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# TEACHING FACULTY THE ART OF MODELLING IN BIOLOGY

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*While a standard calculus course may include some neatly-packaged applications of rate of change or Riemann sums to problems of kinematics, majors from biology and medicine are in urgent need of mathematics taught from a modeling perspective. Yet, the art of modeling is scarce in tertiary mathematics classrooms in part because, much like in schools, many mathematicians may lack (a) the relevant real-world concepts (beyond simple physics and engineering) (b) knowledge of the mathematics from a modeling perspective or (c) confidence to change their classroom practices. To remedy this, we trialed a professional development workshop for faculty to learn to mathematically model biological contexts with dynamical systems. The workshop enacted the field's recommendations for professional development with teachers. We observed gains in faculty's self-reported comfort with mathematics and biology concepts and teaching mathematics with a modeling perspective.*

Mathematical modeling has emerged over the past few decades as a premier means for addressing multiple flaws in STEM education, because it promotes equitable participation in society and students' conceptual knowledge (Julie & Mudaly, 2007). Mathematical modeling supports conceptual development in mathematics and fosters students' appreciation of mathematics through helping them understand real-world situations from a variety of disciplines, including more recently in life sciences (Garfinkel et al., 2022; Steen, 2005). In addition, mathematical modeling promotes STEM persistence due to empirical associations among student interest, self-efficacy, proficiency, and mathematical modeling experiences (Czocher et al., 2019; Hernandez-Martinez & Vos, 2018).

Yet, modeling is still scarce in university mathematics departments. Hernandez-Martinez et al. (2019) reported that a majority of faculty in their international comparison study used real-world examples to illustrate or motivate new theory. But viewing examples and evaluating values for pre-made models in a textbook are not equivalent to the deep conceptual connections students make when they engage in the decision making that produces mathematical models. Faculty consistently report obstacles to innovating their practice, such as students' negative attitudes towards modeling, students' abilities, tight classroom time for overburdened syllabi, and their colleagues' admonitions that modeling is not "proper mathematics" (Czocher & Roan, 2018; Hernandez-Martinez et al., 2019). Departments fret that their instructors could not teach with a modeling approach and that abandoning traditional calculus would hurt students in their STEM courses (Garfinkel et al., 2022). Beyond contextual obstacles, thinking in terms of instructional goals that may not be purely content based can be challenging and even frustrating (Wagner et al., 2007).

Because robust modeling tasks already exist in abundance, the problem remains to disseminate good problems to eager faculty along with evidence-based guidance and support they need to overcome hesitancy in using modeling as a vehicle to teach mathematics. While a great deal of recent scholarship has focused on teacher education for mathematical modeling at the secondary level (e.g., Borromeo Ferri, 2018), the adaptation of these findings as design principles for tertiary education remains untested. To this end, we report on the implementation and evaluation of a faculty professional development (PD) workshop designed in accordance with principles emerging from empirical research on modeling-specific pedagogical content knowledge. The specific context is a tertiary curriculum that teaches a dynamical systems approach in the context of biology and is taken in lieu of calculus.

## EMPIRICAL AND THEORETICAL BACKGROUND

Effective PD should include multiple modalities for generating knowledge related to instruction: engaging in content-focused investigations, encountering information via brief lectures and topical readings, observing demonstrations of pedagogical principles, and analyzing student instructional materials, cases of instruction, and samples of student work (KMD, 2010). The Knowledge Management and Dissemination project (KMD) also advocates for longer duration and more contact hours to increase participants' opportunities to engage in analysis, reasoning, and communication activities.

Specific to PD for mathematical modeling, Borromeo Ferri (2018) elaborated what she termed *competence dimensions* that included theoretical dimensions (information about aims of mathematical modeling, criteria of modeling tasks), task-related dimensions (developing modeling tasks and performing cognitive analysis of them), teaching-related dimensions (planning and conducting lessons using modeling tasks, practicing appropriate interventions in students' modeling processes), and diagnostic dimensions (identifying phases in the modeling process, recognizing common student difficulties, and assessing modeling products). Greefrath and colleagues (2021) confirmed that the facets of "modelling-specific pedagogical content knowledge" included: knowledge about characteristics of suitable interventions in students work, knowledge about modeling processes and the difficulties students encounter, knowledge about characteristics of tasks and their cognitive analyses, and knowledge rationales for using modeling in class. The study confirmed a strong association between beliefs and self-efficacy expectations for mathematical modelling and between these two constructs and modeling-specific pedagogical content knowledge. The results suggest the strands are interdependent and thus point toward programming that would supply faculty with positive experiences engaging students with mathematical modeling alongside addressing content knowledge and knowledge-of-students.

## SETTING & METHODS

Given the obstacles facing university mathematics faculty and the critical role of instructor confidence in choosing to teach with a modeling approach, we designed a PD workshop with the goal of improving participants' content knowledge and self-efficacy for using modeling as a vehicle to teach mathematics for life sciences. We operationalized self-efficacy as an individual's perceived capacity to carry out given activity (Bandura, 2006). The PD took place concurrently with the first week of Harvard's Summer School, a program for high school seniors to study eclectic topics, like Mathematics of Biological Systems. The Summer School was taught by two Harvard preceptors who invited the faculty to give guest lessons. This report focuses on the faculty participants. We documented changes in their self-efficacy for using a modeling approach to teach mathematics for life sciences with a matched pre/post survey design.

## A MODELING-FIRST APPROACH TO MATHEMATICS FOR LIFE SCIENCES

The mathematics and biology content was based on UCLA's Life Sciences 30 (LS30) course, which was adapted to the two-week Summer School program. LS30 class uses a modeling-first approach to focus heavily on "biological themes that resonate deeply with life science students in the class" and form "the core motivation for studying mathematical concepts" (Garfinkel et al., 2022, p. 42) including predator-prey systems, the dynamics of epidemics, feedback control of hormone physiology, gene regulation, and cellular metabolism. There are three interrelated components: the art of modeling, analyzing interesting mathematical phenomena from a geometric perspective, and programming simulations of dynamical systems. The mathematical phenomena include rates of change, accumulation, bifurcations, oscillations, stability, and chaos. Students learn to code Euler's method in Python, which is used to integrate differential equations numerically and obtain solution curves. The goal is to teach students teaches the importance of modeling feedback loops, positive and negative, in ecology, physiology, and molecular biology, all without a calculus prerequisite. Instead, curricular materials develop a geometric approach to studying the evolution of dynamical systems based on the idea that there are states of the system and changes to states.

For example, A canonical system from life sciences is the predator-prey system. In a simple version, there is one predator species that hunts one prey species as the species members are born, live their (possibly furry little) lives, and die; no other influences are placed on the system. The Shark-Tuna system is an example of a system with feedback – the tuna population positively affects the shark population, and the shark population negatively affects the tuna population. The tuna population increases with tuna births, decreases when a shark eats a tuna, and decreases when tuna die due to other causes. The Shark-Tuna system is mapped to a coordinate plane, called the state space, where Tuna population is placed on the horizontal axis and Shark population is placed on the vertical axis. The differential equations $(T', S')$ then indicate, for each $(T, S)$ coordinate pair at time $t$, the "direction" and magnitude of change. The set of vectors $\left(T'\left(T(t), S(t)\right), S'\left(T(t), S(t)\right)\right)$ form what is called the tangent space. There is a duality between the state space $(T, S)$ and the tangent space $(T', S')$. The key conceptual idea is that $(T', S')$ is the vector tangent to the trajectory formed in the state space by the differential equations evolving with time. The solution to the differential equations, geometrically, is the trajectory flowing from an initial condition. The geometric representation allows students to study and classify the behavior of the system subject to differing initial conditions.

The LS30 curriculum represents a radical departure from traditional introductory calculus courses, particularly for future life scientists. Eschewing the abstract, formula-heavy approach of standard calculus, it introduces a practical, geometric interpretation of dynamical systems. This innovative method is accessible even without extensive calculus prerequisites, making it a game-changer in mathematics education for life sciences. LS30 has seen great success at improving biology majors' attitudes towards mathematics, reducing performance gaps for minoritized students, and increasing students' grades in subsequent physical and life sciences courses (Garfinkel et al., 2022). However, it also poses significant challenges to faculty who do not have much experience with biology or the specific mathematical requirements for effective life sciences education. Additionally, faculty might be unsure of how to teach the needed dynamical systems content in an introductory mathematics setting (i.e., without a full suite of calculus and differential equations pre-requisites). This can lead to low confidence in being able to teach modern mathematics to life sciences majors, and therefore a low rate of uptake or low rate of fidelity to the curriculum.

## THE *MASTER CLASS* WORKSHOP

The literature guided our design principles for the week-long PD workshop: (1) opportunities to plan, conduct, debrief with peers, and revise lessons using the life sciences modeling curriculum, where participants could practice appropriate task design and interventions (2) overviews of educational research about the aims of modeling and modeling-specific pedagogical content knowledge tied to the content in the curriculum (e.g., benefits of modeling for students, student reasoning while modeling predator-prey scenarios, modeling processes and covariational reasoning) (3) discussions of interaction between content and pedagogy (e.g., using Euler's method, state spaces, and vector fields to replace concepts of differentiation and integration) and (4) synopses of the relevant mathematics and biology content knowledge along with how mathematical biologists model the biological systems (e.g., feedback and homeostasis in ecological systems; identifying and classifying equilibrium points and the role of bifurcation in predicting ecological mechanisms).

Taken together, the literature suggests that supporting faculty in adopting and sustaining evidence-based and effective instructional practices related to teaching mathematics through mathematical modeling requires a PDE centering faculty's knowledge of students' modeling, leadership of self, and ongoing professional supports. According to the KMD (2010a), effective PDEs should include multiple modalities for generating knowledge related to instruction: engaging in content-focused investigations, encountering information via brief lectures and topical readings, observing demonstrations of pedagogical principles, and analyzing student instructional materials, cases of instruction, and samples of student work. The KMD also advocates for longer duration and more contact hours to increase participants' opportunities to engage in analysis, reasoning, and communication activities. These modalities are reflected in the *Master Class* (see Table 1).

All aspects of the workshop were created in concordance with existing educational research on the teaching and learning of differential equations and mathematical modeling (Blum, 2015; Czocher, 2017; Czocher et al., 2022; Lesh et al., 2000; Maaß, 2010; Rasmussen & Keene, 2019; Rasmussen, 2001) and best practices for professional development for teaching mathematical modeling (Baumert & Kunter, 2013; Borromeo Ferri, 2018; Greefrath et al., 2021; Klock & Siller, 2019; Wess et al., 2021). In the mornings, faculty used targeted observation protocols to observe Harvard Preceptors teach mathematics, biology, and the art of modeling to Harvard Summer School enrollees (sample in Figure 1). Observations were followed by debriefing sessions. After a communal lunch with focused reflection topics, participants studied the mathematics, biology, and pedagogy needed to successfully adapt LS30 to their home institutional contexts.

| Mod | Brief Description | Core Strand | Tangibles & Outcomes |
|---|---|---|---|
| *A* | Modeling Activity | Leadership of Self | Authentic experiences |
| *B* | Overview of MM | Knowledge of students' modeling | Research-based knowledge of best practices |
| *C* | Barriers & Drivers | Leadership of Self<br>Professional Support | Contextual challenges and ideas for navigating them |

| Mod | Brief Description | Core Strand | Tangibles & Outcomes |
|---|---|---|---|
| *D* | Objective Mapping | Knowledge of students' modeling; Leadership of Self | Learning objectives |
| *E* | Knowledge of Students' Reasoning | Knowledge of students' modeling | Research-based interpretations of student work |
| *F* | Assessing Modeling | Knowledge of students' modeling | Assessment Plans |
| *G* | Support Lesson Adaptation | Knowledge of students' modeling Professional Support | Lesson Plans |
| *H* | Observe, teach & revise HSS lessons | Knowledge of students' modeling | Authentic experiences; re-set expectations |

Table 1: Workshop modules addressing core strands.

*Module A* featured the Sharks & Tuna, which was the opening lesson for the HSS students. *Module B* was a mini-lecture overview of mathematical modeling from an educational research perspective; it shared information about the modeling process, key features of modeling tasks, and findings about student outcomes. *Module C* elicited individual, perceived departmental and institutional barriers and drivers for teaching calculus through mathematical modeling, based on the methods from (Shadle et al., 2017). *Module D* provided space and support for faculty to map LS30 content to instructional goals in the typical calculus course for life sciences. *Module E* shared educational research about student reasoning to familiarize faculty with the ways students think about change and the ways student think about arithmetic operations while modeling. *Module F* gave a high-level overview of educational assessment theory, from the perspective of the purposes of assessment emphasizing understanding students' productions (based on the GAIMME report {gaimme}) rather than trying to "prove" that their modeling course is "working." *Modules G* and *H* were interwoven throughout the week to provide opportunities to try LS30-based lessons, reflect on them and revise them. In *Modules G, Module $G''$, Module $G'''$, Module $G^{(iv)}$* and *Modules H, $H'$, $H''$,* participants worked on their chosen lessons and assessment rubrics, implemented their lessons, debriefed and make revisions to their lessons, based on participants' observations and insights and revise their instructional goal maps based on their perceptions of *student experiences of the mathematics*.

| **Guidelines for observation**<br>The goal here is not to evaluate the instructor's teaching. Instead the objective is to document opportunities for student engagement with in the mathematical modeling process. |
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| Individual Observation – Note taking |
| 1. Write down some occurrences of students using the word "model." Include what object or idea they are making reference to with that word.<br>2. Write down some assumptions (either mathematical assumptions or real-world assumptions) that the teacher explicitly made<br>3. Write down some assumptions (either mathematical assumptions or real-world assumptions) that the students explicitly made.<br>4. Write down some examples of the teacher demonstrating that a model (or assumption) was adequate (or inadequate). What was the rationale given?<br>5. Write down some examples of a student demonstrating that a model (or assumption) was adequate (or inadequate). What was the rationale given? (Especially valuable are examples of when the decision is normatively incorrect.) |
| Individual reflections |

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| 1. How do you think the students you observed would define a model? You need not develop an "if an only if" type statement. It could be enough to describe examples of what is and is not a model for these students.<br>2. How explicit were the modeling assumptions in the students' work? Did you notice any assumptions that were left implicit?<br>3. Did you notice any implicit assumptions seem to be agreed upon by everyone? During class, did you notice any implicit assumptions lead to confusions or disagreements? Did you observe anyone object to an explicit assumption? How were disagreements resolved? |
| Group debrief discussion questions |
| 1. Were there opportunities for students to introduce or define variables (or parameters)? Were there opportunities for students to choose to include or ignore variables (or parameters)? Who had these opportunities? What was left open to decide?<br>2. What strategies did you observe students using to decide whether a model (or an assumption) was adequate? How did they decide? |

Figure 1: Sample observational protocol: opportunities to make assumptions and validate

## DATA COLLECTION & ANALYSIS

Faculty participants were recruited from a national advertisement through professional and personal networks. Acceptance to the PD workshop depended on demonstrating departmental support for adapting the course to participants' home institutions. In total, 24 mathematicians (8 men, 16 women) attended the workshop. Of these, 23 consented to participate in research and 14 completed both the pre- and post-surveys. Most (16/23) came from universities with high or very high research activity, and the remaining came from 2- and 4-year degree institutions or state agencies. Most had taught traditional calculus many times, and 10 had never before taught a mathematics for life sciences course. The survey queried participants' self-assessment of their mathematics knowledge (change equations, vector fields, Euler's method, Riemann sums, fundamental theorem of calculus, equilibrium points, classifying equilibrium points, initial conditions, limit cycles & attractors, bifurcations, bistable systems, and hopf bifurcations) on a four-point scale (not knowledgeable, somewhat knowledgeable, knowledgeable from a mathematical perspective, knowledgeable from mathematical and modeling perspectives) and their confidence to teach these topics from a modeling perspective on a 0-100 point scale. The biology topics (three point scale: not knowledgeable, somewhat knowledgeable, very knowledgeable) and self-confidence (0-100 point scale) were feedback loops, types of feedback in ecological systems, homeostasis, oscillation, and HPG systems. Finally, confidence for student-centered tasks of teaching was queried on a 0-100 point scale: discussing the art of modeling, launching group work, monitoring group work, transitioning between large and small groups, orchestrating student work at the board, allowing students to resolve their own mistakes, emphasizing concepts, discussing the benefits of modeling with a colleague, and referring to educational literature on modeling. Scales were formed by summing across knowledge items and confidence items. Due to the small sample size, we used simulation methods to estimate the $p$-values for self-reported gains in knowledge of content and confidence to teach the content from a modeling perspective. We also queried participants as to what they found most to be strengths and weaknesses of the workshop, in keeping with Fan, et al's (2023) view that participants ultimately decide whether professional development was successful for them.

## RESULTS AND CONCLUSIONS

We observed a statistically significant gain in faculty's self-reported knowledge of the relevant biology content ($p < 0.001$), knowledge of the relevant mathematical content ($p < 0.001$), their confidence for teaching the biology ($p < 0.001$) and mathematics ($p < 0.001$) content from a modeling perspective, and their level of comfort carrying out the tasks of teaching a mathematics course from a modeling perspective ($p = 0.025$). We conclude that the *Master Class* workshop was successful in meeting its stated goals of increasing faculty familiarity with and self-efficacy for teaching a modeling-first approach to introductory mathematics for life sciences majors.

To the open-response item querying the strengths and weaknesses of the workshop, participants indicated that they gained insights from the mathematical content sessions which offered novel perspectives on vector fields, bifurcations, and stability. One participant noted that they were previously unaware of these mathematical interpretations. Gaining that awareness led to increased confidence in "tackling a similar course." Another participant mentioned that learning both the rigorous mathematical justification for the interaction term in a predator-prey model *and* educational research about how students think about, decompose, and justify that term "clarified a lot of my own confusions from when I was first learning to understand these terms." Others found that observing and reflecting on the Harvard Summer School lessons was useful to see varying approaches to teaching the course content and that they valued learning about student reasoning revealed by educational research on mathematical modeling. In contrast, some participants desired a more concrete view of how LS30 is taught specifically at UCLA, which uses a larger lecture-based classroom instead of small classrooms set up for group work. Of the 16 participants who responded to the post-workshop questionnaire, 15 indicated they would recommend the workshop to a colleague. We take this as pointing towards potential success of the workshop, though we cannot predict whether participants will be able to follow through on their ambitions to implement an LS30-based course at their home institutions.

One would expect mathematicians to have extensive mathematical knowledge and a high confidence in teaching that knowledge. However, many may not know the content from a modeling perspective and certainly, they may not be aware of contextual uses of mathematics in biology. We found that a week-long professional development workshop featuring sessions about mathematics in a biological context sufficiently familiarized them with the content to increase their confidence in their knowledge for the purposes of teaching mathematics to life sciences majors through mathematical modeling. These results are a positive indicator that empirically-confirmed dimensions of modeling-specific pedagogical content knowledge generated in the secondary school context (Baumert & Kunter, 2013; Borromeo Ferri, 2018; Greefrath et al., 2021; Klock & Siller, 2019; Wess et al., 2021) are appropriate design principles for organizing professional development for faculty at the tertiary level.

We acknowledge that the participants and research setting were exceptional – all of the faculty were eager to attend, and they taught lessons to eager, talented students. However, formulating comparison groups – e.g., withholding successful PD for faculty or effective modeling curricula from students -- would be problematic. Nor does it seem reasonable to compare with PD for other contextual content (e.g., physics or engineering), though it may be that mathematicians would report higher initial knowledge on physics content, since many have taught a version of calculus intended for physicists and engineers or taken first year physics themselves.

We recommend further studies to ascertain the extent to which mathematics faculty feel comfortable with biology and ecology subject matter as well as their beliefs about the role of mathematics content in introductory mathematics courses. This kind of groundwork could begin to identify faculty who are most likely to seek to implement modeling-first curricula for life sciences majors. We also recommend that future work seek to understand the institutional conditions and constraints that lead to successful faculty adoption of modeling-first courses, as much is already known about why faculty choose not to implement evidence-based pedagogical practices and courses (Shadle et al., 2017).

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