"OH! THAT'S INTERESTING!": CAPTIVATING STUDENTS WHO HATE MATHEMATICS WITH MATHEMATICAL IDEAS

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The positive relationship between students' attitudes toward mathematics and mathematics achievement is well documented. Yet there is a worsening problem of low appeal of mathematics especially at the secondary level. Therefore, in this paper we focus on three high school students who report a strong dislike of mathematics. By analyzing student surveys, interviews, and lesson observation data, we examined how some mathematical lessons improved these students' experiences (i.e., their aesthetic dimensions). We found that while student preferences varied, each student was interested in lessons that centered them as sense-makers and in which the content unfolded with suspense. Such lessons led to positive aesthetic responses such as surprise, curiosity, and satisfaction. Thus, lessons can be designed in which even students with the most negative views of mathematics can find mathematical concepts interesting.

Keywords: Affect, Emotion, Beliefs, and Attitudes; Instructional Activities and Practices; Curriculum; High School Education.

There is widespread agreement that lessons that are *captivating* (i.e., those that students find interesting and engaging) increase student learning. For example, across all cycles of the TIMSS Assessment since 1995, there has been a consistent relationship between students liking mathematics and increased mathematics achievement (Mullis et al., 2020). Unfortunately, in the U.S., there is a persistent problem with low appeal of mathematics, especially at the secondary level. For example, by the time U.S. fourth graders in 2015 reached eighth grade, the percentage of those who did not like learning mathematics nearly doubled to 45% (Mullis et al., 2016, 2020). This trend of secondary students disliking mathematics is also becoming worse over time; between 2011 and 2019, the portion of U.S. eighth grade students who do not like learning mathematics increased by five percent (Mullis et al., 2012). This phenomenon calls for an urgent focus on this growing collection of students who do not enjoy mathematics, and for an attempt to connect the types of learning experiences offered in their mathematics classrooms with their *affect*, which we draw from Middleton and colleagues (2017) to define as a students' attitudes, interests, beliefs, and emotions.

Luckily, student affective reactions to mathematical experiences are not fixed and are responsive to the teacher and learning environment. Student interest in a lesson, for example, can be piqued even if the student does not generally find mathematics interesting (Hidi & Renninger, 2006; Middleton et al., 2017). The same is true for other aspects of affect, such as their emotions and engagement (Goldin, 2017; Hannula et al., 2016; Satyam, 2020). Although long-term attitudes, beliefs, and emotional views of mathematics may be relatively stable without much day-to-day variation (Middleton et al., 2017), an accumulation of new types of experiences can eventually lead to shifts of a student's overall affective traits (Hidi & Renninger, 2006; Marmur, 2019; Middleton et al., 2017). This gives each lesson power to shape a student's general disposition towards mathematics—and by extension, their future achievement.

Thus, we're interested in how mathematical experiences potentially impact a student's emotional reaction, what we refer to as the *aesthetic dimension* of a mathematical learning experience. In this study, we analyze the experiences of three students—Cassie, Mateo, and Mei

(pseudonyms)—who each have a strong and stable dislike of mathematics. Despite this mathematical disposition, however, each student found at least one observed mathematics lesson interesting. We analyze their aesthetic responses to these lessons and describe the curricular designs that enabled these positive student experiences. By connecting the mathematical content and lesson design with student affect and interest, we begin to answer the questions: *What aspects of a lesson captivate students who particularly dislike mathematics? What can we learn about the designs of mathematics lessons from these students?*

Theoretical Framework

To connect a student's mathematical experience and their emotional reaction, we draw from theoretical perspectives that attend to the aesthetic dimensions of experiences. Drawing from Dewey (1934) and Sinclair (2001, 2008), we interpret the aesthetic dimension of an experience not as a characteristic of the experience itself, but rather as the way the experience emotionally impacts an individual. Thus, it is the interaction of an individual and the experience (e.g., a theatrical play or mathematics lesson). Although the notion of aesthetic is often associated with artistic beauty, we instead contend that *all* experiences, both positive and negative, provide opportunities for aesthetic impulses which can compel an individual to act (Dietiker, 2015). For example, in a surprising moment, an individual might be compelled to wonder or question. On the other hand, when an experience is uninspiring, this individual might instead feel bored and be compelled to shift their attention to something else. Other aesthetic experiences, particularly in mathematics classrooms, can include a sense of frustration when things persistently don't make sense and the individual does not have an indication that this will change, as well as satisfaction when an individual shifts from confusion to clarity (i.e., when it feels like the pieces are coming together to make a coherent whole). Thus, the aesthetic dimensions of a mathematical experience can motivate an individual to persevere through difficulty or can, unfortunately, lead them to quit in frustration.

Methods

This study is part of a three-year design-based research project, called the *Mathematically Captivating Learning Experience (MCLE) Project*, which focused on better understanding the aesthetic dimensions of high school mathematics lessons. In MCLE, six veteran high school mathematics teachers designed and taught lessons specifically to increase positive aesthetic opportunities such as surprise or suspense. In order to learn about student aesthetic experiences, all participating students were surveyed after each lesson about their experiences, and a subset of students were selected for a post-lesson interview. In addition, to learn whether these MCLE lessons led to improved aesthetic experiences, two to four additional "everyday" lessons were observed, and students were surveyed using the same protocols so that students would not know which lessons were specially designed for the project. Finally, to learn whether the improvement of the aesthetic experiences only occurred for certain students (for example, those who already like mathematics), a dispositional survey was given at the start of the year.

In the present case study, which draws from the 2018-2019 cycle, we analyze three participating high school students who especially dislike mathematics but who also found at least one of their mathematics lessons interesting. We draw from the post-lesson survey and interview data to describe the aesthetic experiences of three students who expressed strong dislike of mathematics in their Disposition Survey. In the rest of this section, we describe the surveys and interviews used for the present study, explain how we identified the three focal students, and finally describe our data analysis.

Data Collection

At the start of each academic year, all participating students took a Disposition Survey, which took approximately 20 minutes to complete. This survey asked Likert questions on the extent to which they agree or disagree with statements about views of mathematics (e.g., "I like mathematics"), their views of activities related to mathematics ("I like to think about math or solve puzzles outside of school."), and their experiences in mathematics classrooms (e.g., "my teacher makes lessons interesting"). It also included an open-response question that asked students to describe how they typically feel during mathematics lessons. The development and testing of the Disposition Survey is described in Riling et al. (2018).

Researchers observed approximately three MCLE lessons and three everyday lessons. For each observation, researchers took field notes. Video cameras were positioned from different angles of the classroom to capture all interactions and student emotional expressions. In addition, audio-recorders were placed around the classroom to capture student and teacher discourse.

After each lesson observation, all student participants were given a Lesson Experience Survey (LES, Riling et al., 2019) which prompts students to select three terms that best describe their view of the lesson from a list of 16 positive, neutral, and negative descriptors (e.g., fascinating, fine, dull). These descriptors were generated by asking high school students questions like, "If you wanted to describe an experience that made you [emotional response, e.g., bored], what word would you use?" The LES also contains Likert questions to measure the degree to which they understood the content (from 1=don't understand, 2=somewhat understand, and 3=understand), were interested in the lesson (from 1=very bored to 4=very interested), as well as multiple four-point questions (1=strongly disagree to 4=strongly agree) on whether they wished more lessons were like this one, felt the content was relevant to their lives, found the content challenging, or felt that the lesson was typical.

In addition, to better understand student experiences, a small number of students (between two and four) were individually interviewed. These students were selected to reflect a variety of perspectives (e.g., engaged, disengaged), while seeking representation across gender, racial, and ethnic identities. During these interviews, students were asked to describe and explain any feelings they had during the lesson and were offered a list of descriptors to choose from.

Participants

From more than 200 participating students of MCLE during the 2018-2019 cycle of the design-based research, we reduced the pool by examining Disposition Survey responses and identifying those students with (a) no positive responses about mathematics and (b) at least three strongly negative responses. We eliminated those with fewer than three completed LESs and any with inconsistent responses with regard to views of mathematics. Of the 12 remaining students, we sought those who provided evidence of interest in at least one lesson and disinterest in at least one lesson. This selection led us to our three cases: Cassie, Mateo, and Mei (see Table 1). These students each had different teachers (Ms. Cherry, Mr. Ash, and Ms. Willow, all pseudonyms). Across the students, there were differences in grade, gender identity, race, and ethnicity. However, all three students are multilingual and were in Algebra 2.

Table 1: Student Demographic Information and Identifications

	School	Teacher	Grade	Gender Identity	Race/Ethnicity
Cassie	B	Ms. Cherry	10	Female	Black or African American
Mateo	A	Mr. Ash	11	Male	Hispanic/Latino
Mei	A	Ms. Willow	10	Female	White, Asian

Data Analysis

We conducted an inductive qualitative analysis (Merriam & Tisdell, 2015) of the Disposition and LES surveys, post-lesson interviews, and lesson observation data. We first built disposition profiles of each student using the responses on the Disposition Survey, writing analytic memos describing their attitudes about mathematics and mathematics class, past experiences, class activity preferences, and beliefs about themselves as mathematics learners. We then examined their LES responses, reading closely and constantly comparing their lesson experiences with their disposition. We analyzed within a single lesson and across each student's four or five LESs, noting both patterns and outliers in their responses. To triangulate, we also reviewed transcripts of the post-lesson interview(s) for each student, looking for both confirming and disconfirming evidence of our interpretations of their disposition and lesson experiences. Throughout this analysis, we noted aesthetic and affective constructs such as when students said they felt "frustrated" or "relieved." We used these responses to point us to key lesson moments.

We next explored the lesson data for all of the lessons our case students experienced (14 lessons overall). For each lesson, we drew from the observation notes, audio and video recordings of lessons, lesson materials, and whole-class LES data to write a detailed memo describing the lesson. For each student, we looked across the lessons they experienced for patterns, comparing again with the disposition profile. We also closely analyzed the aspects of the lesson(s) that were interesting to our students and those that were not, contrasting key differences in design and enactment. Through this process, we consistently returned to each student's voice to ground our analysis and remain close to the data. After determining important aspects of the lessons that attracted each focal student, we completed a cross-case analysis to generate themes across all three cases. All interpretations were discussed and reconciled as a research team and alternative interpretations and disconfirming evidence are included.

Findings

The three students selected for this study all expressed extreme dislike of mathematics. Yet despite this similarity, what they disliked differed. In this section, we illuminate ways that lessons can encourage these students—chosen for their negative views of mathematics—to become interested in and positively affective towards a specific mathematics class experience. To start, for each student, we describe their views of mathematics. Next, we present their reactions to the specific observed lessons collected for this study (both positive and negative) to identify the captivating curricular features. Finally, we present themes across the three cases.

Cassie

Cassie's dislike of mathematics, independent of the classroom, is profound. She thinks mathematics is very boring and does not enjoy learning about it. However, Cassie has strong positive feelings toward her current teacher, Ms. Cherry; she feels that Ms. Cherry respects her ideas, encourages her, and tries to understand how students feel. Nevertheless, Cassie does not look forward to mathematics class. In her current class, she feels stressed and describes how important it is that she feels "knowledgeable" in relation to the other students. She discusses in her interview feeling "disappointed" if she shares an answer and it is wrong, and so is reluctant to share her ideas. Further, Cassie prefers to have consistent support and validation from her mathematics teacher and does not like "when another student describes [the solution to a math problem]. It actually confuses me even more." Cassie is a student who wants to understand, is motivated by others' perceptions, and views the teacher as her primary resource.

Given Cassie's mathematical disposition, it is not surprising that she largely found the observed lessons boring as their sequence of activities generally followed the pattern: warm-up

problem(s), student explanation, small group problem solving, and answer confirmation by Ms. Cherry. Cassie's focus on being correct and having her answers validated before sharing would be a challenge to her participation in this structure, as she would be asked to discuss with her group and share her thinking before knowing if she was right. Overall, Cassie found these lessons uninteresting, irrelevant, and too challenging.

However, Cassie found one MCLE lesson interesting, and this lesson differed from the typical lesson in Cassie's class in important ways. In the warm-up for this lesson, students were asked to describe a graphed translation of a linear function and justify their reasoning. This presented an opportunity for differing arguments; students could describe vertical or horizontal translations. Although Cassie did not explicitly mention this portion of the lesson in her post-lesson interview, given her desire to be perceived as knowledgeable, this opener with a focus on justification—rather than a single correct solution—could have enabled her to participate without fear of being wrong.

Next, students graphed different parent functions and translations by hand and used graphing calculators to check their work. This segment of the lesson was similar, however, to one of the lessons Cassie found uninteresting. In that uninteresting lesson, students contended to find the polynomial equation for a given graph (modeling a roller coaster), using graphing software to verify potential equations. While both lessons asked students to use technology to investigate connections between representations, Cassie's responses to these two lessons differed for key reasons. First, Ms. Cherry posed the roller coaster problem as a whole-class race to see who could find the correct equation first. When another student found the equation and was publicly praised by Ms. Cherry, Cassie describes, "I felt sad because I didn't get to - I wasn't the one who discovered it." That lesson then moved on without an opportunity for Cassie's sense-making. In contrast, in the interesting lesson, Ms. Cherry framed the exploration in non-competitive terms, and Cassie's process was not truncated by a single correct answer. This extended inquiry was important as at first Cassie did not like this portion of the lesson. In her interview, she says, "I was really frustrated … I was really confused." It is possible that Cassie felt uncomfortable with her initially incorrect predictions, but since the lesson was designed without a narrow focus on a single solution and provided ample time for exploration, she remained engaged.

During this exploration, Cassie began to make sense of the connections between the equations and graphs. She explains, "an aha moment, like, it was just like a thought, like 'maybe, this works for this, or this works for that.' But I was … kind of nervous, but what if I'm wrong or what if I'm right?" Since Cassie likes to find answers and feel confident in their accuracy before sharing with a group, this progress beyond her frustration was key for Cassie to remain engaged. Despite her very strong negative views of mathematics, Cassie felt satisfied by this "aha moment." Later in the lesson, Ms. Cherry said something Cassie had found would be discussed in the following class, and Cassie described feeling proud: "I was like, look! … this is similar to what we were looking for, even if it was wrong … I figured out something, and I was like, 'Oh! That's interesting!'" Cassie notes her interest in discovering something—even if it was not completely correct. This is further evidence that Cassie can be engaged with mathematical ideas in ways that disrupt her performance anxiety. When she felt frustrated by not understanding at the beginning of the lesson, Cassie's struggle was productive because she could validate her own conjectures using technology, had the time to make sense of the mathematics, and did not have her thinking cut short by a focus on a single correct answer. Cassie, a student who strongly dislikes mathematics, felt interested and engaged in learning.

Mateo

Based on the analysis of his responses to the Disposition Survey items, Mateo dislikes both mathematics class and mathematics as a subject. While he has had positive and negative experiences in earlier mathematics courses, he strongly disliked his 10th grade class, which is likely highly influential on his current reported views. Mateo's previous experiences also hinged directly on the teacher; both in years he enjoyed and those he did not, Mateo reported that the teacher was the most influential factor. In line with this, Mateo prefers lessons where the teacher takes an active role and prefers lecture over all other class activities. While Mateo's concept of "lecture" is unclear, we can conclude that Mateo is drawn to mathematics classes that have a high content focus and direction by the instructor. Notably, Mateo was the only student of the three that describes feeling "alright" in his current mathematics class. Further, he is the only one who sees mathematics as useful for his career or as potentially being amazing. This suggests that Mateo has some potential interest but has not had the learning experiences to foster this interest. He is bored and dislikes his mathematics classes but not because he is completely apathetic toward the subject; Mateo is a student seeking out mathematical stimulation.

Therefore, it's not a surprise that the lessons that he enjoyed were mathematically suspenseful. The first lesson that Mateo found interesting was an MCLE lesson introducing imaginary numbers. The lesson gave students the expectation that they would be solving quadratic equations to find numbers that have certain sums and products. When a challenge emerged where the solutions to a quadratic equation had an integer sum and product but were not real numbers, students were motivated to solve the mystery. It was not until students had grappled with the idea and concluded that the solution would need to include the square root of a negative number that imaginary numbers were introduced. Mateo reported he understood the topic of this lesson, wished more lessons were like this one, and described it as "intriguing." The intentional design of a lesson around a mathematical mystery captivated Mateo because it caused him to question his mathematical assumptions (i.e., that all numbers are real).

The second lesson Mateo found interesting modeled exponential decay with dice. Mr. Ash distributed a total of 100 dice to the students and prompted them to roll all the dice simultaneously and remove those dice that showed a number divisible by three. This procedure was repeated until all dice were eliminated. The lesson captured students' attention as they waited to see when the final die would be eliminated. The lesson then had students compare the probabilistic results with the experimental outcome. Mateo reported that the lesson was interesting and found it "enjoyable" and "satisfying." We can gain more insight about this lesson from his classmate's post-lesson interview. His classmate called the lesson suspenseful and further stated, "We don't do a lot of stuff like that and so like an activity like that was pretty fun." While Mateo did not find the topic of exponential decay challenging, he wished more lessons were like this one and indicated it was the only lesson that he felt was relevant to his life. This lesson, like the imaginary numbers lesson, elicited an aesthetic response of surprise and suspense through the unfolding mathematical ideas, a quality that seems key to attracting Mateo.

Looking at the lessons Mateo did not like, we can see that Mateo found these lessons uninteresting and was generally apathetic towards them. These three lessons involved a range of instructional strategies including group work, pattern recognition, guided exploration, and lecture. Even though for two of these lessons the majority of other students were interested, Mateo reported feeling unchallenged, bored, and did not find any of the lessons relevant to his life. While the lessons that involved problem solving could provide an element of unpredictability, Mateo did not experience them in that way. In these lessons, the investigations

involved repeated calculations or pattern recognition, which are not particularly suspenseful and did not call Mateo to stretch his sense-making skills. Rather, Mateo is interested when lessons contain mathematically suspenseful activities and keep the important mathematics central.

Mei

Mei exhibits the strongest negative feelings about mathematics of our case students. None of Mei's Disposition Survey responses about mathematics were positive or even neutral. When considering mathematics in school, her teacher is pivotal to Mei's experience. In previous years she credited her teacher and her classmates as a reason she liked or did not like her class. In her current class, Mei typically feels "frustrated and confused." While Mei thinks her current teacher, Ms. Willow, is encouraging and respectful of her ideas, she feels that Ms. Willow fails to make lessons interesting or make the material clear to her. As a result, Mei turns to her peers to negotiate and validate her sense-making in mathematics class. Mei says she pushes herself hard to completely understand her lessons, and yet, she remains stressed, confused, and frustrated.

The lessons Mei did not find interesting were representative of typical lessons in her class that are teacher-led and highly scaffolded, lacking opportunity for student exploration and sense-making. This could contribute to Mei's consistent confusion in class. Additionally, Mei specifically states that she struggles with algebraic abstraction: "I get confused when the letters are involved in math." These non-captivating lessons were heavily dependent on algebraic manipulation with little connection to concrete values or multiple representations.

In the one lesson that Mei found interesting, the design connected the algebraic and numerical representations of logarithms. Near the beginning of this MCLE lesson, students examined a table of logarithm values looking for pairs with integer sums and differences and then used these patterns to discover the addition and subtraction properties of logarithms. Mei said the problems at the beginning of the lesson appeared "scary," stating repeatedly that she finds the abstraction of algebra difficult. However, since the properties of logarithms emerged from patterns in their decimal values, Mei was motivated and had a point of entry. She stated: "I was … pleasantly surprised that I was able to solve the questions … because of the pattern that makes sense." The way the algebraic abstractions were built from numerical patterns interested Mei and fostered her sense-making.

This lesson was also designed for student inquiry; the formal logarithm properties were not made explicit until students had made the connection independently. In other lessons, the new content was provided early in the lesson by Ms. Willow or in a whole-group discussion, where Mei felt lost. In contrast, this lesson was designed to create suspense through the hunt for patterns. Significantly, the lesson also gave Mei ample time to search for these patterns, thereby building suspense and increasing the chances that she accomplishes her goal (i.e., making sense of the pattern). Similar to other lessons, Mei found this lesson challenging and frustrating, but the active role she was offered and the time she was provided allowed her to be successful in her sense-making, which led to her to also describe the lesson as "thought-provoking." Thus, Mei's aesthetic experience of satisfaction and surprise were a result of the lesson's challenging and suspenseful design and contributed to her interest.

Another aspect of Mei's aesthetic response to this lesson was curiosity. She felt a sense of "wonder" and a desire to continue exploring the conceptual foundation for the patterns she discovered. Even a week later, Mei's curiosity continued to impact her experience. Despite finding this later everyday lesson uninteresting, Mei's other responses were uncharacteristically positive. For example, similar to how she described the interesting lesson, she found this later

lesson frustrating, thought-provoking, and satisfying, and it was the only lesson besides the logarithm properties lesson that she wished more classes would be like.

Themes Across Cases

Two central themes emerged across the mathematics lesson experiences of these three students. First, the lessons these students found interesting and engaging all prioritized students grappling with important mathematics, with appropriate time for inquiry. These lessons decentralized the cognitive work away from the teacher, instead positioning the students as sense-makers. Also critical to this success is the focus away from a single correct answer, which turned classmates into competitors and truncated our students' thinking. Instead, the interesting lessons incorporated ample time for students to explore mathematical relationships with the appropriate tools to evaluate their own progress.

A second key finding across the student cases was the centrality of suspense. Not only were the lessons that interested our students open-ended, but they also were crafted so that students could experience surprise, wonder, or satisfaction. These aesthetic responses were due to the lessons' arc—the mathematics was accessible for Cassie, Mateo, and Mei to begin working without understanding the climax that the content was building towards. In Mei's case, this involved searching openly for patterns within a table, but concluded with some of the central logarithmic properties. For Mateo, what began as a sum and difference number puzzle motivated the need for an expansion of the number system. The intentionally suspenseful and surprising aesthetic of these lessons allowed for continued student engagement, satisfying "aha moments," and sustained curiosity even beyond the class period.

Discussion

Cassie's, Mateo's, and Mei's experiences in mathematics class offer a key message for educators and curriculum designers: lessons can be designed to interest and engage even those students who have the most negative views of mathematics. Further, the aspects of the lessons that reach these students are not superfluous or gimmicky. Cassie, Mateo, and Mei were all drawn to lessons that positioned them as sense-makers about important mathematical concepts, and they each fostered a sense of satisfaction from their own mathematical engagement. Although we only focused on three students that had different reasons for disliking mathematics, we were surprised that the captivating lessons shared common characteristics of opportunities for student sense-making as well as structuring the unfolding content so that suspense could be built. We wonder how much their previous mathematical learning experiences had offered satisfaction; we contend that all students have a right to experience meaningful mathematics in ways that enable students to exclaim "Oh!" as Cassie did, as the mathematical ideas come together and make sense. We argue that more research needs to address the particular needs of students like these, in the hopes that all future students can experience the wonder and satisfaction of mathematical inquiry. With repeated positive aesthetic experiences with mathematics, we hope to reverse the trend in poor attitudes toward mathematics in high school.

Acknowledgements

This material is based upon work supported by the National Science Foundation under Grant No. #1652513. We would also like to thank teacher participants and research team members for their help in collecting the data for this study.

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