DESIGN OF META-MATERIALS FOR TAILORED NON-LINEAR STRESS-STRAIN RELATION

Rushabh Sadiwala[1]*, Gang Li[1], Georges Fadel[1], & Nicole Coutris[1]
[1]*Department of Mechanical Engineering; College of Engineering, Computing and Applied Sciences, Clemson University, Clemson, SC, USA*

Summary
This work presents a comprehensive methodology for designing meta-materials with desired non-linear elastic behaviors. The approach employs a modified asymptotic expansion based homogenization method for topology optimization with finite deformation. Design and optimization of meta-materials for targeted non-linear elastic response under various loading conditions is explored.

INTRODUCTION

Meta-materials, which are periodic heterogeneous structures designed to achieve specific responses, have gained considerable attention for various applications, including electromagnetic, acoustic, and elastic domains. This work focuses on meta-materials tailored for a target nonlinear elastic response. The meta-material design process involves a two-scale optimization procedure: solving a structural-level problem to obtain the desirable deformation response and using topology optimization to determine the local meta-material geometry[1][2]. While most previous research has concentrated on meta-materials under infinitesimal deformations, meta-material design for prescribed nonlinear properties has been investigated in several studies in the literature [3], [4]. However, these studies either focus on specific properties such as Poisson's ratio [3] or utilize discrete meta-material constructions such as truss-based systems [4]. In comparison, this work proposes a continuum domain topology optimization approach for designing meta-materials with prescribed non-linear response under general deformation modes. In this approach, at the unit-cell level (fine scale), topology optimization is used to determine the material distribution with local stress-strain responses obtained from structural-level analysis. In addition, a modified asymptotic expansion is employed for finite deformation homogenization.

METHOD

Modified Asymptotic Homogenisation

It is well known that multi-scale methods relying on asymptotic expansion of displacements are limited to addressing infinitesimal problems and lack generalizability to finite deformation problems [5]. Alternative approaches have been developed to overcome this limitation, including computational homogenization methods using RVEs [4], [6] and methods that extend asymptotic homogenization to nonlinear problems. In this work, we employ a method involving a coupled two-scaled problem, achieved by using a Taylor series expansion to modify the asymptotic expansion to accommodate finite deformations [5]. The modified expansion is used to derive a coupled two-scale problem in which the coarse-scale stress-strain relation is established numerically by solving the fine-scale problem(s) for a non-homogeneous structure in lieu of closed-form analytical solutions, as shown in Fig 1.

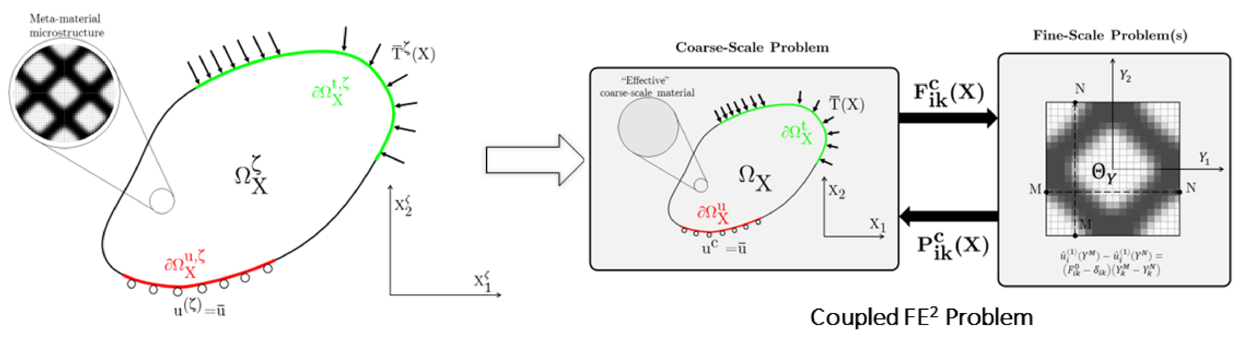


Figure 1. Homogenization in the coupled two-scale problems and their inter-dependencies.

Topology Optimisation

In the topology optimisation of the meta-material unit cells, SIMP, mesh filtering, and hyperbolic tangent projection are employed to mitigate the numerical issues and enhance the robustness of the method. Typically, large deformation

* Corresponding author. E-mail: rsadiwa@clemson.edu

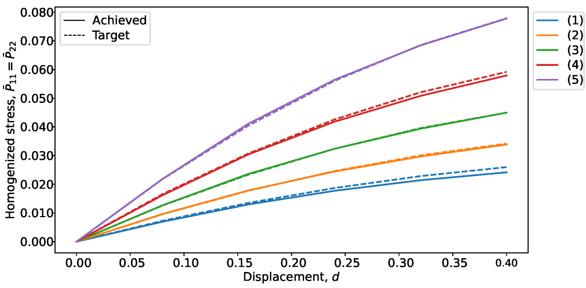
problems describe material behavior through a linear relationship between the second Piola-Kirchoff stress tensor, S, and the Green-Lagrangian strain tensor, E, following the St. Venant-Kirchhoff model. We show that this constitutive relationship is unstable in topology optimization due to issues associated with low-density elements and the inaccurate strain-softening behavior of the St. Venant-Kirchoff model in compression. To address this instability, we describe the base material using the linear hyperelastic relationship between the Kirchhoff stress τ and logarithmic strain $ln(V)$, and employ a projection function to replace the non-linear void elements with linear ones. The nonlinear stress-strain curve is quantified numerically by sampling a set of target stress-strain pairs. The optimization problem is hence formulated as follows:

$$\min_{\rho} V(\rho) \quad \text{s.t.:} \quad R(u,\rho)=0; \quad \frac{\bar{P}_{ij}^{(k)}}{P_{ij}^{(t,k)}}-1=0,\ k=1...n; \quad \rho_{min}\leq\rho_e\leq 1 \tag{1}$$

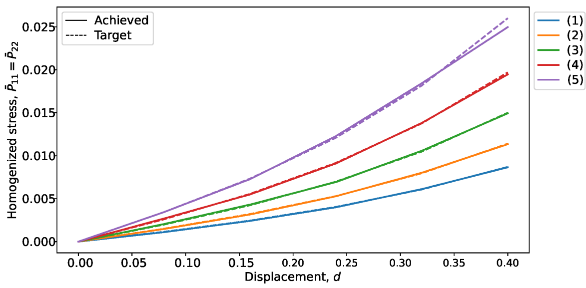
where V is the mean of all elemental densities ρ_e, R are the residual forces of the equilibrium equations. The target stress-strain curve is described using the target stress, $P_{ij}^{(t,k)}$ at given deformation steps where k refers to the deformation step, and t denotes the target values.

RESULTS

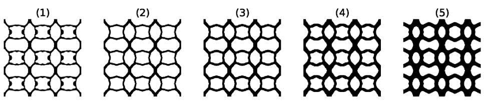
Meta-material designs are obtained by using the proposed approach for various target stress-strain curves with different deformation modes. In the unit-cell topology optimization process, it is assumed that the given local deformation gradient (F_{ij}^c) from the structural-level analysis is uniform across the material design domain X. Several examples of meta-materials generated using this approach for a biaxial tension deformation are shown in Fig 2.



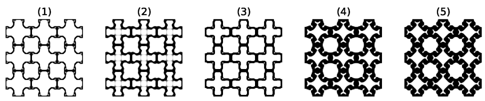
(a) Optimized Force vs deformation curves - softening



(b) Optimized Force vs deformation curves - stiffening



(c) Optimized meta-material for (a), 3x3 UCs



(d) Optimized meta-material for (b), 3x3 UCs

Figure 2. Meta-materials exhibiting prescribed nonlinear deformations under biaxial tension.

CONCLUSION

By employing a nonlinear deformation homogenization and continuum domain unit cell topology optimization, meta-materials are designed for target non-linear stress-strain responses under given deformation gradients. The performance of the proposed approach is demonstrated by numerical results.

REFERENCES

[1] C. Czech, P. Guarneri, J. Gibert, *et al.*, "On the accurate analysis of linear elastic meta-material properties for use in design optimization problems," *Composites Science and Technology*, vol. 72, pp. 580–586, 2012.

[2] Z. Satterfield, N. Kulkarni, G. Fadel, *et al.*, "Unit cell synthesis for design of materials with targeted nonlinear deformation response," *Journal of Mechanical Design*, vol. 139, no. 12, Dec. 2017, ISSN: 10500472.

[3] K. Qiu, R. Wang, Z. Xie, *et al.*, "Optimal design of chiral metamaterials with prescribed nonlinear properties," *Structural and Multidisciplinary Optimization*, pp. 1–17, Oct. 2020, ISSN: 16151488.

[4] F. Wang, O. Sigmund, and J. S. Jensen, "Design of materials with prescribed nonlinear properties," *Journal of the Mechanics and Physics of Solids*, vol. 69, no. 1, pp. 156–174, Sep. 2014, ISSN: 00225096.

[5] J. Fish, *Practical multiscaling*, p. 398, ISBN: 1118410688.

[6] P. B. Nakshatrala, D. A. Tortorelli, and K. B. Nakshatrala, "Nonlinear structural design using multiscale topology optimization. Part I: Static formulation," *Computer Methods in Applied Mechanics and Engineering*, vol. 261-262, pp. 167–176, Jul. 2013, ISSN: 00457825.