processes to describe collegial discourse because these processes align more closely with Evans' (2012) and Zepeda's (2020) conception of collegiality. Specifically, these frame processes are more likely to develop new meanings, examine, and/or disagree with ideas from previous frames in ways to which others in the PLC could respond.

Since our PD model is rooted in the TRU framework, and to help us understand how the teachers reified TRU concepts, in our final level of analysis we aligned each prognostic and motivational frame with a TRU dimension and scored it using a rubric for TRU Talk in PLCs (see Appendix B). Our analysis also considered if connected frames were aligned to different TRU dimensions and if TRU alignment scores were the same or different between connected frames. This rubric is a version of the TRU Math Rubric (Schoenfeld et al., 2014), adapted with permission from Dr. Schoenfeld to fit our context and in collaboration with our project's external evaluator. This rubric partitioned each dimension of TRU into three numeric levels, with level 3 being the highest rating for teacher talk aligned with powerful mathematics classroom activity. When a frame did not clearly align with whole number scores, half-scores were assigned.

Bannister (2015) focused on individual members of a CoP by analyzing changes within prognostic frames related to pedagogical strategies generated by specific teachers. Incorporating frame processes and TRU alignment of proposed teaching moves allowed us to leverage Bannister's model to analyze the CoP as a whole, rather than individual teachers. By analyzing changes in frame processes and TRU alignment scores, we were able to capture patterns of participation and reification by noting how the dialogue as well as PLC ideas evolved within PD sessions.

Results

The results reported here are informed by our analysis of teachers' participation in component (c) of the AIM-TRU PD model focused on PLC participants' reflective discussion of the video cases as they related to TRU and the big mathematical ideas of the lesson. This section addresses the research question that guided this study, namely how teacher learning manifested within a CoP situated in the AIM-TRU PD model. We answer this research question in two ways. First, we articulate our general findings about the ways in which teachers in their PLCs participated in the AIM-TRU PD model across all sessions in the full two-year data set. Second, we illustrate how teachers in their PLCs changed the ways they participated within sessions with descriptions of representative excerpts from our data set.

General Findings: Teachers' Participation in Their PLCs

Over the course of the three PLCs, we coded 226 frames in which an individual teacher participant offered a prognosis for a problem of practice observed in the video case of classroom activity, or a motivation for such a prognosis. These frames occurred as a part of natural conversations among colleagues, and all 30 teacher participants are represented in these frames. Our analysis of this dialogue revealed that teachers participated both congenially and collegially in their PLCs. They also participated by motivating their prognoses and leveraging TRU concepts during conversation. In general, we found that when teachers participated in collegial frame processes, they engaged more often in motivational frames and their conversations aligned more closely to the TRU framework. Furthermore, we found that these types of participation connected to reification of the TRU dimensions, as evidenced by higher TRU scores during collegial dialogue and more connections made between multiple TRU dimensions when compared to congenial dialogue (see Table 2). Teachers engaged most often in congenial conversation, with 77% of teachers' prognostic or motivational frames (174 total frames) being

Table 2

Summary of Teachers' Participation in PLCs

	Congenial Frame Processes	**Collegial Frame Processes**
Prognostic and Motivational Frames (%)	174 (77%)	52 (23%)
Motivational Frames (%)	54 (31%)	33 (62%)
Average TRU Score	2.39	2.82
TRU Dimension Change (%)	14 (8%)	21 (40%)

classified as an articulating, punctuating, bridging, amplifying, or extending frame process. At other times, teachers participated in collegial conversation, with 23% of teachers' prognostic or motivational frames (52 total frames) being classified as a transforming, disputing, or countering frame process. This finding shows that teachers engage in dialogue in various ways within their PLCs and indicates that understanding participation and reification through the lens of congeniality and collegiality can provide important information about the nature of their learning.

Within teachers' congenial and collegial participation, we found two ways teachers participated in the AIM-TRU PD model that were impactful to their learning: by engaging in prognostic or motivational frame types, and by the ways they aligned their frames to TRU dimensions. By proposing an in-the-moment instructional solution or providing an accompanying rationale for an in-the-moment instructional solution, teachers in the PLCs toggled between prognostic and motivational frames, respectively, as a method of sharing their suggestions for teacher moves. When conversations were congenial, teachers' frames were motivational 31% of the time, which means that teachers' frames were prognostic and did not offer a motivation for a proposed solution the other 69% of the time. In contrast, when conversations were collegial, teachers motivated their proposed teaching moves and connections to the big mathematical ideas 62% of the time, which means that teachers' frames were prognostic the other 38% of the time. This finding shows that teachers participating in a collegial environment were doubly likely to justify their prognosis to their peers, compared to when participating in a congenial environment, which suggests the teachers were able to leverage the collegial environment to participate within the CoP more powerfully with agency and motive. Additionally, teachers in the PLCs varied the degree to which their frames aligned to TRU dimensions. When conversations were congenial, teachers' frames were assessed to have an average TRU alignment score of 2.39 out of 3; 8% of these frames were connected to a previous frame and involved a change in TRU dimension alignment (see our Representative Excerpts below for examples of this), which shows an understanding of the interrelatedness of the TRU framework. Alternatively, when conversations were collegial, teachers' frames were assessed to have an average TRU alignment score of 2.82 out of 3; 40% of these frames were connected to previous frames and involved a change in TRU dimension alignment. These findings show that teachers participating in a collegial environment were positing teacher moves that were more closely indicative of ambitious mathematics instruction, compared to when participating in a congenial environment. These findings also show how teachers' reification of TRU concepts, via higher TRU alignment scores and more emphasis on making connections between TRU dimensions, were more prevalent in collegial environments.

Overall, these general findings suggest that when teachers engage in the AIM-TRU PD model, specifically in component (c), collegial dialogue promotes participation in the form of motivational framing and alignment to TRU dimensions. This participation type supports teachers in reifying TRU dimensions and how the dimensions relate to the big mathematical ideas of the lesson. The next section presents representative excerpts from the AIM-TRU PD model sessions. These excerpts will help show ways in which teachers changed their participation within sessions and help us understand teacher learning within these PLCs.

Representative Excerpts: Illustrating the Changes in Teacher Participation in PLCs

The general findings indicate that collegial frame processes appear to have stronger TRU alignment scores and more connections between TRU dimensions, therefore showing evidence of reification of TRU concepts within PLCs. We present three representative excerpts illustrating specific instances of teachers from various communities changing their participation to help describe how teachers' learning patterns may have emerged. Each of the three excerpts below provides a window into a frame process aligned with collegial conversations: countering, transforming, and disputing, respectively. The excerpts were chosen to provide examples from each of the PLCs within the CoP, and to illustrate different frame processes in context. All of these representative excerpts illustrate the duality of changes in participation and reification within PD sessions, and thus, illustrate how teacher learning manifested from a CoP perspective. To help the reader recognize the different frame processes in these excerpts, we highlighted the relevant text in the transcript and in the corresponding analysis that follows according to the color scheme in the Frame Alignment Processes showcased in Table 1.

Excerpt I: Countering and CD in the Context of Quadratic Functions

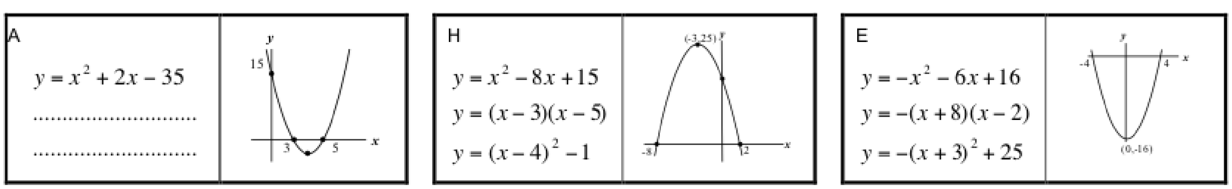
During the first year of the PD model, there were a number of congenial frame processes, but during the sixth session we found evidence that this community of mixed middle and high school mathematics teachers changed their participation to shift into collegial dialogue. Here, the PLC was investigating a video case centered on representing quadratic functions graphically. Teachers discussed whether students were struggling unproductively with a domino lesson activity (Figure 4) in which they created links between quadratic graphs and their algebraic representation. In the following transcript[1], Teachers 1, 2, and 3 prognose teaching moves related to the organization and presentation of the task. Teacher 4 then questions the need for such alterations and prognoses a teaching move:

Teacher 1: I also looked at one of the ideas on the outside of the target: "discussions are answer-focused." So, the students were definitely praising their struggle and being like, okay, progress at the end. But it kind of seemed like they were still like, "I gotta do all these things, and oh my gosh, there's so many cards." I wonder how this task would have changed or their approach would have changed if we just gave them three cards to look at or three totally random cards, you don't even need to connect. But look at these and see what you make of them.

Teacher 2: Even if instead of giving them both sides of the card, maybe just splitting them and giving them just one portion and seeing what they would do with it. Seeing if they could, for certain cards, seeing if you gave them the graph, that they come up with the equation, that they come up with the factored format, that they come up with, just whatever they can pull from

Figure 4

Example Cards from Activity in PD Session 6 (MARS, 2015c)



[1] All transcripts in this paper have been edited to include gender-neutral pronouns.

it. Then if you gave them a side that had some of the equations there or maybe one of them, could they come up with the other pieces, to kind of see how much they know and understand, and how it can interrelate before they get the piece with the picture.

Teacher 3: The FAL, it recommends that you start the kids out with just two cards, A and H, and you give them or you give them like three, I think. And you just give them these three cards and then they talk about matching them and how it works and stuff. So I think . . . if you follow the lesson structure, it sets up the kids, we'll look at one at a time, instead of going all over a little bit.

Teacher 4: I totally understand. But, Teacher 5, I really enjoy watching your class, I thought that they did a phenomenal job even through the productive struggle. But even when we look back at that cognitive demand bullseye, they could very well have started them with three or even just two. But then working through that and pushing through and making reference to their previous notes really shows that they were, they had some type of knowledge on how to maneuver that y-intercept or that x-intercept and substituting it for different numbers. I just thought that if we had more time . . . I think that we would have seen an even more successful lesson where the kids would have been able to do that. I think just them having that mindset of even pushing through, I thought it was a really good job. I understand starting with two or three, but they were not giving up and they were making reference to their notes, and whether the notes were in a

> notebook or on the walls, they knew exactly where to go to find those answers or something that will lead them to an answer.

In this interaction, Teacher 1 articulated a prognosis that the teacher in the video could have reduced the initial number of dominos to combat students feeling overwhelmed by the number of cards within the task. This initial frame from Teacher 1 was coded as Level-2 TRU alignment to CD because Teacher 1 suggested a move that could help keep students productively engaged with central mathematical ideas but scaffolded away some of the challenge. Teacher 2 responded in a second frame with a prognosis to change the activity to a matching activity to help students focus on making a single link between a quadratic graph and equation, rather than making several links across representations. This was an example of a transforming frame process because Teacher 2's new prognosis transformed Teacher 1's original prognosis to focus on a new activity to help keep students productively engaged. This frame was also coded as a Level-2 TRU alignment to CD because Teacher 2 suggested a move that could help keep students productively engaged with mathematics, but also scaffolded away some of the challenge. While Teacher 2's frame is a collegial frame (transforming), the prognosis did not strengthen the TRU alignment score or shift the TRU dimension of focus. Teacher 3 then reminded the community that their suggestions are actually part of the FAL's directions. This frame was coded as an amplifying frame connected to Teacher 1's original prognosis. This frame was coded with a Level-2 TRU alignment score because Teacher 3 did not alter the original prognosis, but rather supported the suggestion by clarifying that the teacher move is embedded in the directions for this FAL.

The final frame in this example was provided by Teacher 4 when they pushed the collegial conversation further by countering suggestions made by the three previous teachers.

Teacher 4 disagreed with the prior diagnosis that students were struggling unproductively in the classroom video, and asserted that teacher intervention was not needed to help students productively struggle. Instead, Teacher 4 prognosed that the teacher could provide the students more time to continue to engage in the mathematical practices that they were using when faced with uncertainty, such as referencing prior resources and displaying the mindset to grapple with the content. Teacher 4 believed that these practices were aligned to a high level of CD, and by providing them with more time, the teacher would see the students successfully navigate the task and make important connections. This countering frame was coded as a Level-3 TRU alignment to CD because the suggestion requires students to continue to engage in mathematical practices without scaffolding away the challenges by providing students adequate time to struggle with the core content. The increase in TRU alignment score indicated reification of the CD dimension within the collegial dialogue.

Excerpt II: Transforming from FA to AOI in the Context of Properties of Exponents

During the third session of the second year in a PLC of middle school mathematics teachers, we found evidence of changing participation and reification via a transforming frame that built on two previous frames, shifted the TRU dimension of focus, and increased the TRU alignment score. The PLC was investigating a video of three students completing a card sort with exponential expressions. In the video, Student 3 relied on a calculator to match equivalent cards and did not respond to suggestions made by two other students (Student 4 and Student 5), who applied exponent rules to match cards. To make a match for the card $6^8 \div 6^4$, Student 4 and Student 5 told Student 3 several times that because the bases were the same, the exponents could be subtracted. Student 3 insisted on evaluating the expression on a calculator first, writing out

$1{,}679{,}616 \div 1{,}296$, and again used a calculator to find this quotient before choosing an equivalent card.

In the following discussion, Teacher 6 suggests a prognosis which is amplified by Teacher 7 before a teacher facilitator poses a question to the community. Teacher 8 then transforms the previous frames:

Teacher 6: When they were writing out 6^8 and they wrote out the big number, as a teacher, what would I say is, is there another way we can represent that 6^8? To help them see and then connect between what Students 4 and 5 were talking about. And what Student 3 was, how they were interpreting that number and then see how it would play out with the division.

Teacher 7: Yeah, I think that piece right there [referring to target] was very powerful, that "Tasks have multiple entry points." Students 4 and 5, I don't know if they understand or memorized the properties that the teacher taught. And Student 3 was able to use a computational [approach]. As a teacher, we could have walked in, and try to get them to make that connection, like that's great what you're doing, Student 3, but what if you don't have a calculator? What can you do to solve this problem for those moments, maybe, and hopefully tie in what Student 4 and Student 5 were thinking? Piece it together to help them make that connection.

Facilitator 1: Just thinking about what even, the comments that we just heard, and even what [Teacher 7] just said about making the connection with evaluating. How can we use all of that to help tie in what that overall big idea should be, and even looking at the notes and things that we've jotted down throughout the session? How can we bring that together?

Teacher 8: I just wanted to add on there, on that [target] where it says "Students have opportunities to explain." They do, and they were making a claim, but Students 4 and 5 weren't following it up with any evidence or reasoning. Maybe having something there for them as a reminder. When you're working in the group that, because they just kept repeating, "because it's the same base, same base!"

During the discussion of possible teaching moves, Teacher 6 articulated a prognosis that the teacher in the video could have asked Student 3 if there was another way to represent 6^8, guiding them to think about the expression using exponent rules rather than using a calculator. This initial frame was coded as an articulating frame because Teacher 6 presented a new prognosis unrelated to previously discussed teaching moves in this session. Teacher 7 then suggested that the teacher could have asked Student 3 how they would make a match if they did not have a calculator. This is an example of a congenial, punctuating frame as Teacher 7 is restating Teacher 6's prognosis, highlighting the need to shift Student 3's reasoning away from the calculator without changing the original prognosed teaching move. These prognoses were both coded as FA Level-2 TRU alignment score because the suggested questioning would elicit student thinking, but plans to build on the student's ideas were not articulated.

The teacher facilitator then probed the community of teachers to think more deeply about their prognoses and to make connections to their generated big mathematical ideas for this lesson. Teacher 8 then responded with a transforming frame by suggesting that providing students with something to remind them to justify their mathematical claims might have helped

Student 4 and Student 5 expand their mathematical explanations beyond just pointing out that both 6^8 and 6^4 have the same base. Teacher 8 then motivated their prognosis:

Teacher 8: If they would have followed it up, just, and shown [Student 3] why it works, that would have maybe helped, or got them thinking on a different strategy.

This is an example of a collegial, transforming, motivational frame because Teacher 8's prognosis sought to address Student 3's over-reliance on the calculator by shifting the focus from the teacher questioning suggested by Teachers 6 and 7 (FA) to encouraging students to take responsibility for explaining concepts to their peers (AOI). This is an example of a change in participation because this was the first time in this exchange that a teacher provided a justification for their prognosis. Additionally, the shift in TRU dimension is an example of how teachers in this community used transforming frames to change the TRU dimension under investigation. This frame was coded with a higher Level-2.5 TRU alignment score in AOI because the suggested move would facilitate students coming to an agreement without the teacher acting as the arbiter of correctness. The shift in TRU alignment and the greater TRU score indicated reification of the interrelatedness of the TRU framework as well as both the FA and AOI dimensions within the collegial dialogue.

Excerpt III: Disputing and EA in the Context of Linear and Exponential Growth

During the fourth session in the second year, we found evidence of changing participation and reification via a disputing frame in the community of high school teachers. This representative example occurs after the community had watched a video of students completing the first card sort of an FAL about representations of linear and exponential growth. In this card sort, students need to match investment plans to formulas that model each plan (Figure 5). The

Figure 5

Example Cards from Activity in PD Session 4 (MARS, 2015b)

P5 Investment: $400 Compound Interest Rate: 8%	F1 $A = 400 \times 1.08^n$
P6 Investment: $400 Simple Interest Rate: 2%	F4 $A = 400 + 8n$

community of teachers is reflecting on the student interactions in the video through the lens of EA. Prior to the excerpt from the conversation, the community discussed that one of the three students does not appear to be participating in the small group discussion. The teachers prognosed multiple teaching moves to address the inequitable participation: holding a conference with students to discuss the exclusion of one student, establishing checkpoint protocols before moving to another card, probing student thinking about what they heard the group say, developing student-to-student questions as a standard practice in the class, and reminding students of class participation expectations. The excerpt below begins with additional prognoses, then transitions into one teacher disputing the general understanding of the community:

Facilitator 1: I just think from an equity point of view. This is not just access, but it's equitable access. If we're letting some kids not participate and we're letting other kids not let them participate. Are there other moves you all can think of that in terms of equitable access you do to try and prevent this kind of thing?

Teacher 9: I used to do this one activity, where students, even in a group, each student would have a different question but relating to the same topic, regardless of what we were studying. So each student had to come up with an answer and their own process first, and then they would compare … Then they would switch questions with other groups. In the end, we were able to have a class discussion based on the same questions, but each student was responsible for one within each set.

Teacher 10: I've done it before, where we've had a group working, and they each have a different role, and then they rotate. One person might be in charge of the explanation, another person would be in charge of recording it, and the other person will be presenting it. Depending on which one they had, they had to be prepared for their own thing…So for the student who might not have been able to develop it, at least they would have to have the understanding of how to explain it if they were chosen to present that . . .

Facilitator 2: Yeah, along those lines, just go to Student 3, and be like, "Hey Student 3, I'd like you to be the one to write on this blank card." And then walk away. Easy way to increase the equitable access in the moment.

Teacher 10: It's important when you're looking at the group . . . is the focus on completing the task? Or making sure that all the people in the task are involved and understand all of the steps? So it doesn't have to be completed if it can be demonstrated that everyone had a say in it and took part in it. Sometimes the difficulty for the students is making sure that they can explain it in a way that somebody else understands it. Not that they

> can demonstrate that they themselves understand. So instead of being the knowledge of the task, the communication of what they're doing might be the focus of the activity for them.

During this interaction, Teacher 9 continued to address the issue of uneven participation among students by suggesting the teacher provide each student a similar, but varied set of problems to give the students a chance to discuss the similarities with mathematical processes. This articulating frame was coded as a Level-2 for EA because the teacher is attempting to develop a structure for equitable participation structures but does not detail how this move could achieve meaningful participation from all students in the group. Teacher 10 then articulated a new prognosis to assign roles for each student: record, explain, and present the group's mathematical thinking. This prognosis was coded as a Level-2.5 for EA because while Teacher 10 provided a teacher move that could achieve broad participation, not all of the student roles can be considered meaningful participation with the mathematical content. For example, a student assigned the role of recorder can passively take notes and not engage with core mathematical practices. Facilitator 2 provided a punctuating frame for Teacher 10's articulation when they suggested that the teacher have the non-participating student be the one to write the equation down. As a set of frames, these talk turns are an example of a congenial conversation. Teachers and facilitators alike articulated new prognoses, politely agreed with each other, and did not challenge each other's thinking.

The general consensus to this point in the discussion was that teacher intervention was needed to have one student participate in group discussions. In Teacher 10's next frame, there is evidence of a disputing frame when they differ from their own previous prognosis as well as those prognoses that came previously by offering a new perspective on the video clip they watched. Each of the previous teacher moves was centered on having all students discuss the outcome and finished product of the card sort and the task. In the final frame presented, Teacher 10 proposed the teacher shift the focus from the completion of the card sort and task to the creation of a learning goal related to group understanding through communication practices. Teacher 10 then motivated their prognosis by claiming that changing the goal of the group to making sure everyone in the group understands the math might encourage students who might not otherwise participate to share their ideas. This disputing frame was coded with a Level-3 for EA because it was a detailed, specific teaching move that has the potential to achieve and support meaningful participation within the group. This new disputing frame transitioned the conversation to a collegial conversation and also increased the TRU alignment score as the disputing frame is connected to previous frames, thus indicating reification of the EA dimension within the collegial dialogue.

Discussion

Our analysis of these PLCs revealed the ways in which teachers participated in the AIM-TRU PD model, the specific participation types that supported reification of TRU concepts, and evidence of changes in participation and reification from teachers within PD sessions. These findings also imply actionable facilitation practices that could inform how teacher leaders support teacher learning within mathematics PD. Iterating on Bannister (2015, 2018), we found evidence of different types of participation patterns through identifying teachers' frames within PD sessions: when the conversation consisted of collegial frame processes, teachers were more likely to engage in motivational frames and TRU-aligned suggestions about teaching moves. These TRU-aligned suggestions also provided evidence of reification of teaching and learning across the five dimensions through connections teachers made from one dimension to another, illustrating the dimensions' interrelatedness (Schoenfeld & the TRU Project, 2016). From a CoP theoretical perspective (Wenger, 1998a, 1998b), these patterns in the nature of the dialogue within PD sessions help illustrate evidence of teacher learning because members of the PLCs demonstrated the duality of participation and reification. Specifically, as members of the PLCs established patterns of participation conducive to collegiality and motivational framing, these styles of discourse were indicative of their learning process about TRU-aligned teaching.

When analyzing the PLC dialogue within PD sessions, teacher learning patterns were most clear when teachers transitioned from congenial to collegial conversation. For instance, we saw evidence of this in all excerpts, but particularly in Representative Excerpt I as Teacher 4 leveraged the TRU On-Target Tool to counter their peers' earlier prognoses and suggest a teacher move more supportive of students' productive struggle with graphs of quadratic functions and their algebraic representations. These instances are indicative of Borko's (2004) and Zepeda's (2020) successful learning communities for teachers, as teachers in their PLCs felt safe to take risks in their dialogue by respectfully challenging each other. It is notable that Teacher 4 prefaced their countering prognosis by referencing the TRU framework (i.e., the CoP's shared repertoire) via the TRU On-Target Tool. Related to Nelson et al.'s (2010) position on leveraging shared repertoires to help elicit collegial ideas and feedback within a PLC, couching a countering frame within the TRU framework made it easier for Teacher 4 to challenge their peers because the teachers' perception of Teacher 4's countering prognosis was not personal, instead it was aligned to TRU concepts. Facilitators can direct participants' attention toward the CoP's shared repertoire, which can provide participants a safe way to engage with each other collegially. In this way, skilled facilitators are imperative for helping shift the PD dialogue from congenial conversation that builds trust to collegial conversation that can create new ideas through constructive disagreement (Burbles & Rice, 1991).

Teacher learning patterns in dialogue were also apparent via collegial, motivational frames, as teachers began to offer rationalizations to their prognoses for a problem of practice. For instance, we saw evidence of this in Representative Excerpts II and III. Specifically, in Excerpt III, we see Teacher 10 justifying their idea to shift the focus from the completion of the card sort task to more student-to-student discussion about the meanings of linear and exponential growth because it encourages all students to participate in sharing their thinking. This motivational frame occurred while Teacher 10 collegially disputed earlier prognoses made by others in the PLC, as well as self-disputing their own previous prognosis. Not only do these types of instances highlight the importance of collegiality, but they also highlight the importance of teachers sharing their motivations for their ideas (Benford & Snow, 2000) about instructional practice within PD. Relevant to teacher leaders, PD facilitators should establish norms during sessions that encourage justification of any and all ideas, perhaps especially ideas that are in discord with others. Finally, these patterns inform teacher leaders about how design elements of PD programs, such as reflecting on ambitious teaching practices via video case analysis (Alles et al., 2018; Borko et al., 2008; Garet et al., 2001), can support such motivations to be shared.

Lastly, teacher learning patterns through dialogue were evident when teachers began to use their suggestions for instruction to make connections between the dimensions of the TRU framework. For instance, in Representative Excerpt II we see teachers suggesting instructional moves aligned with both FA and AOI. Particularly, we see Teacher 8 shifting the focus from FA to AOI by suggesting that the teacher remind students in the video clip to take responsibility for explaining concepts of exponential properties to their peers. Such student-to-student discourse could cultivate new understandings without the teacher acting as the arbiter of correctness. This collegial, transforming, and motivational frame aligned the teacher dialogue more closely to the TRU framework and showcased possible connections between FA and AOI. Following Facilitator 1's prompting to focus on the PLC's big mathematical idea, Teacher 8's prognosis also refocused the discussion on the lesson content, i.e., properties of exponents. These instances underline the importance of collegiality and skilled facilitation within content-focused PD (Darling-Hammond et al., 2017; Garet et al., 2001). Mathematics-content PD can be a challenging place for teachers because they may fear judgment about their content knowledge and withhold their full participation. Encouraging collegiality is especially important in these settings because PD should stimulate discourse about the mathematics content itself, in addition to the pedagogy, to discuss and open up opportunities for growth in content knowledge. Moreover, collegiality is critical for any PD grounded in the TRU framework because the mathematics content is at the center of TRU, and without a deep understanding of the mathematics, no authentic learning can be realized across the other pedagogical dimensions (Schoenfeld & the TRU Project, 2016). Teacher leaders can support such learning within PD by challenging participants to focus their comments on the mathematical content that is the focus of the PD session.

These findings also implicitly contribute to teachers' identity development within a CoP. Lave and Wenger (1991) argue that "learning and a sense of identity are inseparable: they are aspects of the same phenomenon" (p. 115). Although our research question was not directly focused on teacher identity, another way to interpret the duality of participation and reification we found in our study is to view it as evidence of the development of teachers' relationship between themselves and their place of membership in their PLC. Teachers in these PLCs developed their identities as effective mathematics practitioners, as evident by their negotiation of different points of view about how to solve problems of practice and how those solutions align with TRU concepts.

As we have alluded to in the previous paragraphs, our findings implicate action for members of the mathematics education leadership community, namely via the design of PD models. The AIM-TRU PD model studied here was intentionally aligned to the design elements of Garet and colleagues (2001), particularly to focus on rich mathematics content, active learning, coherence, sustained duration, and collective participation. We quickly learned, however, that collegial conversation within PD activities is vital to support teacher learning about ambitious mathematics instruction. PD models need to be intentional about how to cultivate collegial environments and invite productive disagreement aligned closely to students' opportunities to learn rich mathematical content. Furthermore, these findings implicate action for facilitators of such PD models to create opportunities for dialogue to transition from congenial to collegial. Within our larger project, we have been reflecting on these findings and intentionally revising the AIM-TRU PD model, specifically through our facilitation guides. The goal of these guides is to equip facilitators with questions that invite more collegial dialogue among members of the PLCs. It is important for mathematics education leadership groups to find ways to support facilitators in this way. Many researchers have shown the critical role facilitators play in supporting and fostering productive teacher learning (e.g., Borko et al., 2021; Coles, 2013; Lesseig et al., 2017), yet there is a lack of research available on how to support teacher leaders in facilitating PD with their peers.

Limitations and Future Research

This work had some limitations that will inform future research. First, we were only able to make claims about PLCs' changes in participation and reification within PD sessions, not generally across all PD sessions. This is because our scope of analysis for this paper did not consider the specific ways in which PLCs evolved in collegiality over time. Therefore, we cannot presently make a claim about how all the PLCs changed their participation from congeniality to collegiality over the course of the two years. Instead, we focused on making claims within PD sessions, to better understand how collegiality manifested in our context and how it might be useful to help teachers reify TRU concepts. Future research will consider the evolution of collegiality across all PD sessions, and thus, more general statements about the duality of changes in participation and reification in all PLCs.

Related to this point, we used a binary framework of congenial and collegial frame alignment processes (Benford & Snow, 2000), which required us to describe PLC conversations as one or the other. This allowed us to tell only a binary story of the dialogue. In reality, frame process types can vary in congeniality and collegiality (e.g., punctuating frame processes may be more congenial than extending frame processes; disputing frame processes may be more collegial than transforming frame processes). Because our research question was exploratory, we made the decision to binarily consider frame processes that were either congenial or collegial to help us understand how learning was manifesting in PLCs. Future research will consider a finer grain size of congenial and collegial dialogue and contribute to the field by defining and operationalizing a spectrum of congenial and collegial conversation, inspired by Burbules and Rice's (1991) work on the plethora of communicative virtues.

Another limitation of this work is that our research question was not focused on the impact of certain facilitation moves to support teacher learning. Although our analysis of teachers' participation in a CoP helped us infer ideas about productive facilitation, we did not study this directly. It is imperative for the mathematics education leadership community that future research investigate the relationship between facilitation moves and teacher learning, specifically how facilitation moves can support collegial conversations during PD.

In addition to future work motivated by the stated limitations, we plan to conduct larger-scale studies to continue our research. The focus of the current research question did not warrant conducting statistical analyses to show significant differences in our findings. This project is ongoing, and we continue to iterate on this work with the goal of testing a larger sample size of frames for significant differences in the occurrences of collegial frames and their associated TRU scores. Also, we only considered one regional site. This paper is part of a larger project that studies PD in several regions, all with unique settings and needs, and future studies within this project will consider all regions to help make claims about how teacher learning can manifest in different contexts.

Furthermore, the current research question allowed us to solely focus on the evidence of teacher learning that manifested within one component of the AIM-TRU PD model: the (c) sets of video case reflective discussion questions based on the TRU framework. Future research within this project will expand the scope of focus to include how teachers' duality of participation and reification manifests in the model's other components, (a) unpacking a lesson's big mathematical ideas and (b) making observations about video cases demonstrating students engaged in rich mathematical activity, and how those experiences might influence the ways in which teachers posit potential solutions to problems of practice in mathematics classrooms.

Conclusion

The goal of this research was to investigate teacher learning within CoPs focused on ambitious mathematics instruction. We found that teacher learning is supported within a collegial environment where teachers can respectfully disagree on how to solve problems of practice in mathematics classrooms. Our findings highlight how engagement within a PD model can support teachers to change their participation and reification patterns to more often engage collegially, justify their positions, and align their positions to research-based frameworks aimed at ambitious teaching practices. These findings allow us to respond to national calls for PD to center on teacher dialogue about classroom practices and to construct new ideas about mathematics teaching and learning (Hiebert & Stigler, 2017; Kazemi & Hubbard, 2008). Our analytic use of frame processes (Benford & Snow, 2000) and TRU Framework alignment (Schoenfeld et al., 2014) extended the frame analysis work of Bannister (2015, 2018) and afforded us the opportunity to identify three distinct manifestations of collective teacher learning within PD sessions: advancing collegiality, increased motivational framing, and alignment of conversation to TRU concepts. At large, future research in mathematics education leadership should focus on how to intentionally foster collegial interaction in PD to support teacher learning, through facilitation support and design elements, as well as examining how teachers' participation in collegial PD models impacts their actual classroom practice.

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Appendix A

Detailed Example of the Coding Involved in Our Analysis Plan

In this PLC session, teacher participants were discussing a video in which students were struggling to (i) interpret speed as the slope of a linear graph and (ii) translate between the equation of a line and its graphical representation. In each transcript example below, we color coded the teacher's **diagnosis** (**red**), **prognosis** (**green**), and **motivation** (**blue**). We also highlighted frame processes according to the color scheme in the Frame Alignment Processes in Table 1. Teacher A began the discussion below; Table 3 summarizes how we coded Teacher A's frame.

Teacher A: So I know one thing that I'm doing now, when we have word problems with, like, a situation like this . . . I'm seeing that **a lot of students don't really understand the word problem** . . . I've learned by doing that in math when they have a word problem before they even start thinking about the "math" that's in the word problem, it helps them to understand what's going on, like if I just asked, "Who are the characters? What's the conflict?" or "What's the problem? What's the goal at the end? What are they trying to figure out?" And I think that, **it's actually like a list of, like a break sheet of questions that they have to fill out before they actually start solving the problem.**

Table 3

Summary of Coding Teacher A's Frame

Category	Code & Explanation
Diagnosis	Some students do not understand the word problem in the lesson.

Prognosis	The teacher could give students comprehension questions about the situation in the word problem.
Motivation	None
Frame Process	This frame was coded as an articulating frame because this is the first time that this problem of practice is addressed in this PLC meeting.
TRU Dimension	This frame was coded as aligned to **Cognitive Demand** because the suggested teacher move involved scaffolding the task in a way to help create and maintain an environment of productive intellectual challenge.
TRU Rubric Score	This frame was scored as **2.5** because although asking comprehension questions could help students engage with the word problem and does not remove opportunities for productive struggle, it is unclear how such opportunities could help students build understanding of central mathematical ideas or engage in mathematical practices.

Teacher B then built on Teacher A's ideas in the following **connected** frame. See Table 4 for a summary of how we coded Teacher B's frame.

Teacher B: . . . It's like you **do a first read** and you just identify what is the story about. You **do a second read** and you identify what are the quantities and their relationships. Like, what are the numbers and what do they mean in the situation. And then **the third read** is you try to ponder, what question are they going to ask me without knowing the question. **So then that way you're being a problem solver before the problem is already presented to you.**

Table 4

Summary of Coding Teacher B's Frame

Category	Code & Explanation
Diagnosis	Some students do not understand the word problem in the lesson (the same diagnosis as the connected frame).

Prognosis	Teachers could encourage three reads of the word problem: to identify what the story is about, to identify what the numbers mean in context, and to predict/pose the question to be asked.
Motivation	Requiring students to predict what question the problem is asking before reading it will engage students in the process of problem solving before the problem is officially presented to them, which will help them engage more deeply.
Frame Process	This frame was coded as a transforming frame because Teacher B's suggested move leveraged Teacher A's suggestion to generate a new understanding about scaffolding. The motivation provided by Teacher B makes it clear that the "three reads" will not only help students engage with the word problem, but will provide them with an important opportunity to develop their problem-solving practices.
TRU Dimension	This frame was coded as aligned to the **Cognitive Demand** dimension because the suggested teacher move built on the previous frame, involving scaffolding the task in a way to help create and maintain an environment of productive intellectual challenge.
TRU Rubric Score	This frame was scored as a **3** because the "third read" will support students in productively struggling to make connections between the word problem and the mathematical ideas central to the problem situation.

Appendix B

Rubric for TRU Talk in PLCs

Rubric for TRU Talk in PLCs

The purpose of this rubric is to capture the alignment of PLC teachers' talk with the TRU Framework dimensions and the rigor of such PLC teachers' talk.

The Mathematics	Cognitive Demand	Equitable Access to Mathematical Content	Agency, Ownership, and Identity	Formative Assessment
To what extent does PLT teacher talk focus on their own understandings of the accuracy, coherence, and justification of the mathematical content?	*To what extent does PLT teacher talk focus on classroom interactions that create and maintain an environment of productive intellectual challenge that is conducive to students' mathematical development?*	*To what extent does PLT teacher talk focus on supporting all students in equal access to and meaningful participation with the mathematics?*	*To what extent does PLT teacher talk focus on providing students opportunities to conjecture, explain, make mathematical arguments, **and** build on one another's ideas, in ways that contribute to students' development of agency, ownership, and their identities as doers of mathematics?*	*To what extent does PLT teacher talk focus on monitoring and helping students to refine their thinking?*
1: The mathematics discussed is not at grade level, or discussions are aimed at "answer getting." Explanations, if they appear, are largely procedural.	1: PLT teachers suggest and/or agree with classroom activity or teacher intervention that constrains students to activities such as applying straightforward or memorized procedures.	1: PLT teachers suggest and/or agree with classroom activity that leaves some students disengaged or marginalized, and differential access to the mathematics or to the group is not being addressed.	1: PLT teachers suggest and/or agree with classroom activity or teacher interventions that either constrain students to producing short responses to the teacher OR do not address clear imbalances in group discussions.	1: PLT teachers suggest and/or agree with classroom activities that are simply corrective (e.g., leading students down a predetermined path) and the teacher does not meaningfully solicit or pursue student thinking
2: Discussions are at grade level but are primarily skills oriented, with few opportunities for making connections (e.g., between procedures and concepts) or for mathematical coherence.	2: PLT teachers suggest moves (e.g., hints or scaffolds) and/or agree with classroom activity that offers possibilities of productive engagement or struggle with central mathematical ideas but scaffolds away some challenges and/or removes some opportunities for productive struggle with central mathematical ideas and/or engagement in mathematical practices.	2: PLT teachers suggest moves and/or agree with classroom activity that illustrates some efforts to provide mathematical access to a wide range of students; OR the teacher does not support student participation in group activities like student-to-student discussion.	2: PLT teachers suggest moves and/or agree with classroom activity that allow at least one student to talk about the mathematical content, but the teacher is still the primary driver of conversations and arbiter of correctness OR students are not supported in building on each other's ideas.	2: PLT teachers suggest moves and/or agree with classroom activity that solicit student thinking, but subsequent classroom discussion does not build on nascent ideas. The moves and/or classroom activity are corrective in nature, possibly by leading students in the "right" directions.
3: Explanations of and justifications for central grade level mathematical ideas are coherent.	3: PLT teachers suggest specific moves (e.g., hints or scaffolds) and/or agree with classroom activity that support students in productive struggle in building understandings of central mathematical ideas and engaging in mathematical practices without scaffolding away challenges.	3: PLT teachers suggest specific moves and/or agree with classroom activity that would actively support and to some degree achieve broad and meaningful participation from all students; OR to establish participation structures that result in such engagement.	3: PLT teachers suggest specific moves and/or agree with classroom activity that allow at least one student to put forth and defend their ideas/reasoning; AND, students build on each other's ideas OR the teacher ascribes ownership for students' ideas in subsequent discussion.	3: PLT teachers suggest specific moves and/or agree with classroom activity that solicits student thinking, AND plans for subsequent classroom discussion to respond to those ideas, by building on the productive beginnings or addressing possible misunderstandings.

We adapted, with permission, Schoenfeld et al.'s (2014) TRU Math Rubric to fit our context of PLC teachers' talk. See Table 5 for an example of different ratings for a sample TRU dimension.

Table 5

Example of TRU Talk Ratings for Formative Assessment

TRU Talk Rating	Formative Assessment Example
1	"The teacher could correct the student's matching mistake and show them how to correctly match cards in the card sort."
1.5	"The teacher could ask a student to share their thinking about a match they made and then show them how to correctly match cards in the card sort."
2	"The teacher could elicit student thinking by giving them blank cards and asking them to create their own word problem scenario similar to those in the activity."
2.5	"The teacher could elicit student thinking by giving them blank cards and asking them to create their own word problem scenario similar to those in the activity. Then, the teacher could work out one of the problems on the board."
3	"The teacher could elicit student thinking by giving them blank cards and asking them to create their own word problem scenario similar to those in the activity. Then, the teacher could facilitate a whole-class discussion about these scenarios to build on students' thinking and address any misunderstandings."