LINEAR AND NONLINEAR TOPOLOGY OPTIMIZATION USING MORPHING BEAM NETWORKS

Andrew Montalbano[1], Georges Fadel[1], Nicole Coutris[1], & Gang Li[1]
[1]*Department of Mechanical Engineering; College of Engineering, Computing and Applied Sciences Clemson University, South Carolina, United States*

Summary Starting from a network of discrete beams, topology optimized structures are produced by simultaneously optimizing each beam's width and the locations of each node within the network. Due to the sparse nature of a beam network and by utilizing gradient descent results in a drastic reduction in computational cost compared to existing methods. Two different optimization objectives are investigated: minimization of the strain energy occurring from loading, often referred to compliance minimization; and the design of structures with prescribed mechanical responses to an applied load.

INTRODUCTION

Topology optimization is a field of engineering optimization that focuses on optimizing a material distribution within a given domain. The resulting topology optimized structures often carry the visage of being constructed of beams, posing the question of replacing the initial domain of the commonly-used solid elements with a sparse network of beam elements to largely reduce the computational cost while retaining the result fidelity.

The idea to replace solid elements with beam elements has been investigated in the literature [1–4]. Typically, these beam networks feature a fully connected grid of nodes [1, 2], but the dense nature of elements rapidly scales with domain size, restricting them to smaller domains [5]. To rectify the rapid increase in degrees of freedom, elements can instead be generated by connecting them to their neighbors [2]. However infeasible structures can be generated when elements overlap one another. Other drawbacks of these methods include limited design space due to insufficient geometric representation [3, 4] or optimization variables [2], and inefficient optimization algorithms [2, 6]. More important, no attempt has been made to develop a beam network based topology optimization method for targeted nonlinear deformation responses, albeit the potential benefits in engineering application [7, 8]. This work presents a method of creating topology optimized structures utilizing a morphing beam network, reducing the associated degrees of freedom and computational cost which then can be leveraged to explore more complicated topology optimization problems such as optimization for nonlinear deformation.

METHOD

Overview

The networks investigated in this paper use a mix of orthogonal and diagonal elements connecting nodes to their neighbors. Acting as a pseudo 2D optimization, the depth of each beam is constant, while the in-plane dimension, width, is optimized. Alongside element width, the spacial xy location of each node in the network is also optimized, but are constrained to lie within the initial domain of the structure as well as their residing Voronoi cells to prevent overlapping nodes which would result in zero-length elements and unfeasible structures. An example network of a 2×1 element grid may be seen in Fig. 1, with a single element shown on the left.

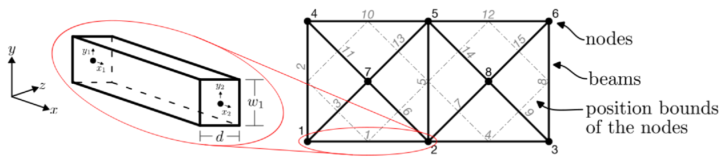


Figure 1. A simple 2×1 element domain has 6 nodes (numbered in black) and 15 elements (numbered in grey). The solid-black lines on the right network show the beams of the structure, while the dashed-grey lines show the position bounds of the nodes. The first element is highlighted to the left to shows the optimization variables, the $x\&y$ locations of both of its nodes, 1 and 2, and the element's width, w_1. The depth of the element is d, which is held constant across all elements throughout optimization

Two different optimization problems were explored with the proposed method. First, to verify the method's ability to preserve the result fidelity found with solid elements, an in-depth investigation of compliance minimization is carried out and the optimized structures are compared with the results obtained with solid elements [9]. The second optimization leverages the computational cost savings obtained by using a sparse beam network to produce structures with prescribed nonlinear load-displacement curves.

* Corresponding author. E-mail: asmonta@g.clemson.edu

Topology Optimization Problems

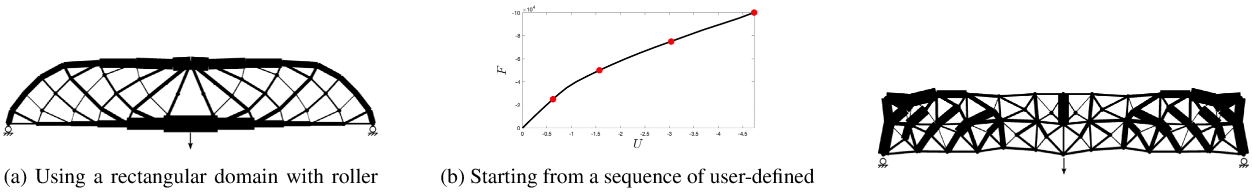
The first problem investigated minimizes the strain energy occurring from loading, often called compliance. Solved with gradient descent, the compliance minimization problem is defined in Eq. (1.1), where $\mathbf{U}$ is the displacement vector of the structure and $\mathbf{K}(\mathbf{z})$ is its calculated stiffness matrix with the optimization variables $\mathbf{z}$, i.e. $\mathbf{z} \in \{\mathbf{x}, \mathbf{y}, \mathbf{w}\}$. The first constraint is the material volume fraction constraint, the second limits the beam width to be nonzero preventing numerical errors, and the last constrains the nodes to lie within the initial domain, Ω. In the second optimization problem, by minimizing the difference between a structure's and a user defined force-displacement curve, structures can be designed to achieve unique and desired deformation behavior. For obtaining the load-displacement response of a structure, a co-rotational formulation [10] is used in nonlinear mechanical analysis. Once again solved with gradient descent, the optimization problem definition is provided in Eq. (1.2), with $\boldsymbol{\delta}$ and $\mathbf{T}$ referring to a set of data points on the generated structure's and the target load-displacement curve, respectfully, and $\mathbf{K}$ being a function of $\mathbf{U}$. Different from compliance minimization, the minimum width is set to be larger than a critical width to prevent buckling, which would result in undefined derivatives.

$$\begin{aligned} \min_{\mathbf{z}} :& \quad c = \mathbf{U}\mathbf{K}(\mathbf{z})\mathbf{U} \\ \text{with} & \quad \mathbf{K}(\mathbf{z})\mathbf{U} = \mathbf{F} \\ \text{s.t.} & \quad \frac{V(\mathbf{z})}{V_0} \leq V_f \\ & \quad 0 < w_{min} < w < w_{max} \\ & \quad (x, y) \in \Omega \end{aligned} \qquad (1.1)$$

$$\begin{aligned} \min_{\mathbf{z}} :& \quad \left(\boldsymbol{\delta} - \mathbf{T}\right)^2 \\ \text{with} & \quad \mathbf{K}(\mathbf{U}, \mathbf{z})\mathbf{U} = \mathbf{F} \\ \text{s.t.} & \quad \frac{V(\mathbf{z})}{V_0} \leq V_f \\ & \quad 0 < w_{min} < w_{cr} < w < w_{max} \\ & \quad (x, y) \in \Omega \end{aligned} \qquad (1.2)$$

RESULTS

A classic bridge topology optimization problem for compliance minimization is presented. The problem features a rectangular domain with rollers on the outer bottom nodes and a load applied at its center. Using a 12×2 grid of unit cells and minimizing the compliance using Eq. (1.1) results in the structure shown in Fig. 2a, which matches the results obtained with other methods at a fraction of the computational cost. Equation (1.2) is used to generate a structure whose load-displacement curve reproduces a user defined curve. Starting from a set of target load-displacements, highlighted in red in Fig. 2b, and using the same bridge domain used for compliance minimization, the network is optimized (Fig. 2c) to produce a displacement of the loaded node following the given target curve.



(a) Using a rectangular domain with roller supports, a topology optimized bridge can be generated requiring only 12×2 unit cells using the proposed method

(b) Starting from a sequence of user-defined load-displacement points (red), a structure (Fig. 2c) can be generated whose mechanical response matches that sequence (black)

(c) Originally a 12×2 grid of unit cells, this structure's load-displacement curve matches a user defined curve shown in Fig. 2b

Figure 2. Morphing beam networks are optimized to minimize compliance (a), and produce a targeted nonlinear deformation (b&c)

CONCLUSIONS

A morphing beam network based topology optimization approach is developed for design of structures with targeted linear and nonlinear deformation responses. Numerical results show that by optimizing the nodal locations and beam widths in a sparse network of discretized beams, topology optimized structures can be produced with largely reduced degrees of freedom and at a largely reduced computational cost required by traditional methods.

References

[1] N. Changizi et al. "Topology optimization of structural frames based on a nonlinear Timoshenko beam finite element considering full interaction". In: *International Journal for Numerical Methods in Engineering* 123.19 (2022), pp. 4562–4585.

[2] M. Pyrz. "Evolutionary algorithm integrating stress heuristics for truss optimization". In: *Optimization and Engineering* 5.1 (2004).

[3] N. Morris et al. "BeNTO: Beam Network Topology Optimization". en. In: *Computer-Aided Design* 156 (Mar. 2023).

[4] A. Asadpoure and L. Valdevit. "Topology optimization of lightweight periodic lattices under simultaneous compressive and shear stiffness constraints". en. In: *International Journal of Solids and Structures* 60–61 (May 2015), pp. 1–16.

[5] OEIS Foundation Inc. *Entry A083374*.

[6] M. Giger and P. Ermanni. "Evolutionary truss topology optimization using a graph-based parameterization concept". In: *Structural and Multidisciplinary Optimization* 32.4 (2006).

[7] Z. Satterfield et al. "Unit cell synthesis for design of materials with targeted nonlinear deformation response". In: *Journal of Mechanical Design* 139.12 (2017).

[8] Nathan Brown et al. In: *Deep reinforcement learning for the design of mechanical metamaterials with tunable deformation and hysteretic characteristics* (2023).

[9] E. Andreassen et al. "Efficient topology optimization in MATLAB USING 88 lines of code". In: *Structural and Multidisciplinary Optimization* 43.1 (2010), pp. 1–16.

[10] L.L. Yaw. *2D Corotational Beam Formulation*. Washington, DC, USA.