

WORTHWHILE PROBLEMS: HOW TEACHERS EVALUATE THE INSTRUCTIONAL SUITABILITY OF CONTEXTUAL ALGEBRA TASKS

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We investigate the beliefs that influence middle and high school algebra teachers' appraisals of contextual problems having diverse mathematical and pedagogical features. We asked six teachers to analyze six contextual algebra tasks and indicate how they would apportion instructional time among the six tasks based on their structure, pedagogical features, and connections to the real world. We recorded small-group discussions in which teachers shared their responses to this activity, and qualitatively analyzed their discussions for evidence of beliefs that influenced their appraisals of the tasks. The teachers' beliefs about contextual problems attended to task authenticity, opportunities for mathematical activity, obligations of tasks, and pedagogy and access. Our preliminary findings can inform future efforts to equip teachers with contextual tasks that develop students' algebraic reasoning and problem solving.

Keywords: Teacher Beliefs, Algebra and Algebraic Thinking, Professional Development

Multiple K–12 mathematics curriculum recommendations and standards documents call for students to learn to apply mathematics in settings beyond school. The National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics* indicate that instructional programs should equip students to "Recognize and apply mathematics in contexts outside of mathematics" (NCTM, 2000). The Common Core State Standards for Mathematical Practice state that "proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace" (NGA & CCSSO, 2010, MP.4); the Texas Essential Knowledge and Skills include a process standard with similar verbiage about applying mathematics to "problems arising in everyday life, society, and the workplace" (TEA, 2012).

While policy documents have underscored the importance of students learning to apply mathematical ideas beyond school, research has also highlighted the potential of contextual problems to help students develop and understand new mathematical ideas. The instructional theory of Realistic Mathematics Education suggests the use of "realistic" situations (which may or may not arise from "real-world" contexts) to stimulate the development of mathematical tools and concepts (Gravemeijer, 2005; Van den Heuvel-Panhuizen & Drijvers, 2020). We view both perspectives on contextual tasks – that mathematics can provide windows into the larger world, and that the world can inspire mathematical thinking and growth – as essential for developing mathematics courses that equip students for success in college, career, and civic life.

Prior research has investigated preservice and inservice teachers' dispositions regarding "real-world" connections in mathematics classes, including connections to issues of social injustice (e.g., Gainsburg, 2008; Girnat & Eichler, 2011; Simic-Muller et al., 2015). Additionally, Sevinc and Lesh (2018) have investigated preservice teachers' views about realistic mathematics problems and suggested interventions that can help teachers think critically about

contextual problems in textbooks from a modeling perspective. However, research on how teachers select and allocate time to contextual problems more broadly – including problems with a tighter focus on curricular content – is relatively scarce. We hope to contribute to understanding of teachers' dispositions toward the use of contextual problems by examining the beliefs about mathematics learning, instructional practice, and obligations of mathematics teaching that might influence teachers' decisions to select a contextual task for classroom use.

Theoretical Framework

A fundamental assumption of our collaboration with secondary algebra teachers is that students benefit when they have opportunities to learn mathematics through problems that authentically reflect the relevance of algebra content beyond the classroom. We are guided by the practicality ethic, which indicates that teachers will only adopt a change proposal if they view it as practical: that is, if the proposal has instrumental content, is congruent with existing practice, and offers benefits commensurate with costs of implementation (Doyle & Ponder, 1977). We therefore seek to understand the beliefs and practical constraints that guide teachers' selection of classroom tasks, including beliefs about the benefits and costs associated with implementation of authentic contextual tasks.

Palm (2006, 2008) describes several dimensions of authenticity for contextual mathematics tasks: whether a task describes an event that is reasonably likely to occur; whether the question posed is one that would likely occur naturally; whether the methods for solving the problem are congruent with a realistic purpose for finding the answer; and whether representations of information in the problem are realistic. For simplicity, in our study we condense these dimensions down to two: authenticity of the event presented, and information provided, and authenticity of the (mathematical) processes involved in solving the problem. In keeping with Vos (2018), we view authenticity as a social construct; rather than imposing our own meaning on task authenticity as researchers, we leave it to our teacher-participants to define what it means for a task to be authentic to real-world considerations. Brantlinger (2022) acknowledged that contextualized mathematics must be in tune with students' experiences, interests, and potential futures in order to be received as authentic. McGraw and Patterson (2019) investigated how authenticity features of contextual tasks influenced secondary teachers' negotiation of problem spaces as they worked on these tasks. A key finding of this work is that features of the task context can widen or constrain opportunities for mathematical thinking; therefore, considerations about task authenticity and about opportunities for mathematical thinking are intertwined.

In a study of elementary teachers creating modeling tasks, Turner et al. (2022) found that teachers tended to start with a context personal to students and build the problems from the context, rather than thinking of a hypothetical situation to fit the mathematics intended (p. 17). This ensured the context was realistic to real-world experiences, and the anticipated mathematics of the problem was evoked in an authentic way. Teachers highlighted the obligation of a modeling task to "deepen their [students'] critical awareness", as their tasks included creating models to highlight environmental and community issues (p. 11). Along with obligation to the students' wider awareness of community issues in creating their modeling problems, teachers attended to grade-level content and standardized test demands, which sometimes generated conflict in how they framed the mathematics within the modeling task. Based on the literature and our own prior experiences in mathematics teacher education, we anticipate that secondary teachers experience similar tensions between relevance to students' lives and adherence to curricular content when selecting contextual problems for classroom use.

Guided by this framework, we aim to address the following research questions:

1. In what ways does task authenticity influence teachers' evaluations of the instructional suitability of contextual tasks in algebra?
2. What other considerations influence teachers' judgments about instructional suitability and the amount of time that should be allocated to contextual tasks in algebra?

Method of Study

The data for this study were drawn from a 2022 workshop conducted as part of the [blinded] project, which studies the teaching of algebra in grades 7–9 and provides professional development (PD) for secondary teachers. Participants in this study included six mathematics teachers from different urban K–8 academies and high schools in the southern United States.

Data Collection

To investigate participating teachers' beliefs about contextual problems, we designed a set of six *Worthwhile Problems* (Table 1) with different task authenticity features and varying alignments with algebra curricular content.

Table 1: The Six Worthwhile Problems

<i>Muffins</i>	<i>Braking</i>	<i>Population</i>
The Edison High School Marching Band is selling muffins to raise money for an upcoming trip. The cost in dollars of producing and selling x muffins is given by the function $C(x) = 50 + 0.25x$, and the revenue in dollars earned from selling x muffins is given by the function $R(x) = 2x$. How many muffins must the band sell in order to make a profit of 1400 dollars?	A comprehensive review of transportation policy by Jack D. Jernigan and Meltem F. Kodaman, written for the Virginia Transportation Research Council, indicates that for an average car traveling s miles per hour, the formula $d = s^2/20$ gives the braking distance d (in feet) needed to come to a stop. What would be the braking distance needed for a car traveling 80 miles per hour to come to a stop? If we saw a set of skid marks 125 feet long made by a mid-size sedan, and knew that no collision occurred at the scene, what could we infer about the car?	The World Population Clock announced that the world human population reached 8 billion on November 15, 2022. Based on the following data from the World Bank, when should we expect the world population to reach 9 billion? [Table shows world population each decade from 1970 to 2020]
<i>Unemployment</i> The following graph, shown on Fox News in 2010, shows the number of jobs that were lost before, during, and after the Great Recession of 2007 to 2009. What does the graph show clearly? What does it not show so clearly? In what way(s) is the graph misleading?  Image source: https://www.businessinsider.com/fox-news-charts-tricks-data-2012-11	<i>Ramp</i> Use the information below to create a wheelchair ramp design for a door at your school or another building that could use one. Laws for new construction and modifications to existing structures are created so that public facilities are as barrier-free as possible. For example, portions of the American Disability Association (ADA) guidelines provide directions for building ramps or using existing space as ramps. Some of these guidelines are: <ul style="list-style-type: none">• Slope and Rise: The least possible slope shall be used for every ramp. The maximum slope of a ramp in new construction shall be 1:12. The maximum rise for any run shall be 30 in (760 mm) [several other technical specifications are given]	<i>Wrapping</i> Taylor is wrapping holiday gifts for their family. Each gift comes in a rectangular box of height h inches, width w inches, and length L inches. Write a formula in terms of h , w , and L for the total area of wrapping paper needed to cover each box on all sides.

In a PD session led by the first author, six participants first worked on the six problems individually and thought about the mathematical work involved in each problem. Each teacher then used an online tool to create a pie chart showing how they would divide up the total amount of time (100%) reserved for contextual problems in their class among the six “types” of contextual problems exemplified by the tasks. Participants were encouraged to apportion time among “types” of contextual problems – focusing on features such as the relationship between the task and the real world, and the mathematical thinking required, rather than specific mathematics content objectives – and not to worry about standards or testing. They then met in groups of three for 45 minutes to discuss their time allocation to each problem before returning to a whole-group discussion. These conversations were recorded and transcribed for analysis.

Data Analysis

Thematic analysis (Braun & Clarke, 2006) was used to analyze the data. All six authors served as coders. In the first phase, each coder watched the two small-group videos to gain familiarity with the discussions. Each coder took notes on beliefs that appeared to influence teachers’ evaluation of contextual tasks to generate an initial set of codes. All coders then met and discussed these initial codes, including examples of each from the transcripts. After reviewing the codes to eliminate redundancy and overlap, we arrived at a set of 16 final codes. We looked for common themes and organized codes into four domains: Task Authenticity (A), Mathematical Activity (M), Obligations of Tasks (O), and Pedagogy and Access (P) (Table 2).

Table 2: Coding Scheme for Teachers’ Beliefs About Contextual Problems

Theme 1: Task Authenticity	
A1. A task should authentically reflect the information (and representation of information) a person would have when working in the context described.	
A2. A task should authentically reflect the (mathematical) processes a person might use in the context described to solve a real problem.	
Theme 2: Mathematical Activity	
M1. A task should address mathematics content.	
M2. A task should encourage students to engage in mathematical practices and processes.	
M3. Students should learn to build, critique, and analyze mathematical models, and analyze ways in which mathematical models approximate (or fail to approximate) reality.	
M4. Tasks that allow students to engage in creative thinking are beneficial.	
Theme 3: Obligations of Tasks	
O1. Students should be exposed to tasks that foreshadow how mathematics appears or is used in the world outside of school.	
O2. Mathematics class should help students to be prepared for classes in other academic disciplines.	
O3. Mathematics class should prepare students for future mathematics courses.	
O4. Tasks that reflect the expectations and format of standardized achievement tests might support student success on these tests.	
O5. Students should be exposed to contexts that intersect with social justice concerns.	
Theme 4: Pedagogy and Access	
P1. Tasks that relate to students’ lived experiences are beneficial for students.	
P2. Tasks that offer “hands-on” experience are beneficial for students.	
P3. Generally, less time should be allocated to tasks that have low cognitive demand.	
P4. Teachers should make an effort to manage the language load of tasks.	
P5. A task that allows multiple approaches or solutions is beneficial because it allows access to the task for more students.	

In the third phase, individual coder re-read the two small-group transcripts and assigned code(s) to each talk turn in a separate spreadsheet. A talk turn may receive one code, more than one code, or no code depending on its content. For example, consider a part of a group conversation about the *Braking* problem:

Denise: It's like, okay, it says 80, put the 80 in there and that was it. Yeah. And not that I didn't think it was relevant. Why may I spend time on that was my thing on that.

Frances: Well, you know, they deal with that in physics.

Denise stated that she would not spend too much instructional time on this problem because it is too simple, while Frances argued that it was relevant to physics, a course many students take in high school. Their turns of talk were coded P3 and O2, respectively. Then each coder collapsed all the belief codes that they assigned each participant for the discussion of each task. For example, Denise's beliefs on *Braking* included P3 and other codes for other talk turns.

In the fourth phase, six coders met to compare the codes for each participant at the task level. If a code appeared in at least five coders' results, we kept it as a consensus code for subsequent analysis. We discarded any codes that showed up in fewer than three coders' results for the same task. For each code that had agreement from at least three coders, we returned to the relevant discussion excerpts and discussed our reasons for assigning (or not assigning) the code until we reached consensus of at least five researchers (or failed to do so and discarded the code). At the end of this process, each teacher had a set of belief codes associated with each task. We report on these consensus codes in the following section.

Results and Analysis

Table 3 shows the consensus codes associated with each teacher and each task, along with the percentage of time for contextual tasks each teacher allocated to each task. In this section we share some key insights gained from our analysis, along with some illustrative examples from the small-group discussion transcripts.

Table 3: Consensus Belief Codes from Small-Group Discussions

TASK (% of time allocated)*	YELLOW GROUP			PINK GROUP		
	Benjamin	Danielle	Viola	Denise	Felipe	Frances
<i>Muffins</i>	M2 (11.9)	O4, P3 (14.3)	A1, A2, O1, P1 (10)	P5 (10)	O1 (25)	M1, O1 (11.9)
<i>Braking</i>	M1, P3 (23.8)	A1, A2, M1, M2, O1, O4, P4, P5 (19)	O1, P1 (15)	M1, P3 (5)	(15)	O1, O2, P3 (23.8)
<i>Population</i>	P5 (23.8)	M3 (28.6)	O1, O5 (20)	M1, O4, O5 (25)	M3 (25)	M1 (23.8)
<i>Unemp.</i>	M2, P3 (8.3)	M1 (4.8)	M2, O4 (25)	M1, O1, P1 (20)	M2 (15)	(8.3)
<i>Ramp</i>	(23.8)	M1, O1, O5, P1, P2 (23.8)	O1, P1 (10)	M1, O1, P2 (35)	O1 (20)	O1 (23.8)
<i>Wrapping</i>	M2, P3 (8.3)	A2, M2, M3, P2 (9.5)	(15)	A2, M2, O4, P2 (5)	A2 (0)	A2, M1, M2, P2 (8.3)

* Benjamin and Frances worked together and completed a single pie chart, so their time allocations are identical.

Authenticity

Considerations of task authenticity influenced teachers' evaluations of some of the six tasks, most notably *Wrapping*. Teachers in both groups (Danielle and Denise) pointed out that *Wrapping* appeared to be a standard surface area calculation task, but that in a real-world scenario, a person wrapping gifts would need to account for overlap and waste (A2). Felipe suggested that the authenticity of the task could be salvaged by changing the context:

A much better surface area question, theoretically, well, because that'd be lateral surface area, would be painting a room. ... Not even paint a room, you have a box. How much paint are you going to need to paint the whole box? ... Yeah, that would be more realistic because it's not like, oh, you're going to need a little bit more. It's just cover it. It's just exact.

In some cases, considerations of authenticity seemed to influence teachers to defend choices made in the tasks provided. Benjamin suggested that *Braking* could be improved by giving students a graph of braking distance as a function of speed and asking them to write a formula for the quadratic function in vertex and standard form; further questioning by the first author confirmed that Benjamin was evaluating the task primarily based on its affordances for helping students learn algebra content (M1). However, Danielle rebutted,

You don't always want to be given a graph because as Viola said, in all the careers that you could use this in, just like this problem where it's giving you research back to, "This is where this equation came from." Everybody in career paths are not generating these equations and these functions, they're using them to determine other things.

This suggests that Danielle considered authenticity of the information in the task (A1) and of the mathematical processes one would use (A2), as compared to what one might encounter in an analogous professional context, as a key factor in determining the ideal task structure.

Opportunities for Mathematical Activity

Participants endorsed some tasks, and rejected others, based on opportunities they appeared to provide for students to learn Algebra I content and engage in mathematical practices.

In some cases, judgments based on the opportunity to engage with grade-level mathematics content (M1) stood in tension with judgments based on opportunities to engage in mathematical processes and practices (M2). Viola viewed *Unemployment* as an opportunity for students to critically analyze a graph; however, Danielle rejected the task for Algebra I because she did not perceive a clear alignment to algebra standards and content (though she did suggest that she would likely allocate more time to the task in a statistics course). While Benjamin recognized the value of the critical thinking in the task (M2), he suggested that the type of function in the graph would not lend itself to sophisticated mathematical work: "... it's critical thinking, this would be good as a warmup right there ... But then I look at the graph because I know it better. It's linear. Come on." As a general principle, Benjamin seemed to evaluate tasks primarily based on the opportunities they created for students' mathematical thinking, and assigned lower priority to tasks whose cognitive demand he considered inappropriately low for algebra students (P3).

One remarkable pattern was that despite concerns about authenticity, teachers in both groups envisioned opportunities for *Wrapping* to engage students in mathematical practices (M2). Danielle and Benjamin pointed out that the task could engage students in constructing and measuring a net for a solid to determine the surface area; we consider this an instance of using representations strategically to solve problems. Denise pointed out that one virtue of the task is

that students can approach it without knowing the formula for surface area, and indicated that she valued students' gaining confidence in their ability to think about novel problems when they do not have relevant procedures readily available more than their fluency in formulas.

Obligations of Mathematics Tasks

At various times, participants appeared to attach positive judgments to tasks that contained important mathematics and also fulfilled other civic or professional obligations that they associated with mathematics teaching. For example, when discussing *Ramp*, Denise and Felipe pointed to the applicability of the task to other professional domains (O1):

Denise: And so when they're asking, when are we ever going to use this, that's exactly a time where, which makes it totally authentic.

Felipe: And it's also like because math leads into construction and engineering, so it's a real world [application].

Their comments suggest that they valued the task for its potential to inform students about where they might use algebra in the future (O1). This was a theme in discussion of multiple tasks.

Additionally, teachers occasionally recognized and praised connections of the tasks to matters of social justice (O5). Denise conspicuously connected *Population* to the politics of replacement theory, which had been in the news around the time of this activity:

Denise: I would take that to social justice too.

Felipe: Yeah. So it's like what could we eventually start decreasing in population?

Denise: Oh no, I don't mean that kind of social justice. I mean that certain populations are afraid that they're being outnumbered now, so they're trying to... strengthen their power. We definitely go there.

Frances: You can also compare our population to other countries.

Denise: Oh, we can do that too. Yeah. Well, and then of the different groups.

We found it notable that despite Felipe's and Frances' bids to connect the problem to less politically heated issues such as global population decrease and different countries' population growth, Denise found the opportunity to connect the task explicitly to an issue related to the politics of racism and xenophobia compelling. This was a remarkable example of a more general belief on Denise's part that tasks should broaden students' awareness of the world around them and illustrate the role of mathematics in the world beyond school.

Pedagogy and Access

Some teachers identified features of tasks that they considered pedagogically advantageous or likely to broaden students' access to mathematical thinking. We took particular note of instances in which teachers spontaneously suggested alternatives to tasks that might accentuate these pedagogical advantages or heighten accessibility and relevance for students. Frances imagined giving her students the materials from the *Wrapping* problem to work on hands on (P2) and reflected back on a past activity where her students grappled with related ideas, "so they had to come up with their measurements to make a little... and they made little boxes, big boxes. So it was kind of interesting." Danielle suggested a similar modification, "You know what I think would be fun with this? Bring in a bunch of different boxes and ask them, 'Which box do you think is going to take the most wrapping paper?' And they all vote and then each group has a box and they have to figure out how much each would give".

Several teachers valued tasks for their potential to be related to students' lived experiences, making the mathematics of the tasks more motivating and accessible for students (P1). Viola pointed out that her own students were conducting fundraisers for an organization they had just

started and therefore could relate to the content of *Muffins*. She also suggested that while students could picture an automobile accident and imagine the skid marks made by a braking vehicle in *Braking*. Of *Unemployment*, Denise suggested that some of her junior students might be trying to find jobs and might find the topic of job availability relatable (P1); she then revised her thinking and surmised that students might not think about their own situation in such a broad social context but could be guided to do so (O1).

Discussion

The goal of our study was to gain insight into how mathematics teacher educators might help teachers select and design contextual tasks that make meaningful connections between algebra and students' lives while creating opportunities for rich mathematical thinking and problem solving. Using the *Worthwhile Problems* activity as a springboard for discussion, we were able to identify four distinct domains of belief that influence teachers' judgments of which types of contextual problems are worthy of instructional time and students' attention. Furthermore, we saw that teachers often take on an "authorship" role when faced with a task that has perceived deficiencies relative to their priorities and professional commitments, envisioning alternative versions of the task that better meet students' needs and curricular goals. We saw this in Felipe's and Danielle's reimagined versions of *Wrapping*, Benjamin's suggested modifications to *Braking*, and Denise's refocusing of *Population* on current social justice issues.

We recognize some limitations of our study associated with our selection of "Worthwhile Problems" for the activity. For example, teachers may have specific beliefs and dispositions associated with mathematical modeling (Sevinc & Lesh, 2018) that inform the complexity and ambiguity that they are willing to tolerate in classroom tasks; we did not have any tasks open-ended enough to help us identify these boundaries on teacher practice. Additionally, while the *Worthwhile Problems* activity helped us map teacher beliefs that influence approval or rejection of contextual tasks, it provided limited insight into how teachers prioritize different beliefs that may be in conflict. Because a sound understanding of teacher beliefs and their implications for classroom practice requires attention to the network within which these beliefs operate (Leatham, 2006), we plan to revise the activity to include more focused discussion prompts that ask teachers to prioritize among different values and beliefs and make design choices.

Overall, our study results suggest that teachers respond positively to tasks that make connections to the world beyond the mathematics classroom, especially when these tasks authentically reflect the mathematical questions and processes that people would pursue in other coursework, careers, and civic life. Teachers sometimes moderate these positive judgments when they perceive that tasks do not contain sufficient opportunities to reinforce grade-level content or develop students' mathematical practice, as seen with *Unemployment*. In future work, we hope to continue refining our map of teacher priorities for contextual tasks and better understand how teachers manage tensions between authenticity concerns about tasks and the need to connect tasks to curricular content, which is often abstract. With additional insight, we expect to be able to help researchers and educators design mathematical problems that are culturally relevant and address contemporary issues of social injustice while providing opportunities for mathematical thinking that teachers expect to see in classroom tasks.

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