Teaching secondary mathematics lessons for joy and wonder

Kayla Scheitlin, Leslie Dietiker and Meghan Riling describe the joy and wonder they find in teaching mathematics.

In high school mathematics classrooms, finding opportunities for students to experience beauty, wonder, or joy is an "important but often neglected" aspect of learning mathematics (NCTM, 2018, p.12). How pleasurable and productive can learning and teaching feel when our students excitedly pose their own questions and drive lessons forward? To understand what makes some mathematics lessons particularly exciting, we find it helpful to think of them as mathematical stories (Dietiker, 2016). With this phrase, we do not refer to word or story problems, but instead refer to the story of the lesson, that is, how the content unfolds with opportunities for growing suspense and/or plot twists. This approach has enabled us to understand how students experience mathematics. Why they might they express boredom, suspense, or curiosity (see for example, Ryan and Dietiker, 2018)? Captivating mathematical stories are those that create opportunities for students to raise questions and anticipate what is going to happen next, thus engaging in mathematical inquiry. We wondered if, in the past, we had revealed information too early in our lessons, eliminating motivation for our students to want to figure anything out. If the author of a mystery novel revealed everything on page one, you would probably stop reading. We wondered how lessons might instead enable students to feel suspense or surprise about mathematical ideas.

As a group of teachers and researchers in the USA who have collaborated to design captivating lessons with this story perspective, we have been exploring how centring students' aesthetic reactions in our design process can result in lessons that are more captivating (Dietiker *et al.*, 2019). In this article, we share a lesson designed by one teacher (Kayla) in collaboration with her co-authors and colleagues. This lesson was enacted with her Year 10 students (aged 14–15) in a non-accelerated integrated mathematics course at a diverse high school in a working-class suburb. When designing this mathematical story about linear functions, this teacher had two purposes: she hoped to have her students engage in rich mathematical discussion even with students' varied proficiencies in English, and she aimed to have students ask nuanced questions about algebraic linear functions, even with limited fluency in related concepts. She aimed to heighten student suspense and curiosity. When teaching this story, she leveraged her lesson design's opportunities to enact a mathematical experience for students marked by joy and wonder.

As you will see, the enactment of this lesson exceeded this teacher's goals and thus provides us an opportunity to share what we learned about teaching for wonder and joy. We first describe the lesson as it unfolded in the classroom to offer insight into how the students may have experienced the lesson. We then describe how the teacher's preparation and in-the-moment decisions enabled a lesson full of student exuberance and engagement with mathematical ideas. We end this article with some take-aways that can inform other teachers' work.

The Mathematics Lesson

At the start of the lesson, I (Kayla) distributed 14 cards to each group of three or four students (see Figure 1). On each card was a representation of a linear function (that is, a graph, a table, an equation, or a contextual description, such as 'I have \$6 in my wallet. Each day, I spend \$0.50'). I introduced the activity by saying:

> I want you to create groups of cards that you think are similar.

As the groups started to decide how to approach this problem, I circulated about the classroom, answering questions and prompting students to get started.

Most groups were eager to get started. Many started comparing pairs of cards and finding matches as shown in Figure 2. ("Alright, no, no, no, no, no, no. These two go together. No, they don't. Oh, my gosh!"). Students naturally began to develop arguments for describing the relationships they were seeing, often focusing on key points, such as the *y*-intercept, and rate of change. For example, one student defended his choice to pair the line graphed on Card I and the description on Card G by focusing on the concept of slope, explaining, "No, no, no. It's G and I, I promise you! Cause look, this one starts at zero, six and then it's creating new points by moving left four and two up so if I start at twelve and I move left four and two up."

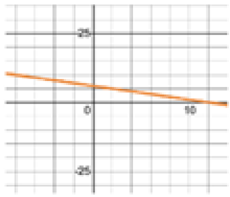
A

$$y = \frac{-1}{2}x + 6$$

B

The line that goes through the points (4, 4) and (10, 1).

C



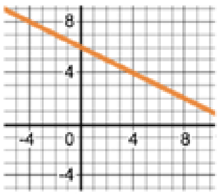
D

x	y
0	6
2	5
4	4
6	3

E

x	y
0	6
1	5.5
2	5
3	4.5

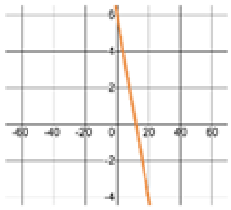
F



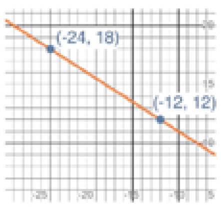
G

Start at (0, 6).
Move left 4
and up 2.

H



I



J

$$y = \frac{3}{-6}x + 6$$

K

The line that goes through the points (522, -255) and (1002, -495).

L

Start at (4, 4).
Move down 3 and right 6.

M

I have $6 in my wallet. Each day, I spend $0.50.

N

x	y
0	6
-24	18
-96	54
-312	162

Figure 1: The linear function representation cards.

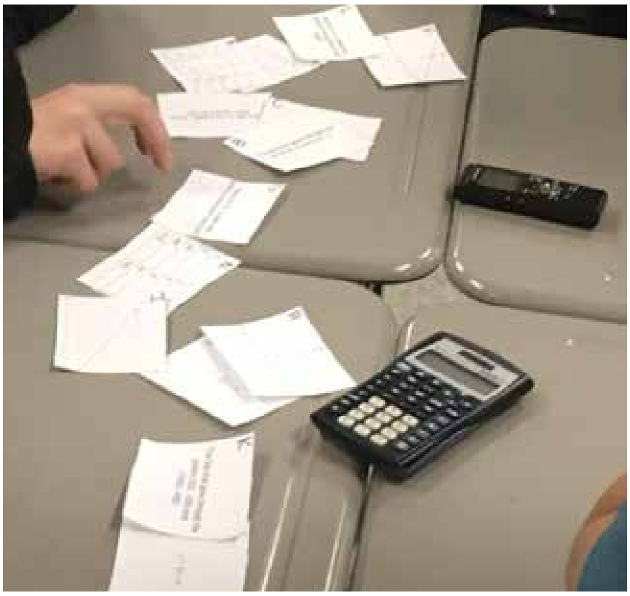


Figure 2. Students consider various groups of cards.

After 10 minutes of similar debates in groups, it became evident that each group of students had at least two groups of cards they were confident they could defend. I pulled the class together, asking: “Who’s got a group that they feel really sure they’re connected, that you’d be willing to share?” As multiple hands raised, I selected Eugene (all names are pseudonyms) who shared that cards E, M, and C formed a group (see Figure 3). I asked, “Why?” Eugene explained, “M relates with E because its going down by point five each time, and C starts around the area of six and it seems to go ... , it’s not very fast in how far it goes down or, how do I say it, like it starts off at six and seems to correlate with where it starts and stops.” This set off a debate as Kevin argued that the graph C should be replaced with the graph F. He explained, “Look, because zero, six, right? Here’s the y-intercept, right? And it matches up! Then you go down to one, and it matches up with five point five, and two is five, three is four point five."

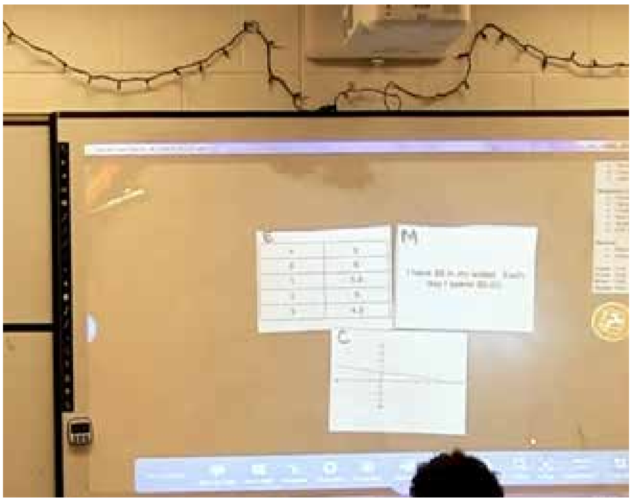


Figure 3. Eugene displays a proposed group of related functions.

As the full class discussion continued, more connections were considered, and seven more students joined in the conversation. We offer this excerpt in its entirety so that readers may experience the robust and enthusiastic student and teacher interactions that show heightened moments of celebration:

Zaya: So, we put M, E, and A together.

Teacher: M, E, and now you think A goes with those? OK, why?

(Students murmur "Oh my goodness!", "No!", "Yes!")

Zaya: Because six is the initial value. And it's going down by a half.

Mandy: Exactly, I said that!

Teacher: OK. Wait, wait. Can someone state what Zaya just said again?

Jessica: Oh! I see it!

Brandon: Yeah, that's facts! (Multiple students laugh and high five)

Mandy: I said that!

Teacher: Ok Mandy, you connected with what Zaya said? (Mandy nods) OK, can you say it one more time? Emir, hold on.

Mandy: So, because it says I have $6 in my wallet, each day I spend fifty cents so... it's like the equation of A.

Teacher: So you think A matches M?

Mandy: Yea.

Teacher OK, how do you see the fifty cents?

Mandy: Because of the one, negative one half... over two.

Teacher: Ok, Emir, do you want to add on?

Emir: I think A goes with K.

Teacher: You think A goes with K?

Lisbeth: Oh, my god!

Teacher: What! You think A goes with K? That's crazy! What do you mean?

Devon: Emir, are you kidding me!

Brandon: (shaking his head) Oh, wait! (starts nodding) Ahh! (now agreeing with Emir)

Emir: (Looks at Brandon and smiles) Nah, I'm right!

Finally, a student suggested that all the cards went together. I feigned disbelief once again, but he insisted that the cards went together. I asked groups to revisit any unmatched cards. During this time, I overheard many excited exclamations (For example, "Boom! Boom! Boom!", "Yo! N and I go together Miss!", and "Yo! Brandon's right! Man that's crazy!") and student celebrations. As a whole class, students provided reasons for how they knew each card had a *y*-intercept of (0, 6), even those that hid this characteristic of the line ("It's not there. The six is invisible, but when you put it on a graph, it's there"). This led to a new question about how slopes that have different numerical forms could be the same. Although we did not have enough time to finish this discussion, students eagerly produced ideas on their potential equivalence and left the classroom with this question to ponder. Even after I wrapped up the lesson, students continued to propose conjectures about how to tell whether the functions' slopes were equivalent.

Teaching strategies that enabled wonder and joy

I purposefully crafted this task, in collaboration with others, to create conditions that would compel students to reason mathematically. I aimed to introduce the potential for my students to wonder about something unexpected, namely, "How could it possibly be that all of these cards represent the same function?" My students had previously learned about linear functions and had some familiarity with slope and *y*-intercepts. Yet, in previous years, I remembered students assuming that when slopes are represented in different numerical forms (such as $\frac{1}{2}$ and $\frac{1}{4}$, the lines would not be parallel. Thus, I hoped this lesson would motivate a need to understand when and why slopes expressed as fractions are equivalent.

My task design deliberately enabled extended mathematical inquiry, increasing the potential for rising tension and suspense. The task played on my students' reasonable assumption that the cards represented multiple functions. Previously in this course, I had asked students to group cards, similar to what is described in Swan (2008), whereby all resulted in multiple groups. Furthermore, I decided to have many cards (14 in all) because I suspected that this would make it difficult for students to immediately see the connections across all of them - while working in their small groups, it is likely that students would need to select only a few cards at a time to compare. This created some tension within groups as students had different ideas about which cards to try, naturally leading to reasoning about slope and points. Since student groups would probably create different matches that involved the same function cards, I predicted that conflict would also arise during the full class discussion as students presented their matches. This tension and conflict, much like in a novel, kept my students anticipating what would happen next. As they reconciled other students' matches against their own, they broadened their understanding of how representations that look very different can actually be the same.

In addition to the design of the task, I recognise in hindsight that some of my in-the-moment teaching strategies enhanced the captivating nature of the lesson. To experience tension, my students first needed to work together in small groups to make their own connections between the cards. Therefore, when I visited these groups, I decided not to evaluate student ideas or interject my thinking into their discussion but instead asked questions that were aimed to make their thinking visible to their peers. I often acted uncertain about what cards went together, saying things like, "I don't know either" and avoided nodding or smiling to indicate agreement with their answers. I asked why students believed certain cards were matches, rather than just which cards matched. When I noticed that one student was dominating a group's conversation, I asked the other group members questions about his thinking to draw them into the problem, too. These responses created the conditions for students to do the questioning, thinking, and evaluating themselves. This extensive student group time enabled wider student participation in our first full class discussion, as all students had some initial ideas.

Finally, I feigned disbelief at several points in the lesson. I used this approach when I sensed that students had enough conviction in their claims and curiosity in what was going to happen next to not simply abandon their ideas due to my quizzical response. For example, when I said, "that's crazy!" to the student's suggestion that all the cards represented the same function, students eagerly waved their hands to contribute ideas, expressed disbelief when a student offered up yet another match, listened to the mathematical thinking of others, and praised their classmates and themselves for presenting important ideas. My expressed disbelief gave students motivation to persevere through the challenging work of explaining why each representation was in fact equivalent. Prolonging the tension allowed for rich satisfaction when students finally proved their claims.

Teaching for captivation

Much like literary authors, teachers play a crucial role in how students experience mathematical stories. We create the conditions for students to potentially experience the beauty, wonder, and joy of mathematics through the decisions we make before and during the lesson. The lesson described here shows that teachers can enact captivating mathematical stories by attending to students' aesthetic experiences, as they already do to students' understanding of concepts and skill development. Teachers can prematurely reveal the "punchline" leaving students with little to anticipate. Consider how changing the prompt to "Explain how each card represents the same function," would have changed opportunities for tension to arise. When teachers choose to withhold information, they can set up a gripping story that sweeps students up in finding out what happens next.

It is important to note that students did not experience this suspense or surprise as spectators, but as active participants in mathematics who struggled, took risks, and devised their own approaches to solving problems. As a result, they learned a great deal of mathematics. Students explained in interviews after the lesson that they felt "really smart" and deepened their understanding of "key words, like initial value, and slope." Throughout the lesson, students expressed pride in their developing understanding and appreciation of the contributions of their classmates. In addition, their enthusiastic verbal reasoning and arguments offered a rich opportunity for formative assessment of student understanding of linear functions.

As mathematics teachers, we too often try to smooth out students' experiences by eliminating frustration. But this lesson shows how allowing for frustration in a supportive environment can motivate learning and offer a joyful, informative experience for students and teachers. Without the frustration, we doubt that students would have felt any suspense or surprise. Any teacher can probe their lessons for sparks of tension that can be further stoked, or ask themselves if they've created any opportunities for students to wonder about mathematics. Too often, the pressure to get through content leads us to avoid these opportunities. We argue that this is a false dichotomy. Intentionally prioritizing student captivation can enable teachers and students to experience delight and wonder as we collectively advance through the mathematical story.

Acknowledgement statement of NSF and fellow teachers

This material is based upon work supported by the National Science Foundation under Grant #1652513. We appreciate the many contributions of the other participating teachers and our broader research team.

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