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Semiotic mediation of gestures in the teaching of early algebra: the case of the equal sign

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Abstract

Gestures are one of the ways in which mathematical cognition is embodied and have been elevated as a potentially important semiotic device in the teaching of mathematics. As such, a better understanding of gestures used during mathematics instruction (including frequency of use, types of gestures, how they are used, and the possible relationship between gestures and student performance) would inform mathematics education. We aim to understand teachers' gestures in the context of early algebra, particularly in the teaching of the equal sign. Our findings suggest that the equal sign is a relatively rich environment for gestures, which are used in a variety of ways. Participating teachers used gestures frequently to support their teaching about the equal sign. Furthermore, the use of gestures varied depending on the particular conception of the equal sign the instruction aimed to promote. Finally, teacher gesture use in this context is correlated with students' high performance on an early algebra assessment.

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Algebra is a linchpin of success in mathematics (e.g., NCTM, 2000; NGA Center & CCSSO,2010). However, scholars have argued that algebra often functions as a *gatekeeper* to higher mathematics (e.g., Kaput, 1999; Schoenfeld, 1995; Stigler et al., 1999), depriving less successful and often more vulnerable students from "opportunities to become productive citizens in our society" (Schoenfeld, 1995, p. 12). Central to all algebra work is the understanding of the equal sign and the equivalence relationship for which it stands. The equal sign makes its appearance in early mathematics and maintains its presence throughout higher level mathematics. Hence, it is important for students to develop a robust understanding of this symbol and avoid misunderstandings that lead to cognitive obstacles (Carpenter et al., 2003; Knuth et al., 2016).

In earlier work, we reported on the success of a large-scale early algebra intervention in Grades 3–5 (Blanton et al., 2019), and a subsequent study of teachers' practices that were associated with improved student performance (Stylianou et al., 2019). One focus that emerged in our work is the semiotic mediation of signs in the form of teachers' use of gestures. Signs as semiotic devices are ubiquitous in the teaching of mathematics, and mathematics classrooms form a rich semiotic environment—a complex "multimodal ecology" (Ginsburg, 2015) that allows an interplay of language, symbols, and inscriptions as well as gestures to bring about mathematical development. We focus our work on gestures as they have received far less attention than the other signs. And yet, as Sfard (2009) notes, "gestures are invaluable" (p. 197) in classroom instruction. Others have noted that "gestures are the very texture of thinking" (Radford et al, 2009, p. 92), as they "jointly support the thinking process of students in a unitary way" (Arzarello et al., 2009, p. 107).

Specifically, in this study, we focus on the use of gestures as another type of semiotic device that may be employed in early algebra. We examine the ways in which teachers use gestures to foster a relational view of the equal sign in the context of open-ended equations. Additionally, we examine the association between teachers' gestures during instruction and student performance in the context of mathematical equivalence.

1 The teaching of early algebra—focus on the equal sign

The equal sign, and the equivalence relationship it denotes, is central to all algebra work. It is one of the most prevalent symbols in mathematics and understanding it as a relational symbol is foundational in solving equations (Baroody & Ginsburg, 1983; Knuth et al., 2005; NCTM, 2000; NGA & CCSSO, 2010). However, developing this relational understanding is not as straightforward as might be assumed.

Research suggests that students typically hold two main conceptions of the equal sign (Stephens et al., 2013): an "*operational* conception" in which students view the equal sign as an indication that they need to perform an operation, or "find the answer." When asked to solve an equation with a missing addend such as "6+3 = _ + 4," students with this conception may place 9 in the blank. The second conception is a *relational* one in which students understand the equivalence of the "two sides" of the equation. Students with this conception would place 5 in the blank in the aforementioned "6+3 = _ + 4." Students with a relational conception might use one of two strategies: a *relational-computational* one, whereby students approach the task by performing computations on both sides of the equal sign, or a *relational-structural* one whereby students approach the task by making use of relationships on either side of the equation (Stephens et al., 2013). In the case of “6+3 = _ + 4,” a student using a relational-structural strategy may notice that 3 on the left side is only one less than 4 on the right side of the equation and use a compensation strategy to determine that the missing value would be one less than 6, or 5.

2 Gestures in mathematics teaching and learning

Gestures are non-verbal forms of communication that involve movements of the hand, body, or head. They usually express or support the expression of an idea. Here we focus on the following four types of gestures.

- *Deictic* gestures are pointing gestures indicating objects as they relate to instruction. A teacher may be pointing with a finger towards each number or symbol in an equation. Pointing may also be abstract as “the speaker [may] be pointing at empty space, but in fact the space is not empty; it is full of conceptual significance” (McNeill, 1992, p. 173).
- *Iconic* gestures are those gestures that “represent meaning that is closely related to semantic content” (McNeill, 1992, p. 13). For example, a teacher may use her hand to make a circular motion around two numbers to indicate that the numbers should be added.
- *Metaphoric* gestures are gestures that stand in, as metaphors to a concept (e.g., motioning with a hand to show the change in slope in a graph). Some researchers consider these to be iconic gestures (McNeill, 1992).
- *Writing* gestures (Alibali & Nathan, 2007) refer to mathematical writing that accompanies speech but resembles a gesture (e.g., underlining an expression, or writing an arrow to point to an expression).

When instruction involves gestures, learning is enhanced (e.g., Cook et al., 2016; Church et al., 2004). Due to the coordination of speech and gesture that reinforces teaching (e.g., McNeill, 1992), gestures are not only helpful, but integral to instruction and learning (e.g., Alibali et al., 2013; Kim et al., 2011; Nemirovsky et al., 2012; Radford et al., 2009; Roth, 2000). Gesture during mathematics instruction benefits students across grade levels (e.g., Aldugom et al., 2020; Alibali et al., 2013). Gestures assist in directing students’ attention to aspects of the mathematics that is taught (e.g., Alibali et al., 2013), and are integral “semiotic resources” in the process of developing mathematical ideas (e.g., Radford et al., 2009; Rasmussen et al., 2004). Nonetheless, more work is needed to further understand the purpose of gestures in instruction, and how different gestures might be used towards instructional goals or in various contexts.

Embodied cognition theory, an extension of semiotic theory, emphasizes that cognition is “embodied,” that is, grounded in the human body and the world that surrounds the body (see, e.g., Nunez et al., 1999). This theory points to the involvement of the body when learning (and teaching) mathematics, the tools in the environment in which we live and think, and in the movements and gestures as we speak and think (again, see Nunez et al., 1999). Gestures are highlighted as one of the ways in which mathematical cognition is embodied and they are elevated as a potentially important semiotic device in the teaching of mathematics. Alibali and Nathan (2007), for example, argued that spontaneous gestures provide evidence for the embodiment of mathematical cognition: pointing gestures and representational gestures.

3 Gestures in the teaching of algebra

Research that highlights the role of gesture in the teaching of early algebra, and particularly the equal sign, is somewhat lacking. Alibali et al. (2014) conducted their work in algebra, albeit in secondary school. They identified four categories of gestures that teachers tend to use when teaching algebra—beats, pointing, representational, and writing (as defined above)—and noted that these gestures are "essential to teachers' ability to conduct their practice" (p. 362). More recently, Sung et al. (2022) specifically looked at the use of gestures in the teaching of the equal sign in early algebra in kindergarten. Their work confirmed the use of three types of gestures that contributed to students' understanding on the equal sign—pointing to either side of the equation, writing to denote combination of terms on each side of the equation, and a representational gesture of "balance." While the work is preliminary and only examined the teaching of a teacher-researcher, their findings shed light on ways in which a teacher might use gesture to support early algebra development. Moreover, we expected to find additional gestures: as the equal sign when viewed through a structural-relational lens involves an abstracted relationship of equivalence, we expected to see some form of dynamic gestural depiction of this relationship—akin to "algorithms in the air" identified when examining gestures in fraction equivalence (Edwards, 2003 as referenced in Edwards, 2009). We seek to extend these findings to further understand how these gestures support student learning of the equal sign for more complex equations.

It is important to note that gestures may be produced by both teachers and students. While a complete semiotic analysis of all classroom interactions including the students' work (utterances and gestures) is important, it is beyond the scope of our work.

4 Research questions

In this study, we examine the use of gestures as semiotic devices when teaching students to think relationally about the equal sign. We hypothesize that teachers employ a variety of semiotic devices to foster mathematical development, with the use of gestures being one of them. We begin by examining the frequency and types of gestures teachers tend to use when teaching lessons related to the equal sign. We continue by examining whether teachers' use of gestures differs, by quantity and type and by the semiotic function of these gestures—that is, their role in instruction. We conclude by assessing whether the teachers' use of gestures is related to students' performance on tasks involving the equal sign, in terms of correctness as well as in terms of both strategy use and conception of the equal sign. The questions that guided our analyses are the following:

1. To what extent do teachers use gestures to support students' conceptualization of the equal sign? What are the types and frequency of gestures teachers employ during instruction about the equal sign within early algebra equation solving tasks?
2. How does teachers' use of gestures and the type of gesture vary in relation to the conception of the equal sign the teacher is supporting?
3. Does teachers' use of gestures, in terms of type and frequency, correlate with students' performance on equation solving tasks in terms of correctness and the conception of the equal sign displayed by students when solving these tasks?

5 Methods

5.1 Setting and participants

The data for this study are taken from a larger study examining the effectiveness of an early algebra intervention in Grades 3−5 (see Blanton et al., 2019). Here, we focus on a lesson taught by 19 different teachers in Grade 3. This lesson was selected from a sequence of lessons focused on the equal sign because we had the most videotaped observations for this lesson.

6 Lesson

This lesson of focus included the following task:

Find the missing value: $14 + \_\_\_ = 15 + 6$.

The purpose of this task and others like it was to help develop students' ability to think relationally about the equal sign using open equations. Similar tasks were posed during the previous two lessons.

6.1 Coding for teachers' use of gestures

Video analysis techniques included notating content logs with time stamps and descriptions of teachers' instructional moves, segmenting videos, transcribing audio and visual content captured in the videos, and conducting cycles of review to code, compare codes, and revise codes (Derry et al., 2010). Prior to coding video, the authors previewed videos and noted the time stamps at which the task was addressed and completed in the class. These instructional segments were then transcribed by an author, verified by another, and then coded by three authors in several iterations using video and transcript simultaneously.

We then coded the types of gestures enacted by teachers. Gestures were segmented and separated from one another as the hands changed shape, position, and placement. A working coding manual was developed using a theory-driven approach (Syed & Nelson, 2015), based on the work of McNeill (1992) and Alibali et al. (2013). However, as the coding progressed, new, data-driven codes emerged and existing codes were disregarded if they were not useful in our data (e.g., the use of "beats[1]"). Each video episode was coded by two team members and compared. Differences were reconciled through discussion.

We searched for themes that would allow us to categorize the gestures we identified. Each gesture was then classified into one of three categories characterized by both form and function; some of these categories were further split into subcategories (see Fig. 1).

Much in the manner of Alibali et al. (2013), to account for the degree of saturation that a teacher's communication utilized gestures, we did not account only for the raw number

[1] Beats are "rhythmic, up and down hand movements that are aligned with the prosody of speech" (Alibali et al., 2013, p. 215). Even though beats were coded and counted in other studies, we did not find a clear meaning to supporting students' understanding of the relational view of the equal sign.

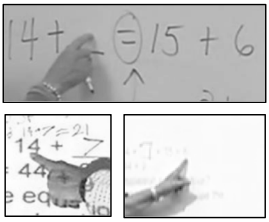
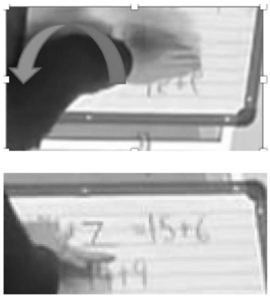
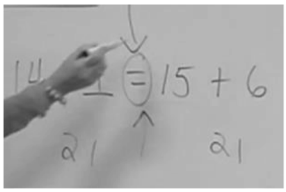
Gesture description	Image	Code
1. Deictic/ Pointing (PTG) – indicating objects/locations/symbols, to bring attention, to clarify what is referred to during conversation. This was a static gesture.		**PTG**
2. Representational/Iconic (RIG) – literal or metaphorical representation of the concept of equality or for grouping objects that were further classified into three categories:		**RIG**
a. Metaphors of equality		
▪ BAL - Balance gesture (teacher holds their arms out moving up and down)		**BAL**
▪ BKF - Back-and-forth gesture (teacher holds out a finger going back-and-forth as if tracing a windshield wiper motion to either side of the equation)		**BKF**
b. GPG - Grouping Icons: tracing a line in the air or tracing with a finger under an equation; tracing circles, clouds, or boxes in the air to suggest grouping; doing a fist gesture to show "grabbing" two numbers together as a group.		**GPG**
c. LKG - Linking gestures: gestures that delineate relationships, such as suggestions to consider comparing two objects, or to connect two objects, further suggesting an "actionable relationship". These "arrow/arch" movements of hands across different terms of the equation, did not appear in the previous literature and could not be captured by the existing codes, hence it's specific to our work. These are akin to the "algorithms in the air" dynamic gestures suggested by Edwards (2003).		**LKG**
3. WTG - Writing gestures - explicit writing or drawing acts that have a pointing or indexical function. This writing occurs with co-speech. Writing gestures can be similar to representational gestures (e.g., circling a number, underlining, drawing an arrow), while leaving a trace with a marker, pen, or chalk. Writing gestures do not include writing symbols or numbers or any writing that can be considered part of solving the task.		**WTG**

Fig. 1 Three broad categories of types of gestures, characterized both by form and function, used by teachers to support conceptualization of the equal sign

of times a teacher used gestures, but rather the in terms of rate of gestures per 100 words (summing word utterances for each teacher's episode).

6.2 Coding for teachers' emphasis of solution conception and strategy

In the final phase of coding, we identified the conceptions of equivalence that were addressed by each teacher in solving the open-ended Eq. $14 + \_\_ = 15 + 6$. We noted that solutions presented fell into one of three categories as shown in Fig. 2. We operationalized these definitions based upon the teachers' utterances and solution steps.

6.3 Student assessment coding

As part of the larger study, students completed a test twice: a pre-test at the beginning of the school year, prior to the early algebra intervention, and a post-test at the end of the school year. This test was designed and validated in our prior work to align with the algebraic thinking practices that were the focus of the intervention (see Blanton et al., 2019). We included four tasks that pertained to the equal sign, as shown in Fig. 3.

Student responses were coded for correctness and for strategy use, particularly for strategies indicating operational or relational approaches. For example, if a student indicated in their explanation for item 1 that they compared numbers on each side of the equation (e.g., "4 is one more than 3, so the number in the blank must be one less than 7"), the strategy was coded as relational-structural. If they computed the numbers on the right ($7+3=10$) and then subtracted 4, the strategy was coded as relational-computational. Finally, if they wrote "10" in the blank, the strategy was coded as operational.

To examine the correlation of teacher gestures and students' performance, we used the Pearson correlation (r) test. For this, we considered student performance on the four equality tasks in each class. Subsequently, we examined the correlation between the use of representational gestures (RIGs) and student performance.

As a second step, we examined the association of teachers' gestures and student strategy use. Note that for this step, we only used item 1, as student responses allowed for

Equality Conception	Example
Operational misconceptions (i.e., the incorrect solution "14 + _1__ = 15 + 6"),	None observed during instruction
Relational – computational (RC) (i.e., computing the right side, 14 + __ = 21, and then noting that "7" satisfies the equation),	*Well how did you go about solving this problem?* *You know what about the two sides of that equation?* *So it's gonna be the same amount, it's gonna balance out, like a scale example you're giving me?* *So they both equal 21, so is that a true statement?*
Relational – structural (RS) (i.e., noting that the 14 is one less than the 15 so the missing addend must be 1 greater than 6),	*You got 15 over here* in the first number, and *then on this side of the equal sign you have 14* *So this side is one less, Than this side so far.* And *since you already have 1 more on this side* *And 1 less on this side?*
Not indicative of a particular conception (NI).	So, how do we go about solving this problem? How do we balance this equation?

Fig. 2 Equal sign conception and strategy observed during instruction

Item 1:
Fill in the blank with the value that makes the number sentence true.
7 + 3 = ____ + 4
Explain how you got your answer.

Item 2:
Circle True or False. Explain how you got your answer.
2a) 12 + 3 = 10 + 5
2b) 57 + 22 = 58 + 21
2c) 39 + 121 = 121 + 39

Fig. 3 Test items

coding of strategy. Additionally, we looked at the change in student strategies from pre- to post-test. We first examined the association of teacher gesture and change in students' use of an operational approach from pre- to post-test, and subsequently, we examined the association of teacher gesture and change in students' use of relational strategies (computational and structural) from pre- to post-test. For example, in one teacher's classroom where 20 students completed pre- and post-tests, if 12 students out of 20 students used an operational approach on the pre-test, and 4 out of 20 on the post-test, then the proportions of operational approach would be 0.6 and 0.2, respectively. This difference between the proportion of operational approach in the pre- and post-tests, 0.4, was used as the dependent value for the teacher. In this analysis, we considered the correlation between the density of teachers' instructional gestures around computational and structural approaches, and the decrease of students' use of an operational approach. Finally, to examine any correlation between the density of teachers' instructional gestures in computational and structural approaches and the increase of students' use of either computational or structural strategies, we used the proportion difference of computational and structural strategies between pre- and post- tests as the second dependent variable.

7 Results

We present the results of our analysis by research question. In each section, we describe the findings that pertain to each of the three research questions. Overall, we examine the frequency, type, and function of gestures we observed in instruction.

7.1 1. To what extent do teachers use gestures to support students' conceptualization of the equal sign? What are the types and frequency of gestures teachers employ during instruction about the equal sign within early algebra equation solving tasks?

We examined 19 episodes that varied from 40 s to 16 min, with an average of 2:45 min. When episodes are cast in terms of number of words uttered by the teachers, episodes varied from 37 to 369 words, with an average of 150 words per episode. Overall, we observed 256 gestures across all 19 episodes. On average, each episode had 13.5 gestures (min: 4, max: 36). When cast in terms of gestures per words, episodes varied from 5 to 17.8

gestures per 100 words. On average, teachers produced 9.5 gestures per 100 words. Is this a gesture-rich context? Alibali et al. (2013) found that teachers produced an average of 13 gestures per 100 words in gesture rich environments; hence, the gesture production in this study was comparable, even more so as we did not consider one of the three types of gestures—beat gestures—that were considered by Alibali and her colleagues (Table 1).

As shown in Table 2, teachers produced a total of 256 gestures. Among these, 135 (52.7%) were deictic (pointing-PTG), 98 (38.3%) were representational (RIG), and the remaining 23 (9%) were writing gestures (WRG).

RIGs, using hand motion to represent actions that teachers highlighted or asked students to perform, were further categorized into metaphors—balance (BAL) and back-and-forth (BKF) (25–9.8% of the total gestures), grouping GPG (35–13.7%), and linking LKG (38–14.8%):

- "Balancing" (BAL): two arms extended and moving up and down to represent the balance of the two sides of the equation—a metaphor for the equal sign.
- "Back-and-forth" (BKF) motion: often a finger tracing an arch motion back-and-forth.
- "Grouping" (GPG) motion: often tracing a line or circle around two numbers to indicate that they form a group.
- "Linking" (LKG): like BKF, traces one motion, but it has a specific start point and an end point, from one part of the equation to another, unlike BKFs that went back-and-forth repeatedly without a specific start or endpoint. It is important to note that LKG gestures have not been observed in previous work. They are akin to Edwards' (2003) "algorithms in the air," in that they denote an invitation to participate in a dynamic relationship, in contrast to other, more static gestures.

We took a closer look at the types of gestures favored during each type of instruction (Table 2, columns 3, 4, and 5). Overall, PTGs were present in all types of instruction; teachers were almost equally likely to point at a number or the equation, or a representation regardless of whether they scaffolded instruction towards a relational-computational or relational-structural strategy.

Representational gestures (RIGs) (Table 2, rows 3, 4, and 5) and writing gestures (WTGs) (Table 2, row 6) followed a different path, particularly as we looked at RIG subcategories. Teachers were more likely to use balance (BAL), back-and-forth (BFK), and grouping (GPGs) as they scaffolded a relational-computational (RC) strategy, encouraging students, through gestures, to group together numbers. Similarly, teachers used WTGs where they wrote (not just gestured or traced) objects such as clouds, circles, arrows, and rectangles, again, mostly to group them together, when they were scaffolding a RC strategy. At the same time, our data suggest that teachers who encouraged relational-structural (RS) strategies systematically produced linking gestures (LKGs). LKGs were *exclusively* used during RS discussions, and they were used during *all* RS discussions. In other words, whenever a teacher encouraged students to consider finding a relationship in the numbers

Table 1 Gestures across all instructional sessions

Average instruction	Min-max	Mean gestures	Min-max	Gesture density	Min-max density
2:45 min	40s–16 min	13.5	4–36	9.5 gestures/100 words	5–17.8 gestures/100 words

Table 2 Gestures across teachers

	Gesture	Relational computational (RC)	Relational structural (RS)	Not indicative of a particular conception (NI)	Total	Total
Deictic	Deictic/pointing (PTG)	6	1	4	11	135 (52.7%)
	- At the equal sign	54	47	23	124	
	- At numbers					
Representational/ Iconic (RIG)	Metaphors (specific to equal sign)	11	5	5	21	98 (38.3%)
	- Balance gesture (BAL)	2	1	1	4	
	- Back-and-forth (BKF)					
	Representational/iconic grouping gestures (GPG) (e.g., underlining, whole palm/fist, circles/clouds)	20	14	1	35	
	Representational/iconic linking gestures (LKG) (arrows, lines)	0	38	0	38	
Writing	Writing gestures (WTG) (clouds, circles, boxes, balance scale, caret, underline)	19		4	23	23 (9%)
		112	106	38		256

on either side of the equal side, they all used an LKG to focus students' attention to specific numbers.

The same pattern emerged when examining gesture use *within* each teacher. All teachers used pointing gestures throughout the episodes. As shown in Table 2, 52.7% of all teacher gestures are deictic (PTG) gestures. Similarly, teachers used representational gestures across their instruction—38.3% of all gestures are representational. However, when teachers switched from relational-computational-supporting to relational-structural-supporting instruction, they incorporated linking gestures (LKG). When teachers switched from relational-structural -supporting to relational-computational-supporting, they shifted from linking gestures to other representational gestures such as grouping gestures. Table 2 shows that the teachers used metaphor gestures or grouping gestures but *no* linking gestures when they covered any RC-supporting strategies. Hence, types of gestures are not teacher-specific, but mathematics-concept specific.

7.2 2. How does teachers' use of gestures and the type of gesture vary in relation to the conception of the equal sign the teacher is supporting?

To further examine the use of gestures, we present two episodes, each paradigmatic of the instruction that scaffolded the relational-computational and relational-structural strategies. Note that the two episodes were chosen based on their rich use of gestures so that we can best illustrate our findings. We do not claim that these episodes are exemplary of "high leverage practices" (as per Ball & Forzani, 2011). We present these as illustrative of *how* we saw gesture use. Furthermore, we do not privilege one relational strategy over the other; both are used in mathematics depending on the type of task and activity performed.

In the first episode, (Fig. 4), the teacher facilitated the development of a relational-computational strategy by noting that both sides of the equation must equal 21 and leading students through a series of computations to reach the solution. In the second episode, (Fig. 5), the teacher facilitated the development of a relational-structural strategy by encouraging students to notice structural relationships among the corresponding addends on each side of the equal sign.

In the first episode the teacher started by writing the Eq. $14 + \_\_ = 15 + 6$ on the board and focused the students' attention on the missing value, by pointing at it. She then pointed at the equal sign and circled it. Furthermore, she drew an arrow towards the equal sign as she continued to talk about it. She asked about the meaning of the equal sign while making two representational gestures: BKF (at 1:20 min) while asking "what about the two sides of that equation?", and subsequently BAL (at 1:25). The teacher returned to the computation by pointing to the entire right-hand side (1:40 min) with a full palm to capture the expression $(15 + 6)$. Notice the "indicating" nature of the gesture (Singer & Goldin, 2005) as the teacher tacitly guided students' attention towards the computation she wanted them to perform.

The teacher scaffolded an RC strategy in a coordination of speech and gestures. She encouraged students to add the numbers on each side of the equation and guided them towards the computation without explicitly telling them what to add. However, with a series of PTGs towards the equal sign and WTGs, she did not let them lose sight of the equal sign. She also engaged students in thinking about the equal sign in a metaphorical sense as a "balance" of the two sides, with BAL and BKF gestures. And while students were asked to maintain a focus on the relation, the equal sign, the expectation to complete a computation was a static one, highlighted by the repeated pointing at the missing number.

Time Stamp	T/S	Verbal transcript	Gestures Description
00:30	T	Can you solve that and can you tell me, is it true or is it false. Okay, what did you put here?	[Teacher starts by writing the equation on the whiteboard as " 14 + __ = 15 + 6 "] PTG: Pointing to blank space.
00:41	S	No audible student response.	
00:51	T	Well how did you go about solving this problem?	
00:55	T	What does this mean *right here*?	**WTG**: Writing. Motions by circling the equal sign
1:08	T	You're thinking?	
1:09	S	A scale, scale	
1:10	T	Okay, *so this symbol right here,* which is called what?	**PTG:** Points to equal **WTG**: Writing. Teacher points towards equal sign and draws arrow
1:14	S	Equal	
1:16	T	The equal sign, so *when you see that*, you know what? You're getting there. You know what about the two sides of that equation?	**PTG**: Pointed at the equal sign **BKF**: Back-and-forth
1:25	T	*What you're saying about the scale?*	**BAL**
1:27-1:32	S	(Inaudible)	
1:34	T	So it's gonna be the same amount, it's gonna balance out, like a scale example you're giving me?	**BAL**
1:40	T	First of all, *should you solve that* first maybe?	**GPG:** Pointing as grouping. Pointed at the RHS of the equation. Flat palm under "15+6"
1:43	S	Yes	
1:44	T	Okay, so what would that be?	
1:49	S	22	
1:51	Ss	21	
1:54	T	21. So *(1) what needs to* go up here so *(2) this sign* is true?	**PTG:** Pointed at the blank **PTG**: Pointed at the equal sign
1:57	S	7	[Teacher wrote 7 in the blank]
2:07	T	Is that correct? So they *(1) both equal 21*, so *(2) is that* a true statement?	PTG and WTG Pointing followed by drawing another arrow above the equal sign
2:10	Ss	Yes	
2:11	T	Yes, alright	

Fig. 4 First episode—relational computational

Time Stamp	T/S	Verbal transcript	Gestures Description
15:05	T	Who'd like to come up and fill in the blank for me and tell us how you got that.	Teacher wrote the problem on the white board, students worked in groups for about 12 minutes and the teacher invited them to share solutions and strategies
15:15	S1	[student comes to the board, writes 7 and explains process]	
15:46	T	So ***on this side you have 15***	Right hand palm straight under 15+6 **PTG** and **GPG** (pointing and representational "underline")
15:48	T	And ***on this side you have 14, you're saying that it's 1 less?***	Left hand moves under moves to left side **PTG, GPG, LKG** (pointing, representational grouping/underline and linking)
15:51	S1	Yes	
15:52	T	Minus one? Okay, alright.	
15:53	S1	Then the 6, that's what the 15 is ,, 15 is one more than 14, so 14 is 1 more less so 7 is 1 more	
16:07	T	Okay. can I make sure I understand what you're saying? You're saying that ***you got 15 over here*** in the first number	Repeats: Right hand palm down under 15+6. **PTG, GPG**
16:13		and *then on this side of the equal sign you have 14*	Repeats: Left hand moves under the left side **PTG, GPG, LKG**
16:16	T	and *you're saying that's one less* .	Right hand moves to the left, too pointing at 14 **LKG**
16:18	T	*So this side is one less*	Tapping left side with right hand. **PTG** (repeated pointing)
16:19	T	*Than this side so far.* (To the whole class) Is that what he's saying?	Moving right hand under right side**, PTG, LKG**
16:21	Ss	Yes	
16:24	T	*Let's just see if we can figure this out.*	Both hands tapping under both sides of the equation **PTG** (with both hands)
16:26		Then you're saying, *then you add (1&2) 6 over here (3)*	Circling the right side with right hand **GPG** (circle gesture) Put right hand under the right side **PTG**

Fig. 5 Second episode—relational structural

Teachers often repeated the same gestures a few times in what McNeill and Duncan (2000) called "catchment gestures," to emphasize important ideas or to provide cohesion in instructional discourse. In this case, the repeated pointing towards the equal sign and the BAL representation maintained a focus on the equal sign, even as students were performing computations, presumably avoiding the common operational pitfall (filling in the blank with "21").

16:29		And *since you already have 1 more on this side* *And 1 less on this side*	Grouping the 15+6 by gesturing a closed fist under right side, **GPG** Pointing at the left side with left hand **PTG**
16:34	T	What do you try to do? *You're trying to even it out*	**BAL** Balance
16:38	S1	Yes then I added 1 more to make it equal	
16:40	S2	I think he took the one from the 15 and make 15 14, then added the one to the 6 so make it 7.	
16:45	T	(To S1) is that what you did?	
16:46	S1	(Shook his head) No	
16:51	S2	(S2 comes to the board) he took the 1 from the 15 and added it to the 6, which makes that (pointed at 15) 14 and that (pointed at 6) 7.	
17:00	T	Okay, I think you're about the same thing, you just did it a little bit differently. (To S2) I think you're really close.	
17:06	T	Are you saying that *you have 1 more on this side* with your first number and you have 1 less on this side	**GPH** underlining the right side **GPG** underlining the left side
17:11	T	So *in order to even it out*	**BAL**
17:13		You (1) *have to have* (2)*1 less* for the second number and (3) *one more* for the second number over here, is that right?	(1) Circling the right side with right hand **GPG** (circle gesture) (2) Put right hand under the right side**, PTG** (3) Put left hand under the left side, **PTG**
17:19	T	(To S2) You did the same idea, you just (1) *moved it* (2) *within the same part*	(1) Pointed at 6 - **PTG** and **LKG** (2) Pointed at 6, 15 and 6 - **PTG** and **LKG**
17:25	T	You just took it (1) *from here* and you (2) *put it over here* to make (3) *this side equal*	(1) Pointed at 6 - **PTG and LKG** (2) Pointed at 15 - **PTG and LKG** (3) Put left hand under the right side**, GPG**
17:29	T	(To S2) Is that what you did? You think so? Okay	

Fig. 5 (continued)

The teacher in the second episode followed a relational-structural approach. She focused students' attention on the right-hand side of the equation by holding a straight palm under the expression "15 + 6," as she said, "on this side you have 15." She continued, "and on this side you have 14," while simultaneously moving her left hand in a linking gesture towards the left and, again, with a straight palm, "underlining" the left-hand side—both a pointing gesture and grouping gesture. While maintaining the students' attention on the numbers, her linking gesture indicated a relationship between the two numbers: 15 and 14. She repeated the process of underlining-pointing to each side, while also making a linking gesture from right to left. She emphasized that further by lifting her right hand and, with another linking gesture, she moved her right hand and positioned it on the left side on top of the left hand. She proceeded to tap the left side of the equation with her right hand while focusing students' attention on a comparison of the two sides: "This side is one less." The teacher returned the right hand to the right side, again pointing each hand to its respective side (left hand to the left, right hand to the right), and gestured a circle-motion, suggesting grouping, for the right side, to talk about "adding 6 over here." She pointed again to both sides, and then did a balance gesture suggesting to "even it out."

Another student suggested a compensation strategy. This student suggested adjusting "15 + 6" into "14 + 7" by breaking the first addend into 14 + 1 and moving the 1 over to 6, hence making it easier to compare with the left-hand side of the equation. The teacher took on the suggestion and scaffolded that strategy once again through a series of pointing, grouping, and linking gestures. This was a relational-structural approach, and the teacher followed the same pattern of gestures.

Notice the use of the linking gesture that was unique to the relational-structural approach. While pointing to the two sides of the equation, the teacher repeatedly formed a linking gesture, asking students to compare the two sides, particularly the two numbers, 15 and 14. We surmise she wanted students to both compare specific numbers, and to maintain a broader focus on the whole equation, and on each of the two sides as a whole. This is evidenced by her pointing while keeping a straight palm under the entire left side expression and the right side expression. It was a dynamic interplay of both keeping the big picture of the equivalence in mind and looking at the specific elements of the equation. This stands in contrast to the previous episode's static character of focusing students' attention on the computation that needed to be performed.

The teacher above, however, did not stop there. She continued by briefly bringing into the discussion a relational-computational approach (Fig. 6). This is illustrative of those cases in which the same teacher chose to use both a relational-computational and a relational-structural approach. And while this teacher had consistently used pointing and linking gestures while scaffolding the relational-structural strategy, once she transitioned to a relational-computational strategy, we saw a change in her gestures: she now used a sequence of pointing, grouping, and writing gestures, completely omitting any linking gestures. The teacher pointed to each side and drew rectangles around each expression on either side, inviting students to consider adding the numbers and in the process of doing so, she used balance gestures maintaining a focus on the equal sign.

Note, however, that this teacher swiftly shifted back to a relational-structural approach by inviting students to do a comparison of methods: "Did you have to do it this way? […] you can see how these numbers are related." At this point, the teacher resumed a linking gesture to connect the left side to the right and to invite that comparison between the two expressions. This last part of the episode is illustrative of the difference in gesture use not only across teachers but within teachers. Change of strategy brought upon a change in gesturing as well.

Previous literature suggests that many types of gestures play a positive role in students' mathematical development. For example, Goodwin (2007) made a compelling argument that gestures allow teachers to direct students' attention to particular elements of mathematics. Here, we take a step further in this direction, examining the role that gestures played in the context of developing students' understanding of the equal sign. Table 3 summarizes this discussion.

- PTG—Pointing gestures: As other researchers suggested, in a complex visual field, pointing helps focus attention to the most relevant aspects of the task, without explicitly telling students to do so. PTGs were used throughout the instructional segments.
- RIGs—Grouping (GPG): We found representational gestures to be devices that facilitate conceptual understanding. RIGs can play a similar role as visual repre-

17:40	T	Did anybody else (1) ***do the little boxes like this*** that we did earlier where you actually added up (2) ***the values on each side***?	(1) Drew a box around each side of the equation - **WTG** (2) Balance gesture **BAL**
17:51	T	Okay, if you did it that way, what did you get ***on the right side***?	(1) Pointed at the **RHS** **PTG**
17:55	Ss	21	
17:56	T	Okay, so you got ***21 over*** there, you have to get ***21 over here***?	WTG WTG
18:01	T	Alright, that's good. Did you have to do it that way?	
18:04	Ss	No	
18:05	T	Did you have to actually (1) ***solve this part*** in order to (2) ***get this part***?	(1) Grouping the 15+6 with a hand, **GPG** (2) Grouping the 14+7 with a hand , **GPG**
18:10	Ss	No	
18:11	T	Why not? Why didn't you have to actually do the computation? (Call on one student)	
18:20	S3	Because there is 15 on the other side and a 14, and 14 is one less, there's a 6 on the other side so you just need to add one more to the 6 and get 7.	
18:35	T	Okay, so you didn't even need to (1) ***figure it out***. You can just see (2) ***how these numbers are related.*** Good job guys.	**BKF** (2) Pointed at each side with each hand **PTG**
18:41	T	Either way you did is okay. There is no wrong way to do that there. But some of you almost took like a little shortcut. You kind of studied the numbers and you said well (1) ***this side is one more than this side so far***, in order to even it out (2) ***I have to make this side*** (3) ***more***. And that way (4) ***it will be equal***. Good job.	(1) Pointed at the right side with right hand and the left side with left hand **PTG** (2) Pointed at the left side**, PTG** (3)—Pointed at the right side with open palm **PTG** (4) **BAL**

Fig. 6 Second episode continued

sentations in that they might create a common ground for a shared understanding by the community. Going beyond earlier studies, we found three types of representational gestures: metaphors about the equal sign, general grouping gestures, and linking gestures. These GPGs signaled to students that two numbers or objects needed to be brought together—an action. GPGs could have a deictic function as they often suggested an expectation of an operation to perform, or act as metaphors of a new object (the expression) as Edwards (2009) suggests. These gestures were used the most during times of exploration phase, as students engaged in solving the task.

- RIGs—Metaphors of the equal sign: Balance (BAL) and back-and-forth (BKF) gestures. Teachers frequently asked, "What does the equal sign mean?" while making a balancing gesture, and subsequently rewarding as correct responses that mentioned the words "balances out the equation." In that sense, BAL and BFK gestures supported developing a relational understanding of the equal sign. Teachers used them at all stages of instruction, but particularly so when students suggested operational misconceptions while solving the task.

Table 3 Summary on the function of gestures during instruction on the equal sign

Gestures	Role—how do they function in instruction	Time—when are they used?	Conception—structural vs. computational
Deictic	Focusing students' attention to a particular feature of the task at hand Ex. Fig. 2, line 7—pointing to equal sign	At all stages of instruction	Both
Metaphors	Focusing students' attention on the meaning of equal sign Ex. Fig. 2, line 10—balance metaphor	At all stages of instruction	Both
Iconic – grouping	Focusing students' attention on the computation Ex. Fig. 2, line 13—grouping the right side of the equation	During computation and when asking students to check correctness or completion of the work	Encouraging students to use computational conceptions/strategies
Iconic - linking	Focusing students' attention on relationships Ex. Fig. 3, line 10—linking 15 on the right side of the equation and 14 on the left side	During discussions comparing strategies and solutions	Encouraging students to use structural conceptions/strategies
Writing	Focusing students' attention on the computation Ex. Fig. 2, line 22—Drawing an arrow Fig. 3, line 33—drawing rectangles around both sides of an equation	During computation and when asking students to check correctness or completion of the work	Encouraging students to use computational conceptions/strategies

- RIGs—Linking: This set of gestures was used when teachers were attending to RS strategies. They involved a level of dramatization and movement that went beyond the static pointing or grouping. Rather, they often involved connecting two parts of the equation. These gestures are akin to discussions of visual representations as "referential objects" (see Ochs et al., 1994). LKGs created an environment in which teachers "dramatized" their own understandings, inviting students to become co-participants by not only looking at an object or performing a computation, but by thinking dynamically about the relationships or connections between different objects.
- WTGs—Writing gestures: Teachers used WTGs as conscription devices—as a space for new strategies and actions to be performed. As teachers underlined or circled equations both as a gesture and as an action, they invited students to share ideas and to take further actions. They tacitly gave permission to students to bring to the fore ideas that may have not been part of the conversation. WTGs were also an extension of the representational gestures in that they were performed simultaneously.

7.3 3. Does teachers' use of gestures, in terms of type and frequency, correlate with students' performance on equation solving tasks in terms of correctness and the conception of the equal sign displayed by students when solving these tasks?

Finally, we looked at students' understanding of the equal sign and examined the relationship between the use of instructional gestures and students' understandings and strategies. To this end, we used the four aforementioned items from the final early algebra assessment and looked at the correlation between instructional gestures and student performance.

As a first step, a Pearson correlation coefficient was computed to assess the correlation of the density of instructional gestures (all gestures per 100 words) and students' performance.

We found a moderate positive correlation between the density of gestures ($m=9.31$, $sd=3.32$) and student performance on the four items ($m=0.67$, $sd=0.19$) as shown in Table 4, albeit this correlation was not significant ($r=0.41$, $p>0.01$) as shown in Table 5. In other words, our work confirmed the positive relationship between learning and gestures; however, this relationship may be more nuanced than that. Our earlier results suggested that not all gestures are used just as effectively. As a result, it seemed reasonable to unpack how specific gesture types correlated with student performance. Pointing gestures were distributed almost uniformly across lessons; hence, we decided to examine more closely the representational gestures ($m=4.42$, $sd=3.65$) and student performance on the

Table 4 Mean and SD value of student performance and teacher gestures

Variables	Mean value	SD
Student performance on items 1, 2a, 2b, 2c	0.67	0.19
OP strategy reduction (item 1 only)	−0.27	0.30
RS strategy (computational and structural) increase (item 1)	0.43	0.21
All instructional gestures' density (gestures per 100 words)	9.31	3.32
RIG density (per 100 words)	4.42	3.65

Table 5 Correlation between student performance on equal sign conception and teacher gestures

Variables	All instructional gestures' density	RIG density
Student performance on items 1, 2a, 2b, 2c	0.414	0.658***
OP reduction (item 1 only)	−0.590***	−0.635***
RS (computational and structural) increase (item 1 only)	0.587***	0.775***

*** $p < 0.01$

same four items. The correlation between the density of representational gesture use and student performance was moderate but significant ($r=0.658$, $p<0.01$). In other words, increases in representational gestures specifically were associated with greater gains in student performance on the four items.

Furthermore, we examined student conceptions of the equal sign, along with strategy use, as these related to teachers' gestures. Specifically, we looked at students' operational, relational-computational, and relational-structural strategies and their change related to teachers' gesture use. Our results indicate that greater use of representational gestures ($m=4.42$, $sd=3.65$) is significantly related to decreased operational strategies by students ($r = -0.59$, $p<0.01$). That is, when teachers used more representational gestures, students were less likely to respond that "10" is the missing number in the equation "7 + 3 = ___ + 4." At the same time, greater use of representational gestures is correlated with an increase in students' use of relational strategies (computational and structural) from pre- to post-test ($r=0.587$, $p<0.01$).

8 Discussion

The equal sign affords opportunities for a gesture-rich instructional context. Teachers in our study deployed multiple gestures during instruction, making instruction on the equal sign comparable to other gesture rich topics in mathematics education literature. At 9.5 gestures per 100 words, we found instruction on the equal sign to be comparable in gesture density to other gesture-rich environments (e.g., Alibali et al., 2013). There was great variation in gesture use and in density of gesture use between teachers. Furthermore, teachers produced a variety of gestures, some of which are specific to this context. We may infer that teachers find the use of these gestures helpful in developing their students' understanding of the equal sign. While our study did not address teachers' motivation for using gestures, our statistical analysis resonated with earlier research suggesting that density of gesture use, specifically *representational* gestures that relate to the context of equal sign, is correlated with higher student performance.

It is notable that teachers used different gestures when aiming to foster different conceptions of the equal sign. While there was abundant pointing during all instruction, other gestures were specific to either the relational-computational or relational-structural strategies. In other studies, researchers have found that gesture use is impacted by the type of information that needs to be conveyed (e.g., Holler & Stevens, 2007). Our findings resonate with these studies and take this idea further by noting that teachers' gestures are impacted not only by the topic but also by the specific conception and strategy they are aiming to foster. Specifically, teachers employed distinct gestures when aiming to facilitate different strategies within equal sign instruction. If teachers wanted to promote a computational strategy, they tended to use more writing and "capturing/grouping" gestures. Conversely, when they were supporting a structural conception, they employed linking gestures.

These differences in gesture use do not appear to be idiosyncratic to teachers, namely a personal preference or even based on teachers' own repertoire of gestures: When the same teachers shifted from focusing on one conception of the equal sign to a another, they altered their gesture use. Hence, we argue that gestures are not specific to individuals; they are aligned with mathematics concepts. Teachers adapt, or even tailor their gestures to align with conceptions they are promoting.

Similarly, we found gestures aided teachers in making connections, linking mathematical ideas and student strategies, and for eliciting student solutions. To the best of our knowledge, gesturing was a relatively spontaneous instructional choice the teachers made that varied in teachers' practice. Although our participating teachers received professional development on early algebra that included the modeling of gestures such as the balance gesture, they were not explicitly taught how or when to use or adapt these gestures. This adaptive use of gestures came seemingly naturally to teachers during instruction. As teacher educators, we can capitalize on this natural tendency and provide support by explicitly modeling how gestures can be used productively. As a first step, making teachers aware of the richness of their own gesture use and its effectiveness may increase teachers' motivation in using gestures effectively.

Our findings also indicate that gesture use correlates with increased student performance and change in conceptions of the equal sign—a symbol that maintains a central position throughout mathematics. Earlier work by Carpenter et al. (2003) and Knuth et al. (2005) suggested that success in algebra requires students to overcome operational conceptions of the equal sign and adopt a relational conception—aligned with both computational-relational and structural-relational strategies. Our findings suggest that use of representational gestures is, at a minimum, one tool that moves students in that direction. While earlier studies suggested that gestures may advance student learning (e.g., Aldugom et al., 2020), this study provides further evidence in this direction, in the case of the equal sign. It extends earlier findings that show gains in student performance on items that focus on the equal sign by suggesting that gesture use that is specific to the equal sign (representational gestures) supports improved and more flexible conceptions of the equal sign.

This work can be further viewed through the lens of the embodied cognition theory. One of the ways in which mathematical cognition is embodied lies in the use of gestures. As one of the highlights of our work was the extensive use of both pointing and representational gestures, we can further discuss gesture use through this lens:

- Pointing gestures helped focus students' attention on the most relevant mathematical information, particularly when students appeared to struggle. Teachers pointed to the numbers that needed to be attended to (actual or abstract), and to the equal sign to help students keep it at the center of their thinking and work. It did not suffice to remind students that they needed to attend to the equal sign; pointing provided an added reference that grounded students' thinking.
- Representational gestures were also common and varied. Providing a gestured motion of the "balance" using both hands, and the back-and-forth motion of maintaining equivalence on both sides helped embody the meaning of the equal sign. These same gestures when written, the writing gestures, could potentially create space for students to extend their thinking. When teachers motioned a circle around

two numbers and then even drew the circle, they gave a tacit invitation and a space for students to create a new computation and extend their thinking.

- The final type of gesture was the linking gesture. We maintain that this gesture, too, provided evidence of embodied cognition by inviting students to connect two distant ideas, numbers, or symbols. The physical, dynamic motion of tracing a curve from one side of the equation to another minimized that distance and allowed students to consider them together and to consider the symmetry of the equation structure. It possibly helped students consider two otherwise unrelated numbers in the same space. Hence, the abstraction of that request to search for a relationship was minimized through the physical action and grounded into a more manageable space. Linking gestures that were specific to the structural conception of the equal sign created a dynamic imagery that highlighted the relationship among parts of the equation. In this sense, like other signs in mathematics education, such as visual representations, gestures were used as a ‘‘stage on which scientists dramatize understanding” (Ochs et al., 1994, p. 10), where teachers and students became co-participants in the work they were doing.

One of the limitations of this study is the exclusive focus on *teachers’* gestures. While a coordinated analysis of both teachers’ and students’ gestures would inform our understanding of how students develop their understanding of the equal sign, our data did not afford us the opportunity to examine students’ gestures. Furthermore, this study only investigated teachers’ *gestures*, leaving unanswered questions about the ways teachers integrated their gestures with other semiotic productions (e.g., drawings; utterances). To be clear, while speech and other representations informed our analysis, they were not centered. A possible next step in this line of work would be to consider multiple semiotic productions (speech, gestures, written signs)—semiotic bundles—and how these interact to create the complex web of understandings that students develop over time.

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Declarations

Conflict of interest The authors declare no competing interests.

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