Stress Testing Spillover Risk in Mutual Funds

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Abstract

We develop a framework to quantify the vulnerability of mutual funds to fire-sale spillover losses. We account for the first-mover incentive that results from the mismatch between the liquidity offered to redeeming investors and the liquidity of assets held by the funds. In our framework, the negative feedback loop between investors' redemptions and price impact from asset sales leads to an aggregate change in funds' NAV, which is determined as a fixed point of a nonlinear mapping. We show that a higher concentration of first movers increases the aggregate vulnerability of the system, as measured by the ratio between endogenous losses due to fund redemptions and exogenous losses due to initial price shocks only. When calibrated to U.S. mutual funds, our model shows that, in stressed market scenarios, spillover losses are significantly amplified through a nonlinear response to initial shocks that results from the first-mover incentive. Higher spillover losses provide a stronger incentive to redeem early, further increasing fire-sale losses and the transmission of shocks through overlapping portfolio holdings.

Key words: mutual funds, liquidity mismatch, fire-sale externalities, first-mover incentive, systemic risk.

JEL Classification: G01, G23, G28

1 Introduction

The mutual fund industry has experienced strong growth in the past decade and holds an increasingly large portion of financial assets. As such, the possibility of a threat to financial stability from the mutual fund sector has become a prominent concern for regulators. In particular, the liquidity transformation provided by open-end funds has been identified as a potential source of vulnerability: investors may redeem their fund shares at the end-of-day net asset value (NAV), even if the fund holds illiquid assets that can only be liquidated over multiple days and at distressed prices. Referring to funds that hold less liquid assets, former Bank of England Governor Mark Carney

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famously stated in June 2019 that "these funds are built on a lie, which is that you can have daily liquidity for assets that fundamentally aren't liquid."

Mutual funds have been implicated in the "taper tantrum" of 2013 and in the disruption of bond markets early in the Covid-19 period. The liquidity mismatch between shareholder claims and fund holdings has resulted in the collapse of individual funds (prominent examples include the Third Avenue Focused Credit fund and the Woodford Equity Income Fund), leading to concerns for the broader impact on financial stability. The mutual fund structure creates a first-mover advantage among investors, because investors who withdraw early are shielded from the adverse impact of asset liquidation. This first-mover advantage can produce a run on a fund that amplifies fire-sale losses to other investors.

The objective of our study is to build a framework to quantify ex ante the vulnerability of mutual funds to fire sales, accounting for the first-mover incentive created by the liquidity mismatch. Our model reflects the fact that investors' redemptions are paid at an NAV that has not yet accounted for the cost of subsequent asset liquidations incurred to meet redemption requests. Furthermore, building on Capponi et al. (2020), we posit that some investors redeem fund shares in anticipation of the impact that their (and other investors') redemptions have on *future* fund performance, instead of responding only to realized shocks. We refer to these investors as first movers, and their inclusion is the key feature that distinguishes our analysis from prior work on the financial stability implications of the mutual fund structure. Funds that hold illiquid assets are more sensitive to the impact of fire sales, and their investors have a stronger incentive to exit the fund early. Early redemptions in turn increase the cost of remaining invested in the fund, and prompt additional redemptions. This creates a downward spiral of investor withdrawals, price impact, and investment losses that can substantially amplify an initial price shock.

We apply the framework to quantify the vulnerability of mutual funds in the United States to spillover losses. We take institutional investors as a proxy for first movers — the investors that exploit the liquidity mismatch. This premise, which we explore further in later sections, is in keeping with the Security and Exchange Commission's (SEC) regulatory treatment of retail and institutional money market funds (MMFs)[1] and with the empirical findings of Jin et al. (2022).[2]

We measure the aggregate vulnerability of mutual funds using the Spillover Loss Ratio (SLR), defined as the ratio between spillover losses and the initial losses due to an exogenous shock. Using a time series spanning the years 2010 through 2022, we document the growing fragility of the mu-

[1]As stated in the SEC Release No. IC-34441, "institutional investors frequently scrutinize liquidity levels in money market funds [...] facilitating rapid redemptions when a fund's liquidity begins to decline." Since 2014, institutional prime and municipal MMFs "are required to use a floating NAV because their investors have historically made the heaviest redemptions in times of market stress and are more likely to act on the incentive to redeem if a fund's stable price per share is higher than its market-based value". The SEC proposed rule "Money Market Fund Reforms", released in December 2021, proposed that these institutional funds be required to adopt swing pricing, a provision aimed at mitigating the first-mover advantage, because institutional investors are more likely to exploit this advantage. In its 2023 final rule, the SEC imposed liquidity fees on institutional funds for the same reason.

[2]They find (p.35) that, "in times of market stress, institutional investors sell more when the fund uses the traditional pricing. This indicates that, in such funds, retail investors are systematically disadvantaged. After the fund switches to swing pricing, institutional investors are more likely to alter their behavior and stay with the funds in times of stress." Thus, institutional investors act like the first movers that swing pricing is intended to target.

tual fund system over time and the increasing contribution of the first-mover incentive to spillover losses. Our empirical analysis indicates that, for flow-to-performance sensitivities within the range estimated by Goldstein et al. (2017) — specifically, between about 40% and 80% — without accounting for the first-mover incentive systemic risk would be underestimated by 20% to 60% over the time frame of our study.

We show that the first-mover incentive creates a nonlinear dependence of spillover losses on exogenous asset shocks, and this nonlinear relation has a compounding effect on losses. In more detail, we construct a *systemicness matrix* to quantify the relation between an exogenous shock and the drop in value of fund shares due to ensuing redemptions. If the spectral radius of this matrix is well below unity, then the first-mover incentive is immaterial; as the spectral radius approaches one, the first-mover incentive becomes stronger, and spillover losses become increasingly large compared to a system with no first movers.

The nonlinearity stemming from the first-mover advantage has implications for financial stability. First, a concentration of first movers in fewer funds increases the system's vulnerability. As a consequence, fund liquidity management measures that unintentionally alter the distribution of first movers across funds, e.g., by prompting them to migrate and concentrate into fewer funds, might increase the fragility of the system. For example, patchy adoption of swing pricing (a tool to remove the first-mover incentive) may inadvertently reduce the system's ability to withstand shocks, instead of strengthening it. Second, because spillover losses do not scale linearly with model inputs, small changes in asset liquidity or investor base can substantially alter the vulnerability of the financial system. This implies that historical evidence on mutual fund resilience may severely underestimate or fail to predict future fragility. The same asset shock may cause spillover losses of different magnitudes in different market environments. Third, the nonlinearity reinforces fire-sale contagion across mutual funds and asset classes. Forced liquidations can spread losses across funds and assets through overlapping portfolios. As the prospects of widespread contagion increase, so does the incentive to redeem early.

Our work provides a new framework to design macroprudential stress tests and measure vulnerability. Prior studies have analyzed the mechanism that renders mutual funds vulnerable to runs (Allen et al. (2009) and Gennaioli et al. (2013)), and provided empirical evidence for this fragility (Chen et al. (2010), Goldstein et al. (2017), Jiang et al. (2022)). The empirical study of Johnson (2004) shows that short-term fund shareholders pay for less liquidity than they demand, and thus impose liquidity costs on the long-term shareholders because of the liquidity mismatch. Our work differs from most prior studies, because we focus on measuring the impact of the first-mover incentive created by the mutual fund institutional structure.

Our paper is related to models of fire sales caused by propagation of shocks across balance sheets of constrained banks (see Greenwood et al. (2015), Duarte and Eisenbach (2021), and Capponi and Larsson (2015)). In these models, banks liquidate part of their holdings in response to an exogenous shock to satisfy leverage requirements. The spillover losses due to deviation of market prices from fundamentals are a measure of the banking sector's vulnerability to fire sales.

The studies of Fricke and Fricke (2021) and Cetorelli et al. (2016) adapt the banking fire-sales model of Greenwood et al. (2015) to mutual funds. They conclude that vulnerability to spillover losses is significantly lower for mutual funds. These studies recognize that poor fund performance leads to forced sales and depressed prices, but do not account for the amplifying effect of funds' liquidity mismatch — the mismatch between the liquidity promised to the funds' investors and the liquidity of the funds' assets. This liquidity mismatch can create greater fire-sale losses through mutual fund ownership than would be incurred if investors held the funds' assets directly. From the perspective of financial stability, it is the key feature that differentiates mutual fund investing from direct ownership of the fund's assets. Hence, macroprudential frameworks that do not incorporate the first-mover advantage, such as those discussed above, may underestimate mutual fund vulnerability.

Choi et al. (2020) study the impact of fire sales caused by fund flows in the corporate bond market. They conclude that the impact of fire sales is low because corporate bond funds maintain significant liquidity buffers to manage redemptions. The bond liquidity measure of Chernenko and Sunderam (2020) is also based on the observation that cash buffers can counterbalance low market liquidity. Cash buffers can help mitigate costly liquidations, but funds still sell non-negligible amounts of illiquid assets — for every 1% of outflows, corporate bond holdings decrease by 0.84%[3] — and cash buffers are eventually depleted. As argued above, we cannot extrapolate from the historically low impact of fire sales triggered by fund flows because the first-mover advantage is highly nonlinear in periods of market stress and low liquidity.

Ma et al. (2022*a*) demonstrate the growing significance of mutual funds as intermediaries in providing liquidity to investors. To evaluate this liquidity provision across various institutional structures, they develop the Liquidity Provision Index (LPI). This index measures liquidity transformation by comparing the contractual value of an investor's claim to the liquidation value of the fund's portfolio. A key finding of their study is that funds implementing swing pricing offer enhanced liquidity. This improvement is attributed to reduced early redemption by investors, enabling funds to maintain smaller cash reserves and hold more illiquid assets. Consequently, this lowers the portfolio's liquidation value and elevates the LPI. Our study complements theirs by examining the systemic effects of such liquidity transformation, proposing a novel systemic risk measure. We highlight the distinct advantages of swing pricing in improving the resilience of mutual funds against market shocks.

To analyze these dynamics, we extend the analytical framework of Capponi et al. (2020), which examines the feedback loop between asset illiquidity, mutual fund performance, and redemption flows. We expand this model to incorporate multiple assets and funds, and analyze the distributional effect of first movers across funds. Furthermore, we complement the empirical findings of Jin

[3]Li et al. (2023) conduct a similar investigation to Choi et al. (2020), but on municipal bonds. They conclude that fire sales due to fund outflows have a significant impact on prices. During the Covid-19 period, bonds held by municipal funds fell more than bonds held primarily by retail investors. Yield spreads between the two types of bonds persisted even after market conditions reverted to normal, suggesting the presence of a fire-sale premium for bonds held by mutual funds.

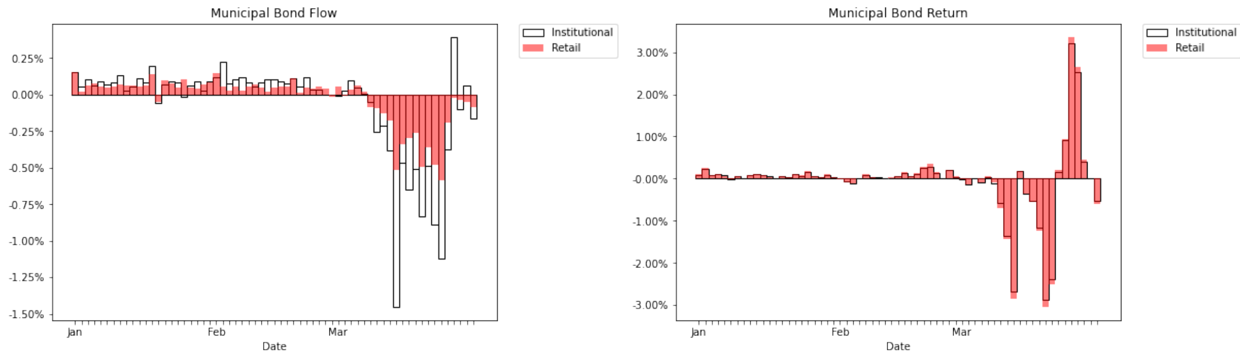

Figure 2.1: Aggregate daily flows (left panel) and average daily return (right panel) for institutional and retail fund share classes in U.S. open-end municipal bond funds during Q1 2020. We source data from the Morningstar database. Municipal bond funds posted positive returns after the Fed announced that the Money Market Mutual Fund Liquidity Facility would accept certain U.S. municipal bonds as eligible collateral on March 20, 2020.

et al. (2022), who investigate the role of swing pricing in mitigating vulnerabilities within mutual funds. We contribute to this discourse by demonstrating potential risks associated with the uneven adoption of swing pricing. Specifically, we illustrate how such uneven implementation could lead to an excessive concentration of first-mover investors in a limited number of funds, potentially undermining the financial stability of the wider mutual fund system.

The rest of the paper is organized as follows. In Section 2, we use municipal bond fund data to motivate our use of institutional investors as a proxy for first movers. In Section 3, we develop our framework and specify our measure of mutual fund vulnerability. In Section 4, we apply the model to a dataset of mutual fund portfolio holdings. In Section 5, we consider variations of our baseline framework. In Section 6, we use our framework to quantify the effectiveness of policies aiming for mutual fund stability. Section 7 connects our approach with the theoretical literature. We conclude in Section 8. Proofs of technical results and robustness checks are relegated to the Appendix.

2 Evidence from Municipal Bond Funds

As motivation for our framework, we use municipal bond funds data to test the hypothesis that institutional investors react faster to a drop in bond prices than retail investors. Municipal bonds are less liquid than many other assets held by funds, and therefore the effect of funds' liquidity transformation is stronger. This evidence lends support to our choice of using institutional investors as a proxy for first-mover investors.

As the Covid-19 shock hit financial markets in March 2020, municipal bond funds experienced a spike in outflows. While average returns for institutional and retail fund share classes were virtually indistinguishable, institutional investors were significantly more likely to run for the exit (see Figure 2.1). Institutional investors are arguably more active in monitoring market conditions and have the technical skills to anticipate how selling pressure exacerbates the impact of a market shock on prices. As a result, we expect them to be more likely to withdraw funds early.

We run the following panel regression using daily data on U.S. open-end municipal bond funds from the Morningstar database for Q1 2020, a period that covers the Covid-19 market shock:

$$\begin{aligned} Flow_{i,t} = \alpha + \beta_1 Return_{i,t-1} + \beta_2 Return_{i,t-1} \times I_{\{Return_{i,t-1}<0\}} \\ + \beta_3 I_{\{Return_{i,t-1}<0\}} + \gamma Controls_{i,t} + \varepsilon_{i,t}. \end{aligned}$$

Here, $Flow_{i,t}$ is the flow for fund share class i on day t, as reported in the Morningstar database. The $Controls_{i,t}$ variables are the lagged flow (the flow over day $t-1$), Log(TNA) (the logarithm of total net assets held by the fund on day t) and IlliqFund (an indicator variable equal to one if the fund invests in long-term bonds and zero if it invests in short-term or medium-term bonds).[4] The above specification has been adapted from the linear model of Goldstein et al. (2017). The primary distinction in our approach is the utilization of raw returns, instead of returns exceeding a sector benchmark as in Goldstein et al. (2017). Our choice to use raw returns is driven by our focus on the systemic implications of mutual fund flows, particularly the impact of sector-wide outflows, rather than isolating the effects of outflows from funds that underperform relative to a sector benchmark. This approach is especially pertinent in light of the Covid-19 pandemic, during which we observed significant outflows from most funds, not solely those with negative returns compared to their sector benchmarks.

We run the regression separately for institutional and retail share classes. The regression findings presented in Table 2.1 reveal that a 1% rise in negative returns correlates with a more than 50% larger increase in outflows for institutional investors, compared to retail fund share classes (evidenced by a comparison of 0.094 = -0.033 + 0.127 for institutional share classes against 0.060 = -0.014 + 0.074 for retail share classes). Conversely, the component of outflows independent of negative return size remains almost the same between institutional and retail investors (illustrated by -0.036 = 0.044 - 0.080 for institutional share classes versus -0.037 = 0.014 - 0.051 for retail share classes). These findings support our hypothesis that institutional investors are more reactive to negative market shocks. In Appendix A, we show that our results are robust to the observation frequency, i.e., they remain qualitatively the same if we use quarterly data from the CRSP database.[5] Our results are also consistent with prior studies, including Schmidt et al. (2016), who compare flow patterns in money market mutual funds around the collapse of Lehman Brothers in September 2008. They provide evidence that large and more sophisticated institutional investors have a stronger reaction to negative shocks than retail investors.

[4]Results are nearly identical if IlliqFund is defined to include long-term and medium-term bond funds.
[5]Unlike Morningstar, the CRSP database does not include daily mutual fund flow data.

Table 2.1: Results from regressing flows on previous day returns for municipal bond funds. Returns are computed after accounting for paid fees. Both flows and returns are measured in percent. We use daily data from Morningstar for institutional fund share classes (left column) and for retail fund share classes (right column) during Q1 2020. The dependent variable is the proportional flow of fund share classes on day t. The return variable is the daily return on day $t-1$. Lagged Flow is the flow on day $t-1$. Log(TNA) is the natural logarithm of total net assets on day $t-1$. Flows are winsorized at the 1st and 99th percentiles.

	Institutional	**Retail**
Constant	0.044***	0.014***
	(0.01)	(0.00)
Return	-0.033**	-0.014***
	(0.01)	(0.00)
Return$\times I_{\{\text{Return}<0\}}$	0.127***	0.074***
	(0.01)	(0.01)
$I_{\{\text{ Return}<0\}}$	-0.080***	-0.051***
	(0.01)	(0.01)
Lagged Flow	0.190***	0.150***
	(0.01)	(0.01)
Log(TNA)	-0.001***	-0.001
	(0.00)	(0.00)
IlliqFund	0.040***	0.013***
	(0.01)	(0.00)
N	19,960	37,379
Adj. R^2	0.078	0.042

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

3 Framework

We begin with the design of a reference model that does not account for the first-mover incentive. The sequence of events is as follows: (1) Asset prices are subject to an exogenous initial shock; (2) investors redeem shares in response to funds' (negative) returns; (3) funds liquidate assets to repay redeeming investors; (4) forced sales drive down market prices; (5) further fund redemptions and asset sales are triggered, i.e., steps (2)–(4) are repeated. This reference framework is related to that proposed by Cetorelli et al. (2016) and Fricke and Fricke (2021) for mutual funds based on the banking model of Greenwood et al. (2015). Our reference model differs primarily in accounting for multiple rounds of share redemptions and asset sales.

We then extend the reference model to the full model, which accounts for the liquidity mismatch in the mutual fund structure. The full model differs from the reference model in two crucial aspects: some investors are fast and redeem before the fund liquidates assets, and thus get repaid at an NAV that does not yet account for liquidation costs; and those investors respond not only to realized returns but also to anticipated liquidation costs that will result from further redemptions by other investors.

We assume that a fund liquidates assets in proportion to its holdings. This is the most commonly adopted liquidation strategy in the fire sales literature (e.g., Greenwood et al. (2015), Duarte and Eisenbach (2021)), and the one implicitly assumed by the SEC in its proposed rule "Money Market Fund Reforms".[6] Jiang et al. (2021) also find that funds tend to liquidate proportionally in stressed scenarios, in order to prevent the liquidity level of their portfolio from deteriorating excessively. Furthermore, funds often have mandates that restrict them from deviating widely from a target mix of assets. We consider alternative liquidation rules in Section 6.2 and Appendix C.

We use lowercase letters to denote quantities for individual funds or assets, and uppercase letters to denote vectors or matrices that summarize quantities for multiple funds or assets. The system consists of N mutual funds, indexed by $i \in \{1, \ldots, N\}$, and K assets, indexed by $k \in \{1, \ldots, K\}$. We use a_i to denote the dollar value of fund i's asset holdings, and A to denote the $N \times N$ diagonal matrix with entries $A_{ii} = a_i$. The weight of asset k in fund i's portfolio, m_{ik}, is the ratio of the dollar value of fund i's holdings in asset k to a_i, and M is the $N \times K$ matrix of portfolio weights. The asset holdings of each fund i are divided into q_i^0 identical portfolio units. One portfolio unit comprises a pro rata amount of each security, i.e., a portfolio unit of fund i consists of m_{ik} shares of each asset k. We normalize the initial price of a share of each asset to \$1. Hence, by construction, the initial value p_i^0 of a unit of fund i's portfolio is equal to \$1. For each fund i, there are n_i^0 outstanding shares. The initial value of a share of fund i, s_i^0, is also normalized to \$1. Therefore, $a_i = n_i^0 = q_i^0$.

[6]The swing factor in the proposed rule was to be computed as the cost of liquidating a pro rata amount of each security in the fund's portfolio. The same procedure applies to the liquidity fee in the final rule.

3.1 Reference Model without First Movers

We outline the sequence of events and actions in the reference model of mutual funds where no first mover is present. Throughout the paper, we use $\top$ to denote the transpose of a matrix.

1. Exogenous shock and investors' redemptions. The assets are hit by negative shocks $\Delta F^0 := (\Delta f_1^0, \ldots, \Delta f_K^0)^\top$. The magnitude of shock Δf_k^0 is smaller than the price of asset k, so asset prices remain positive. The value of a portfolio unit of fund i decreases by

$$\Delta p_i^0 = \sum_{k=1}^{K} m_{ik} \Delta f_k^0. \tag{1}$$

Therefore, the change in value of each fund's portfolio is given by the vector $\Delta P^0 = M \Delta F^0$. The change in value of a share of fund i is

$$\Delta s_i^0 = \frac{q_i^0}{n_i^0} \Delta p_i^0 = \Delta p_i^0. \tag{2}$$

Let U be the $N \times N$ diagonal matrix with $U_{ii} = \frac{q_i^0}{n_i^0}$. In vector form, the change in fund share value is $\Delta S^0 = U \Delta P^0 = U M \Delta F^0$. Because $n_i^0 = q_i^0$, U is the identity matrix.[7]

We assume a linear relation between fund performance and net fund flow. Let b_i be the flow-to-performance sensitivity of fund i, i.e., following a change in fund i's share value Δs_i^0, investors redeem

$$\Delta w_i^0 := -a_i \cdot b_i \cdot \Delta s_i^0 \tag{3}$$

shares of the fund. B is the $N \times N$ diagonal matrix with $B_{ii} = b_i$. In vector form, $\Delta W^0 = -AB\Delta S^0$ is the number of redeemed shares per fund.

2. Asset liquidation. Funds liquidate assets to raise cash to repay redeeming investors. We assume that funds sell their holdings proportionately to their portfolio weights. In other words, each fund sells some number of its portfolio units. This pro rata liquidation strategy is the most commonly adopted assumption in the fire-sale literature;[8] it posits that funds aim to hold the same portfolio mix before and after asset liquidation.

Each fund i sells Δq_i^0 units of its portfolio to meet Δw_i^0 redemptions, with Δq_i^0 determined by

$$\Delta q_i^0 \cdot (p_i^0 + \Delta p_i^0) = \Delta w_i^0 \cdot (s_i^0 + \Delta s_i^0);$$

the expression on the left is the cash raised through the sale, and the expression on the right is the cash required. Because $p_i^0 = s_i^0$ and $\Delta p_i^0 = \Delta s_i^0$, it follows that $\Delta q_i^0 = \Delta w_i^0$. Since fund i

[7]In the full model, the number of portfolio units and that of fund shares may instead deviate, and it is therefore more convenient to express quantities using the matrix U.

[8]For example, Greenwood et al. (2015), Duarte and Eisenbach (2021), Fricke and Fricke (2021) make this assumption.

sells $m_{ik}\Delta q_i^0$ shares of asset k, the total number of shares of asset k liquidated across funds is $\sum_{j=1}^N m_{jk}\Delta q_j^0$. In vector form, ΔQ^0 is the number of sold portfolio units per fund, and $M^\top \Delta Q^0$ is the number of sold shares per asset across all funds.

3. Price impact. Asset liquidation has a linear impact on asset prices. After a sale of Δh shares of asset k, the price of asset k declines by $l_k \cdot \Delta h$. L is the $K \times K$ diagonal matrix with price impact coefficients $L_{kk} = l_k$.

The number of shares of asset k sold by all funds is $\sum_{j=1}^N m_{jk}\Delta q_j^0$, so the price of asset k declines by $l_k \sum_{j=1}^N m_{jk}\Delta q_j^0$. The change in value of a portfolio unit of fund i due to liquidation costs is then

$$\Delta p_i^1 = -\sum_{k=1}^K m_{ik} l_k \sum_{j=1}^N m_{jk}\Delta q_j^0.$$

In vector form, $\Delta P^1 = -MLM^\top \Delta Q^0$. Hence, the change in value of fund i's share due to liquidation costs is

$$\Delta s_i^1 = \frac{(q_i^0 - \Delta q_i^0)(p_i^0 + \Delta p_i^0 + \Delta p_i^1)}{n_i^0 - \Delta w_i^0} - s_i^0 - \Delta s_i^0.$$

Since $p_i^0 = s_i^0$, $\Delta p_i^0 = \Delta s_i^0$, $q_i^0 = n_i^0$ and $\Delta q_i^0 = \Delta w_i^0$, we obtain that $\Delta s_i^1 = \frac{q_i^0}{n_i^0}\Delta p_i^1$.[9] Hence, in vector form, $\Delta S^1 = UMLM^\top UAB\Delta S^0$.

4. Further rounds of redemptions and asset liquidation. The change in funds' share values due to the price impact of fire sales triggers further redemptions. Investors redeem an amount $\Delta W^1 = -AB\Delta S^1$ of additional fund shares, funds liquidate $\Delta Q^1 = -UAB\Delta S^1$ portfolio units, which in turn drives down the value of each portfolio unit by $\Delta P^2 = MLM^\top UAB\Delta S^1$, and results in the fund share change in value $\Delta S^2 = UMLM^\top UAB\Delta S^1$. The total fund share value change due to both fire sales and the initial exogenous shock is $\Delta S^\infty := \sum_{n=0}^\infty \Delta S^n$, where $\Delta S^n = (UMLM^\top UAB)^n \Delta S^0$ is the change in value after the n-th round of redemptions. (Recall that in the reference model, U is the identity matrix. We have included it here in preparation for the full model.)

3.2 Full Model

In this section, we describe the steps and actions in the full model, which accounts for the presence of first movers in the funds. We refer to all other investors as second movers. Recall that B is the $N \times N$ diagonal matrix with $B_{ii} = b_i$, where b_i is fund i's flow-to-performance sensitivity, and L is the $K \times K$ diagonal matrix with price impact coefficients $L_{kk} = l_k$.

1. First movers' redemptions. Following the initial negative shock $\Delta F^0 = (\Delta f_1^0, \ldots, \Delta f_K^0)^\top$ to asset prices, the value of fund i's portfolio unit declines by Δp_i^0 in (1), and the fund's NAV also

[9] In the reference model, $\frac{q_i^0}{n_i^0} = 1$. We include this coefficient for notational consistency with the full model.

declines by Δs_i^0 in (2). We write Δs_i^∞ for the total change in fund i's NAV due both to the initial exogenous shock and subsequent fire sales. We do not yet know Δs_i^∞; it will be determined as a fixed point as we iteratively update the funds' NAVs through subsequent rounds of redemptions and liquidations. We write Δs_i^* for an initial guess of the total NAV change Δs_i^∞.

The proportion of first movers among fund i's investors is π_i, and Π is the $N \times N$ diagonal matrix with $\Pi_{ii} = \pi_i$. Fund i's first movers withdraw their investments in response to the anticipated (as yet unrealized) NAV change Δs_i^* and redeem

$$\Delta w_i^{fm} := -a_i \cdot \pi_i \cdot b_i \cdot \Delta s_i^* \tag{4}$$

fund shares. In vector form, $\Delta W^{fm} = -A\Pi B\Delta S^*$ represents the quantity of shares redeemed by first movers, and ΔS^* is the vector of anticipated NAV changes, Δs_i^*, per fund. Equation (4) captures the key feature of first movers: they anticipate that liquidation costs will drive down the fund's NAV, and they redeem shares in anticipation of this decline. In contrast, the redemption orders in (3) respond only to the realized decline Δs_i^0.

2. Asset liquidation to repay first movers. When mutual fund investors redeem shares, they receive a price per share equal to the NAV at the end of the day that they submitted their redemption orders. As the fund sells assets to meet these redemptions, it incurs liquidation costs that are borne by investors who remain in the fund. In particular, first movers do not bear the liquidation costs they impose on the fund. Each share of fund i redeemed by first movers is repaid at the price $s_i^0 + \Delta s_i^0$, and fund i sells Δq_i^{fm} units of its portfolio to meet first movers' redemptions. Since funds sell assets in proportion to their initial allocations, the total amount of shares of asset k liquidated across all funds is $\sum_{j=1}^N m_{jk}\Delta q_j^{fm}$, and the price of a share of asset k declines by $l_k \sum_{j=1}^N m_{jk}\Delta q_j^{fm}$. The cash raised by each fund i from asset sales is $\Delta q_i^{fm} \cdot (p_i^0 + \Delta p_i^{fm})$, where

$$\Delta p_i^{fm} = \Delta p_i^0 - \sum_{k=1}^K m_{ik} l_k \sum_{j=1}^N m_{jk}\Delta q_j^{fm} \tag{5}$$

is the change in value of fund i's portfolio unit due to both the exogenous shock (reflected in Δp_i^0) and asset liquidation (reflected in the double sum in (5)). In vector form, ΔQ^{fm} is the number of portfolio units sold to repay first movers, and $\Delta P^{fm} = \Delta P^0 - MLM^\top \Delta Q^{fm}$ is the resulting price change. In order to meet first movers' redemptions, the number of portfolio units Δq_i^{fm} sold by fund i must satisfy

$$\Delta q_i^{fm} \cdot (p_i^0 + \Delta p_i^{fm}) = \Delta w_i^{fm} \cdot (s_i^0 + \Delta s_i^0). \tag{6}$$

The expression on the left is the cash raised through the sale, and the expression on the right is the cash required to redeem Δw_i^{fm} fund shares. Hence, the vector ΔQ^{fm} is the solution to the system

$$Diag[\Delta Q^{fm}](P^0 + \Delta P^{fm}) = Diag[\Delta W^{fm}](S^0 + \Delta S^0), \tag{7}$$

where $Diag[x]$ is the diagonal matrix whose j-diagonal entry is x_j. Recall that ΔW^{fm} and, as a consequence, ΔQ^{fm} and ΔP^{fm} are functions of the (as yet unknown) total NAV change ΔS^*.[10]

3. NAV change due to first movers' redemptions. The share price $s_i^0 + \Delta s_i^0$ received by first movers does not incorporate the liquidation costs they generate because $s_i^0 + \Delta s_i^0 = p_i^0 + \Delta p_i^0 > p_i^0 + \Delta p_i^{fm}$, and therefore $\Delta q_i^{fm} > \Delta w_i^{fm}$ in (6). As a result,

$$n_i^{fm} := n_i^0 - \Delta w_i^{fm} \geq q_i^0 - \Delta q_i^{fm} =: q_i^{fm}. \tag{8}$$

Here, n_i^{fm} is the number of fund shares remaining after the first-mover redemptions, and q_i^{fm} is the number of portfolio units remaining after the asset sales used to meet these redemptions. Fund i's NAV after first movers' redemptions is $s_i^{fm} = \frac{q_i^{fm}}{n_i^{fm}}(p_i^0 + \Delta p_i^{fm})$, which is the ratio of the fund's assets to the number of fund shares outstanding. The change in NAV observed by remaining investors is $\Delta s_i^{fm} = s_i^{fm} - s_i^0$. Let U^{fm} be the $N \times N$ diagonal matrix with diagonal entries

$$U_i^{fm} = \frac{q_i^{fm}}{n_i^{fm}}. \tag{9}$$

The NAV change due to both the exogenous shock and first movers' redemptions is

$$\Delta S^{fm} = U^{fm}(P^0 + \Delta P^{fm}) - S^0, \tag{10}$$

where the vectors P^0 and S^0 are, respectively, the initial value of a portfolio unit and of a fund share. The NAV change ΔS^{fm} is a function of ΔS^*.

4. Second movers' redemptions. The remaining iterations mirror the reference model. Fund i's second movers observe the NAV change Δs_i^{fm} and redeem $\Delta w_i^{0,sm} = -a_i(1-\pi_i)b_i\Delta s_i^{fm}$ fund shares, which parallels (3). In vector form, $\Delta W^{0,sm} = -A(1-\Pi)B\Delta S^{fm}$. Following the same steps as in the reference model, redemptions force funds to sell assets, further depressing asset prices and fund NAVs. More precisely, the impact of second movers' redemptions on each fund's NAV is

$$\Delta S^{1,sm} = U^{fm} M L M^\top U^{fm} A(I - \Pi) B \Delta S^{fm}.$$

This NAV change triggers further rounds of redemptions by second movers. The total change in each fund's NAV is

$$\Delta S^\infty = \sum_{n=0}^{\infty} \Delta S^{n,sm} \tag{11}$$

where $\Delta S^{n,sm} = (U^{fm} M L M^\top U^{fm} A(I - \Pi)B)^n \Delta S^{fm}$.

[10] In our numerical calculations, we truncate (5) and (6) so that prices never become negative and funds never sell more assets than they own. In our theoretical analysis in Appendix B, we show that these caps are unnecessary for sufficiently small price impact coefficients $(l_k)_k$.

5. Total NAV change. The total NAV change $\Delta S^\infty(\Delta S^*) = \sum_{n=0}^{\infty} \Delta S^{n,sm}$ computed in the previous steps depends on the initial guess ΔS^* through ΔS^{fm}. But recall that we assume that first movers correctly anticipate the full NAV impact of the initial shock and subsequent liquidations. This holds when $\Delta S^* = \Delta S^\infty(\Delta S^*)$; that is, when the anticipated NAV impact is a fixed point of the mapping defined by (11). The next proposition establishes the existence of such a fixed point.

Proposition 1. *Assume that M has nonnegative entries, each price impact coefficient l_k is sufficiently small, $b_i < 1$ for each i, and $\Delta s_0^i > -s_0^i$ for each i.*[11] *Then there exists a unique fixed point of the mapping $\Delta S^* \to \Delta S^\infty(\Delta S^*)$ defined in step 5 of the above procedure.*

3.3 Aggregate Vulnerability Measure

We measure the aggregate vulnerability of the mutual fund sector as the total amplification of losses through the sector. We measure this amplification through the ratio between the endogenous losses, due to fund redemptions and fire sales, and the exogenous losses caused by the initial shock only. Formally, we define the Spillover Loss Ratio as

$$SLR := \frac{\sum_i a_i \Delta s_i^{sl}}{\sum_i a_i \Delta s_i^0},$$

where the sum is over funds, a_i is fund i's asset value, and $\Delta s_i^{sl} := \Delta s_i^\infty - \Delta s_i^0$ is the NAV change due exclusively to the feedback loop between fund redemptions and fire sales of assets needed to meet these redemptions.

We impose a cap on both the number of portfolio units that each fund can sell and the price impact imposed on each asset. A fund cannot sell more portfolio units than it owns, so the total number of liquidated portfolio units Δq_i is capped at a_i. A fund *fails* if it liquidates all of its assets. Furthermore, asset prices cannot become negative as a result of price impact from sales.

4 Mutual Fund Aggregate Vulnerability

In this section, we apply the model to the system of U.S. mutual funds and estimate the system's Spillover Loss Ratio from data.

4.1 Data Description

We use quarterly mutual fund holding data from the CRSP Survivor-Bias-Free US Mutual Fund Database spanning the period Q1 2010 through Q4 2020. For each date, we remove from the database ETFs, index funds, money market funds, funds with missing information, and funds with less than $5 million in total net assets. The database includes the total net asset value of each fund,

[11] An initial shock $\Delta s_0^i = -s_0^i$ implies that fund i's asset holdings become worthless, and its fire sales are inconsequential. Hence, the fixed point of the system could be computed without including fund i.

and groups each fund's holdings into the twelve asset classes listed in Table 4.1. We divide funds into nine types, according to their CRSP Style Code. The types are equity domestic (ED), equity foreign (EF), fixed income municipal (IU), fixed income corporate (IC), fixed income government (IG), fixed income foreign (IF), other fixed income (I), mixed fixed income and equity (M), and other (O). For each type, we work with the 100 largest funds, and we combine the holdings for the remaining funds into a single aggregate fund.[12] Hence, for each quarter the system consists of at most 909 funds, and we use these funds to construct the matrices A and M from the CRSP data.

We use price impact parameters estimated under stressed trading conditions by Bouveret and Yu (2021).[13] To account for time varying liquidity, we construct a price impact matrix L_t that depends on time t. The parameters in Table 4.2 pin down the matrix $L_t = L^*$ at the initial date of our analysis (the benchmark date), which is Q1 2010. The price impact matrix is then renormalized by the size of the financial sector on subsequent dates to capture the idea that the pool of potential buyers of fund assets varies over time. For this calculation, we follow a similar approach to Duarte and Eisenbach (2021). As a proxy for the wealth w_t of potential buyers of liquidated assets, we take the value of assets held by the U.S. financial sector and U.S. households minus the value of mutual fund shares they hold. We source this data from the "Financial Accounts of the United States".[14] The price impact matrix at date t is $L_t = \frac{w^*}{w_t} L^*$, where w^* is the value of w_t at the benchmark date.

The CRSP database classifies every fund share class as either institutional or retail. We measure the proportion of first-mover investors π_i in fund i as the proportion of total net assets held by institutional share classes within fund i. This identification is supported by the empirical evidence and discussion in Section 2. We will also investigate the effect of varying the proportion of first movers. Observe that our measure π_i depends on the quarter t.

Prior research has studied the relationship between fund flows and performance. For example, Franzoni and Schmalz (2017) find that the sensitivity of flow to performance strongly depends on the state of the market and can range from 20% to around 70%. These estimates cannot disentangle the direct response measured by the coefficient b in our model from the combined effect of first- and second-mover redemptions. We will therefore examine the impact of different values of b, holding this parameter constant across funds.

We apply shocks of different magnitudes to different asset classes, based on their relative volatilities. For example, to translate a 10% drop in stock prices to an equally severe shock to municipal bonds, we would use a drop of 3.981%, based on the relative volatilities in Table 4.2. To calculate the relative volatilities under stress, we use daily returns during Q1 2020 (the Covid-19 shock) on

[12] We have verified that aggregating funds at different levels of granularity does not significantly affect our results. Aggregation may even understate vulnerability, because it removes the first-mover heterogeneity within each aggregated fund.

[13] Greenwood et al. (2015) assume that a net trade of 10 billion euros leads to a price change of 10 basis points, regardless of the liquidated asset. Duarte and Eisenbach (2021) consider heterogeneous price impact parameters implied by the Net Stable Funding Ratio of the Basel III regulatory framework.

[14] The corresponding codes are FL794090005 (Domestic financial sectors; total financial assets), FL154090005 (Households and nonprofit organizations; total financial assets), FL793064205 (Domestic financial sectors; mutual fund shares; asset), FL153064205 (Households and nonprofit organizations; mutual fund shares; asset).

	Domestic Equity	Foreign Equity	FI Corporate	FI Foreign	FI Government
Total assets ($ billions)	1,783	647	46	75	73
Institutional investors (percent)	37.67	48.79	35.64	62.36	47.92
Portfolio shares (percent):					
Cash	2.19	2.12	2.03	4.84	1.99
Common Stocks	85.32	84.51	0.38	0.11	0.03
Preferred Stocks	0.23	0.91	0.49	0.11	0.01
Convertible Bonds	0.15	0.04	0.65	0.10	0.00
Corporate Bonds	1.94	0.89	57.86	24.08	5.51
Municipal Bonds	0.08	0.03	2.89	1.11	0.29
Government Bonds	3.33	1.65	16.46	59.09	72.26
Asset-Backed Securities	0.33	0.07	4.92	2.25	6.12
Mortgage-Backed Securities	0.79	0.14	9.39	2.30	10.85
Other Equities	2.40	7.15	0.09	0.06	0.00
Other Fixed-Income Securities	0.34	0.12	2.49	1.52	1.68
Other Securities	2.90	2.38	2.36	4.44	1.24

	FI Muni	FI Other	Mixed FI & Equity	Other
Total assets ($ billions)	213	566	570	110
Institutional investors (percent)	26.29	55.48	27.18	51.82
Portfolio shares (percent):				
Cash	1.21	2.67	2.78	6.84
Common Stocks	0.07	0.51	51.93	6.23
Preferred Stocks	0.04	0.43	0.59	0.13
Convertible Bonds	0.00	0.28	1.23	0.34
Corporate Bonds	2.06	43.74	13.98	16.19
Municipal Bonds	95.38	1.52	0.61	0.48
Government Bonds	0.14	18.99	13.91	8.69
Asset-Backed Securities	0.02	9.42	1.64	9.02
Mortgage-Backed Securities	0.02	14.63	4.52	24.39
Other Equities	0.01	0.19	2.26	0.34
Other Fixed-Income Securities	0.45	4.74	1.76	20.72
Other Securities	0.59	2.87	4.77	6.61

Table 4.1: Summary of the balance sheet data used to compute aggregate vulnerability. The table shows average total net assets, proportion of assets held by institutional fund share classes, and aggregate portfolio composition for each fund type over the period from Q1 2010 to Q4 2022.

Asset Class	Price Impact	Relative Volatility
Cash	0	0
Common Stocks	$2.8{\times}10^{-13}$	1
Preferred Stocks	$2.8{\times}10^{-13}$	1
Convertible Bonds	$7.7{\times}10^{-13}$	0.8710
Corporate Bonds	$7.7{\times}10^{-13}$	0.3169
Municipal Bonds	$14.5{\times}10^{-13}$	0.3981
Government Bonds	$0.3\ {\times}10^{-13}$	0.1905
Asset-Backed Securities	$0.5{\times}10^{-13}$	0.1829
Mortgage-Backed Securities	$0.5{\times}10^{-13}$	0.1829
Other Equities	$2.8\ {\times}10^{-13}$	1
Other Fixed-Income Securities	$0.3\ {\times}10^{-13}$	0.3169
Other Securities	0	0

Table 4.2: A price impact of 10^{-13} indicates that a $10 billion net trade leads to a price decline of 10 basis point. The second column is the relative daily volatility, over Q1 2020, of an ETF representative of each asset class compared to that of equity.

representative ETFs for each asset class. We use the Vanguard Total Stock Market ETF (VTI) for common and preferred stocks and other equities; the iShares Convertible Bond ETF (ICTV) for convertible bonds; the Vanguard Total Bond Market Index Fund ETF (BND) for corporate bonds and other fixed-income securities; the iShares National Municipal Bond ETF (MUB) for municipal bonds; the iShares US Treasury Bond ETF (GOVT) for government bonds; and the iShares MBS ETF (MBB) for mortgage-backed securities and asset-backed securities. Unless otherwise specified, we apply the exogenous shock simultaneously to all asset classes, excluding the "Other Securities" class, to which no shock is applied.

4.2 Mutual Fund Vulnerability in the Reference Model

We begin by measuring spillover losses in the reference model without first movers and then measure the impact of accounting for first movers. Through portfolio overlap, as reflected in M, fire sales can spread from one asset to another. We refer to the matrix $MLM^\top AB$ as the *systemicness matrix*. The total change in each fund's share value is then given by the vector

$$\sum_{n=0}^{\infty}(MLM^\top AB)^n \Delta S^0,$$

At each round of redemptions, the vector of shocks is multiplied by the systemicness matrix. If its spectral radius is smaller than 1, then the spillover losses of each round are eventually smaller than losses from the previous round of redemptions. If instead the spectral radius is larger than 1, the vector of NAV shocks ΔS^0 can get amplified in each iteration. The spectral radius of the systemicness matrix is therefore a measure of aggregate fund exposure to fire sales caused by redemptions.

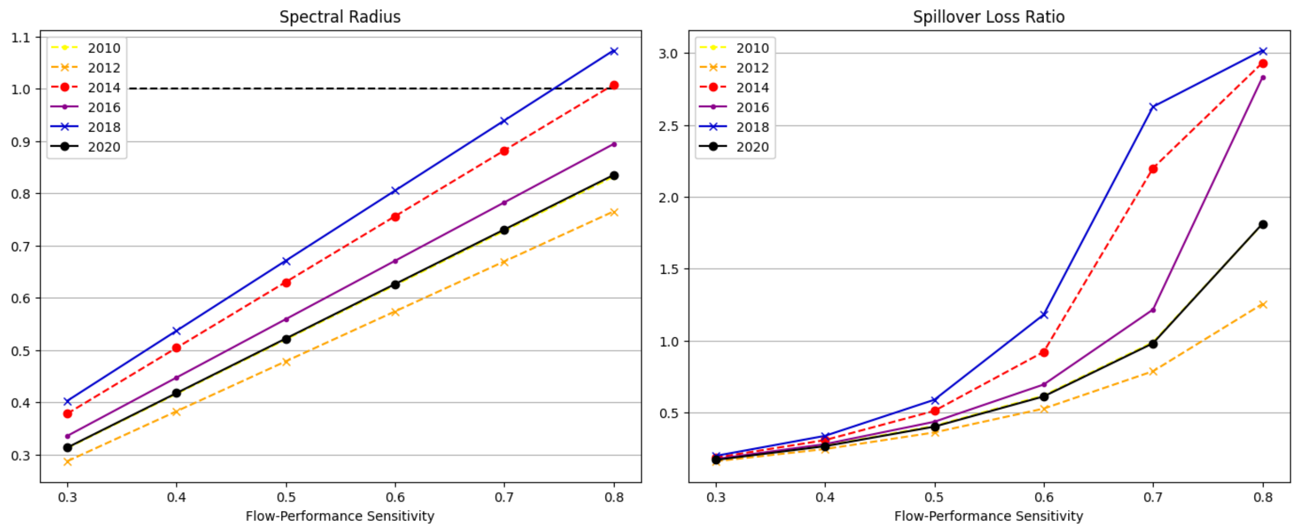

Figure 4.1: Flow-to-performance sensitivity is assumed constant across funds, data refer to the first quarter of each year. The left panel shows the spectral radius of the systemicness matrix for different values of flow-to-performance sensitivity and different years. The right panel shows the Spillover Loss Ratio for different values of flow-to-performance sensitivity and different years.

The systemicness matrix can be decomposed into three factors, analogously to the decomposition of aggregate vulnerability in Duarte and Eisenbach (2021): $MLM^\top$ is the *illiquidity concentration*, A is the *size* of the system, B is the *flow-to-performance sensitivity*. The (i, j) entry of the illiquidity concentration matrix $MLM^\top$, $\sum_{k=1}^{K} l_k m_{ik} m_{jk}$, is the liquidity-weighted portfolio overlap of funds i and j. Recall that the entries of the diagonal matrices A and B are, respectively, the net total assets and flow-to-performance sensitivity of each fund. The spectral radius is therefore larger, and the system more vulnerable, if large funds with high flow-to-performance sensitivity have significant portfolio overlap on illiquid assets.

The magnitude of the SLR is directly related to the value of the spectral radius. In the right panel of Figure 4.1, we calculate the SLR for an initial exogenous shock of -5%, scaled by the relative volatilities listed in Table 4.2, specific to each asset class. By comparing the two panels in Figure 4.1, we see that spillover losses dwarf initial losses if the spectral radius is close to 1 or larger.[15] Moreover, in recent years, the spectral radius has exceeded 1 for large, yet plausible, values of flow-to-performance sensitivity.

Investors that hold their assets directly, rather than through a mutual fund, may also liquidate them if their portfolios are subject to a negative shock. As a result, they would drive down asset prices. If holding a portfolio directly or through a fund does not affect investors' sensitivity to performance, the spillover losses quantified using the reference model would remain in the absence of mutual fund intermediation. However, as we demonstrate in the next section, spillover losses would be greater if the assets are intermediated by the fund, after accounting for the first-mover advantage.

[15] Observe that spillover losses are finite because of the imposed caps discussed in Section 3.3.

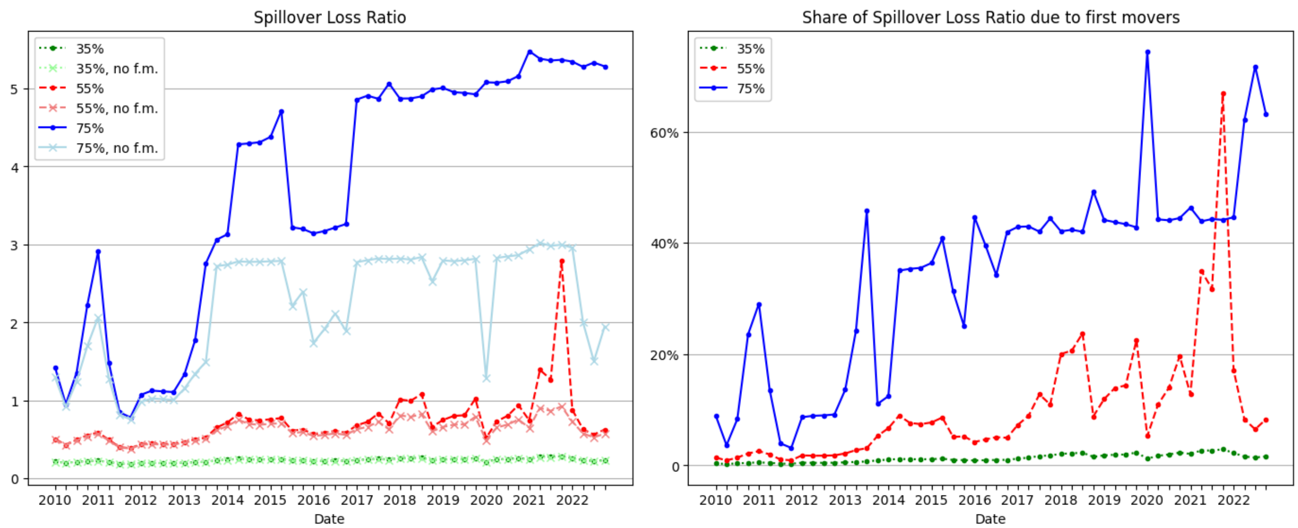

Figure 4.2: The left panel shows the Spillover Loss Ratio over time for different values of flow-to-performance sensitivity with/without first movers. The right panel shows the proportion of Spillover Loss Ratio due to the presence of first movers.

4.3 Impact of First-Mover Advantage

We now quantify the share of spillover losses that can be attributed to funds' liquidity mismatch and the resulting first-mover advantage.

4.3.1 Spillover Loss Ratio over Time

The presence of first-mover investors exacerbates the vulnerability of a fragile system but has minimal impact on a resilient one. In fact, first movers have limited incentive to exit a fund early, if asset liquidation costs are low, i.e., when the spectral radius of the systemicness matrix is significantly below 1. However, the first-mover advantage has a strong destabilizing effect on a system that is already vulnerable: if first movers expect funds to face significant spillover losses, then they benefit from redeeming their fund shares early, accelerating a systemic fire-sale spiral. In Figure 4.2, we compute the SLR with and without first movers for an initial price change of -5% multiplied by the relative volatilities in Table 4.2 for each asset. If in the absence of first movers the system would be resilient to spillover losses, e.g., if flow-to-performance sensitivity is low, then the impact of first movers is negligible. However, the fragility of a system that is moderately vulnerable without first movers may deteriorate when accounting for the first-mover advantage. As shown in Figure 4.2, after the year 2017 and assuming a flow-to-performance sensitivity of 55%, the SLR in the full model is often at least 20% larger than in the reference model.

4.3.2 Contributing Factors to Spillover Losses

The vulnerability of the mutual fund system is sensitive to several factors (see Figure 4.3). The first factor is the size of the U.S. mutual fund industry relative to the whole U.S. financial sector.

Over time, funds have accounted for an increasingly large share of the whole financial market. To see this, compare the first quarter of 2010 when assets held by mutual funds accounted for less than 12% of all financial assets, with the last quarter of 2019 when this proportion grew to more than 16%.

A second factor is the concentration of fund holdings in illiquid assets. Let a^m be the aggregate asset value held by mutual funds and a^{tot} the total value of assets in the whole financial system. The matrix $C := \frac{a^{tot}}{a^m} \cdot MLM^\top A$ quantifies the impact that portfolio overlap in illiquid assets has on each fund.[16] Notice that this matrix is independent of the size of the system: the entries of $\frac{A}{a^m}$ are the weights of each fund in the system, and the entries of $a^{tot}L$ are (approximately) a size-independent measure of each asset's illiquidity. (Recall that our specification of price impact is such that assets are more liquid as the size of the whole financial system increases.) We measure the amplification effect due to portfolio concentration in illiquid assets using the spectral radius of the matrix C. Notice that accounting only for the impact of C on the initial vector of shocks ΔS^0 does not capture vulnerability due to portfolio commonality. This is because we consider multiple rounds of redemptions and fire sales and, in each round, the vector of realized shocks across asset classes may be different compared to the previous round. As seen from the top right panel of Figure 4.3, the impact of illiquidity concentration on the system's vulnerability has increased steadily since 2013.

A third factor is the propensity of investors to redeem fund shares in response to a decline in fund NAV. The stronger the reaction of investors to negative NAV shocks, the more vulnerable the system to asset fire sales. As shown in Goldstein et al. (2017), funds that hold more illiquid assets have a higher sensitivity of outflows to bad performance. Even if our analysis assumes that the flow-to-performance sensitivity b_i is the same across funds, a fund holding illiquid assets is subject to more redemptions after a negative initial shock than a fund holding liquid assets because first movers anticipate the higher spillover losses and therefore have a stronger incentive to redeem early.[17] Hence, our model is consistent with the finding of Goldstein et al. (2017).

The fourth factor is the proportion of institutional investors among holders of fund shares. Early redemptions by first movers increase asset liquidation pressure and, hence, spillover losses. The presence of more first movers creates additional feedback effects, as other first movers account for their withdrawals and hence redeem additional fund shares. It can be seen from the bottom graph of Figure 4.3 that the fraction of assets held in institutional fund share classes, our proxy for the proportion of first movers, has increased to nearly 50% in the year 2020. Even though we presented the proportion of first movers as a separate factor that affects the system's vulnerability, we cannot disentangle the impact of illiquidity concentration from that of first movers. This is because we consider a system with first mover heterogeneity: fragility is magnified if funds holding concentrated portfolios have a higher proportion of first movers. Even in an otherwise homogeneous system, if

[16] Portfolio overlap is measured here in terms of asset categories rather than individual securities.

[17] Even in the absence of first movers, redemptions would be higher for illiquid funds because of the feedback loop between price declines and redemptions. The presence of first-movers significantly amplifies the feedback loop between spillover losses and number of redemptions.

Figure 4.3: The left panel plots the proportion of assets held by institutional fund share classes (our proxy for first movers) over time. The right panel plots the spectral radius of the matrix $C = \frac{a^{tot}}{a^m} \cdot MLM^\top A$ over time. The bottom panel plots the size of the U.S. mutual fund industry relative to the whole U.S. financial sector over time. The systemicness matrix is defined as $\frac{a^m}{a^{tot}}CB$. Therefore, differences in the magnitudes of the spectral radius of C and that of the systemicness matrix are due to the relative size of the mutual fund industry $\frac{a^m}{a^{tot}}$ and to the flow-to-performance sensitivity matrix B (set as a multiple of the identity matrix in all examples in the paper).

first movers are concentrated in fewer funds, the system would be more fragile (as discussed in Section 4.3.3).

4.3.3 Nonlinearity of Spillover Losses due to First Movers

We demonstrate how the nonlinearity introduced by first-mover incentives exacerbates the impact of first-mover concentration and initial shocks on spillover losses.

To analyze the impact of the first-mover distribution across funds, we split every fund into two identical funds, each holding half of the assets of the original fund. We compare two system configurations for the distribution of first-mover investors. In the first configuration, we set the

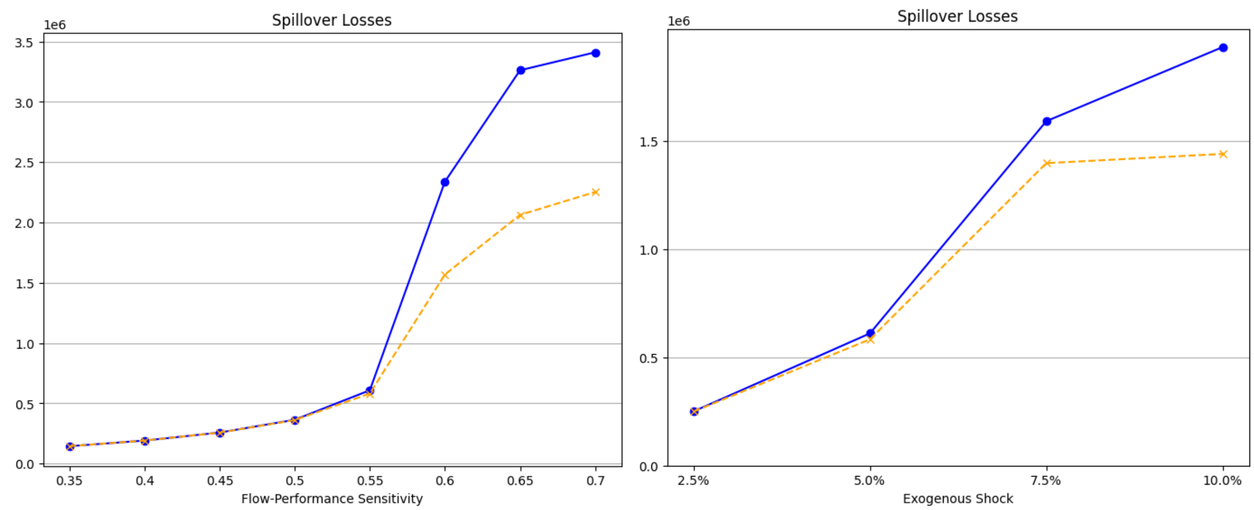

Figure 4.4: Spillover losses when 90% of first movers are concentrated in half of the funds and 10% on the other half (solid line) and when first movers are evenly distributed across funds (dashed line). In the left panel, the exogenous shock applied to each asset equal is equal to a price change of -5% multiplied by its relative volatility specified in Table 4.2. In the right panel, flow-to-performance sensitivity is set to 55% and the exogenous shock to each asset is obtained by multiplying the price change on the x-axis by each asset's relative volatility. We use fund holdings data from Q4 2019.

proportion of first movers in every fund equal to 50%. In the second configuration, for each pair of identical funds, 90% of the first fund's shares are owned by first movers, and 90% of the second fund's shares are owned by second movers. Hence, the total number of first movers is the same across the two configurations, but in the second configuration first movers are more concentrated in half of the funds in the system.

Figure 4.4 illustrates the spillover losses for each of these two configurations using fund holdings data from Q4 2019. The system in which first movers are highly concentrated in fewer funds is more fragile than the system in which first movers are evenly distributed across funds (a formal statement is provided in Proposition 2). The higher fragility is explained by the nonlinearity in spillover losses created by the first-mover advantage: the feedback between fire sales and fund redemptions is stronger in funds with a high proportion of first movers, and the resulting downward pressures imposed on asset prices may also hit funds without first movers. The difference in vulnerability between the system with first mover concentration and the system in which first movers are evenly spread across funds is small in the market scenarios where the flow sensitivity to performance is low. In these market scenarios, the incentive to run is small, and thus fire-sale losses are not impacted much by the distribution of first movers in the system.

We next study the amplification of initial shocks created by redeeming first movers. In the reference model, given the linear assumptions on price impact and flow sensitivity to performance, spillover losses scale linearly with the size of the initial exogenous shock; the SLR increases in proportion to the initial shock. But the reference model fails to capture the incentive to run observed with first movers. Figure 4.5 shows that spillover losses grow faster and nonlinearly in

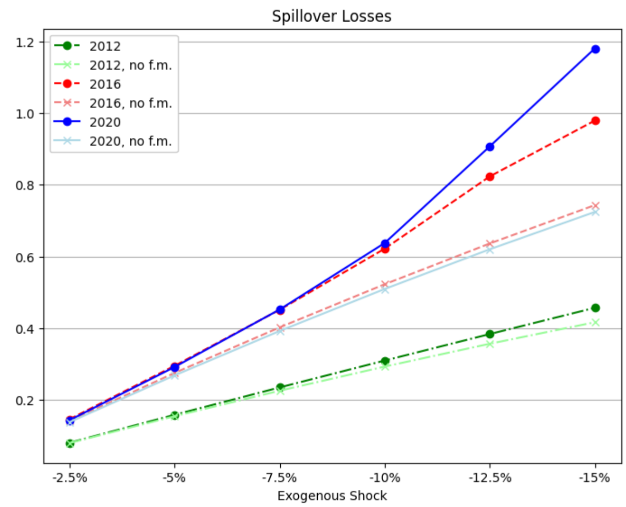

Figure 4.5: Spillover losses (in trillions of dollars) for different initial exogenous shocks with/without first movers. Flow-to-performance sensitivity is set equal to 60%. We use asset holdings data from each year's first quarter.

the size of the initial exogenous market shock once we account for the first-mover advantage.

In Figure 4.5, assets are subject to initial shocks ranging from -2.5% to -15% times their corresponding relative volatilities in Table 4.2. Consider, first, the results using parameters for 2020. As we increase the the exogenous shock from 7.5% to 12.5%, the SLR for the reference model increases linearly, as expected. Over the same range, the SLR accounting for first movers grows far more, with an inflection in the growth rate at a 10% shock.

For the year 2012, we observe little impact of first movers on spillover losses in Figure 4.5. This can be explained by the measures plotted in Figure 4.3, where it can be seen that mutual funds represented a smaller share of the U.S. financial sector, exhibited lesser portfolio overlap in illiquid assets, and had a reduced proportion of first movers. Such a comparison highlights that solely extrapolating from the conditions prevalent in 2012 would overlook the heightened vulnerability of the system in 2016 and 2020. This insight emerges as the key takeaway from Figure 4.5. In a system sufficiently fragile, a critical threshold exists for the initial shock, beyond which a cascade of redemptions is triggered, markedly amplifying spillover losses. As a consequence, spillover losses in ordinary times serve as an inadequate gauge for assessing the aggregate vulnerability of the system or of the potential magnitude of spillover losses in a highly stressed economy. These effects become evident only after accounting for the first-mover incentive.

4.3.4 The Interactions of Portfolio Commonality and First Movers

Price shocks can spread across mutual funds and asset classes through portfolio commonality. A fire sale by one fund drives down the share price of other funds holding the same assets; and a decline in the price of one asset class can force a fund to sell off other assets to fulfill redemption requests, which in turn may depress the prices of these other assets. These contagion effects and sales by first movers can reinforce each other.

To study the joint impact of portfolio commonality and first movers on financial fragility, we consider a benchmark system in which funds of type i[18] are not connected to other funds in the system. In such a system, asset liquidation by funds of type i does not impact others in the system, and vice versa. We then compare the benchmark with the original interconnected system, both with and without first movers.

Figure 4.6 shows that isolating a fund type from the rest of the system can significantly reduce the total spillover losses, either because it shields some large funds from fire-sale externalities, or because it reduces the spread of the shock across asset classes. For Q4 2019, spillover losses due to portfolio commonality are significantly higher in the presence of first movers. It is the first-mover advantage that fuels the spread of shocks through the system via the contagion channel stemming from portfolio commonality. If we consider asset holdings data from the end of Q1 2020, when prices were already severely depressed by the Covid-19 shock, the impact of first movers would be less severe (see the right plot of the figure).

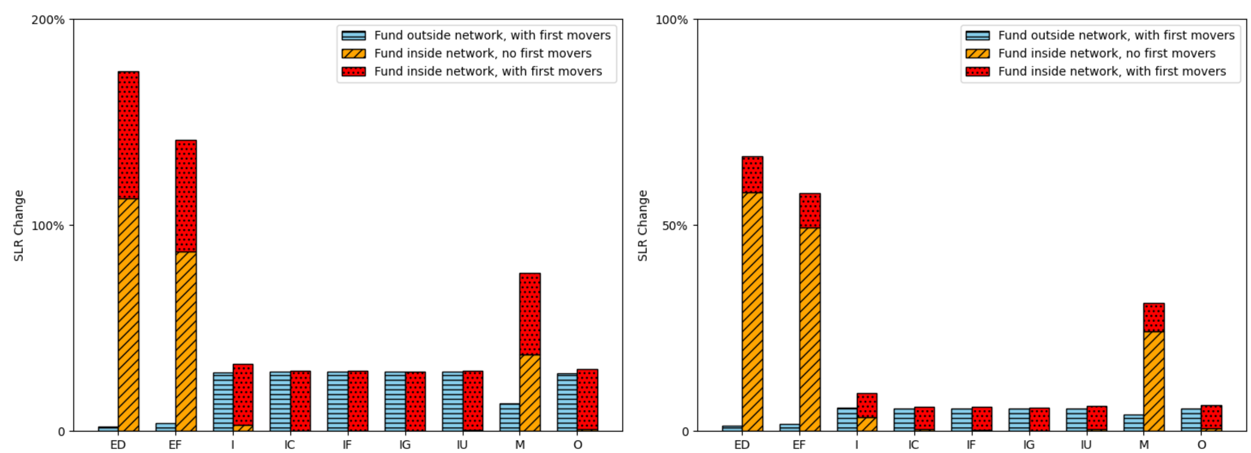

Figure 4.6: Change in spillover losses when each fund type is isolated from other fund types, with and without first movers. For each fund type, the bar with horizontal lines shows the increase in spillover losses due to first movers if funds of this type are isolated from others. The bar with diagonal lines and the dotted one show the increase in spillover losses if these funds are connected to the rest of the system — respectively with and without first movers — relative to the case in which they have no portfolio commonality with other funds and there are no first movers in the system. We set the flow-to-performance sensitivity to 55%. We apply initial shocks to all assets equal to $-5\%$of their respective realized relative volatilities. We consider portfolio holdings in the fourth quarter (left plot) of 2019 and first quarter (right plot) of 2020.

Next, we explore the transmission of shocks across asset classes via the portfolio commonality channel, examining both the aggregate fund level and the specific dynamics within individual fund types.

[18]The fund types are equity domestic (ED), equity foreign (EF), fixed income municipal (IU), fixed income corporate (IC), fixed income government (IG), fixed income foreign (IF), other fixed income (I), mixed fixed income and equity (M), and other (O).

To assess the impact at the aggregate fund level, we consider a scenario in which a few assets are subject to a large initial shock, and we aggregate funds within each of the nine types. Figure 4.7

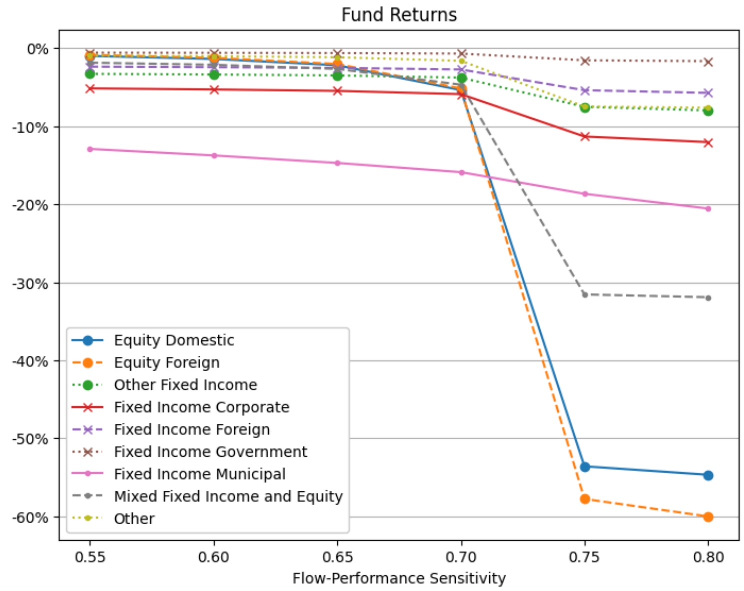

Figure 4.7: Fund returns for different values of fund-to-performance sensitivity for an exogenous shock to convertible, corporate and municipal bonds, equal to a price change of -20% multiplied by the realized volatility of each asset. We use asset holdings data from Q4 2019.

shows that for low levels of flow-to-performance sensitivity, spillover losses are inconsequential and the shock does not spread across the system. In fact, the fund sectors that are most impacted are those holding the assets subject to the initial shock. This is not the case if investors react more strongly to fund performance. Large redemptions at funds that hold both fixed income assets — affected by the exogenous shock — and equity assets may lead to sell-offs in asset classes not hit by the initial shock, and cause widespread spillover losses through the system. As the flow-to-performance sensitivity increases, equity funds become the most vulnerable to spillover losses, even though we applied the initial shock exclusively to fixed income assets. This is because the initial shock spills over to the equity asset class via the portfolio overlap of mixed funds.

To assess the vulnerability of smaller segments within the mutual fund industry, we compute the SLR for various categories without accounting for spillover effects between different sectors. These categories include equity funds (ED, EF), fixed income funds (I, IC, IF), government bond funds (IG), municipal bond funds (IU) and other funds (M, O). Figure 4.8 shows that the SLR is the highest for equity funds, which is attributable to the substantial size of mutual funds that specialize in equity investments.[19] The municipal bond sector exhibits the second highest SLR, despite its considerably smaller scale. This higher SLR is primarily due to the low liquidity characteristic of municipal bonds. In our analysis, the first-mover advantage is relatively minor within the municipal

[19]With even higher values of flow-to-performance sensitivity spillover losses magnify significantly in the equity fund sector.

bond sector. This is likely because institutional investors, which we use as a proxy for first movers, represent a smaller proportion of investors in municipal bonds. The SLR is minimal for government bond funds, reflecting their holdings of highly liquid assets.

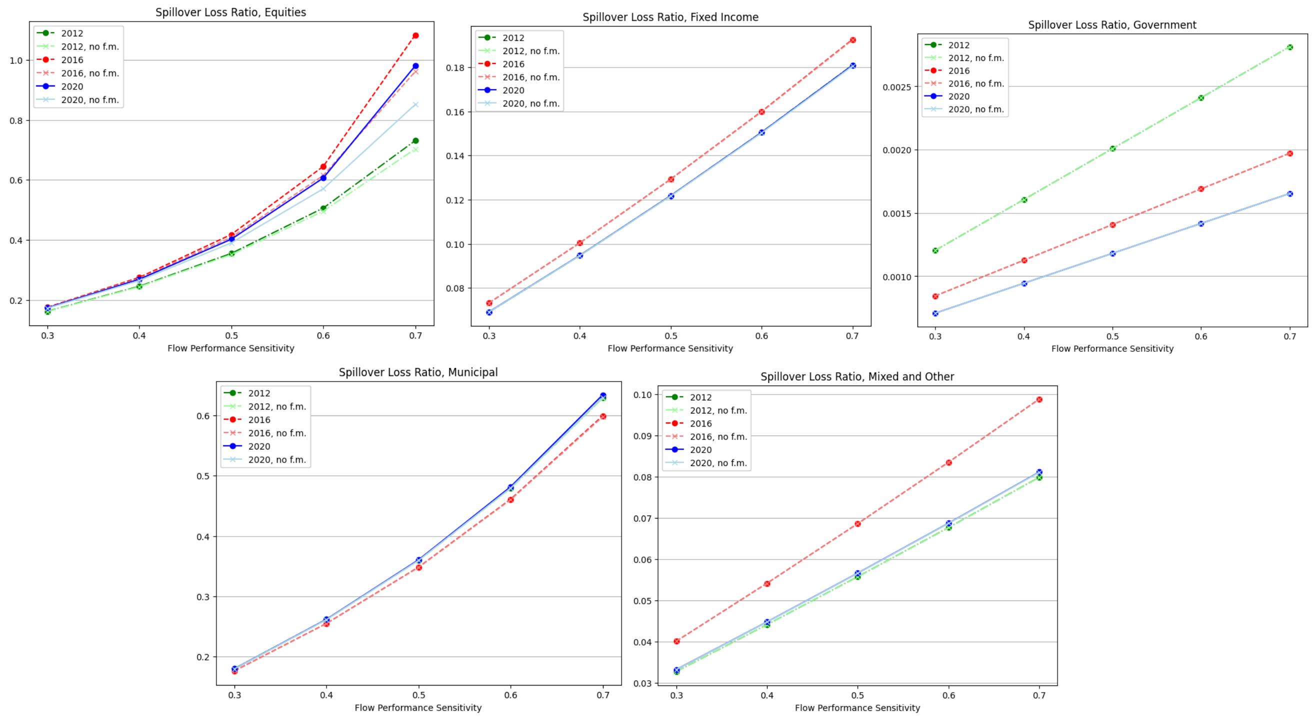

Figure 4.8: Spillover Loss Ratio for different values of flow-to-performance sensitivity and different years with/without first movers. Each panel shows the Spillover Loss Ratio for different subsets of mutual funds: equity funds (ED, EF), fixed income funds (I, IC, IF), government bond funds (IG), municipal bond funds (IU) and other funds (M, O). We apply initial shocks of -5% times the corresponding realized assets' relative volatilities. We use asset holdings data from each year's first quarter.

In Figure 4.9 we compare the weighted average of SLRs for the different segments of the mutual fund industry and the SLR of the mutual fund sector as a whole. Specifically, let $\mathcal{I}$ be the collection of subsets of mutual fund industry detailed in Figure 4.8 and for every subset $I \in \mathcal{I}$ define the weight $w_I := \frac{\sum_{i\in I} a_i \Delta s_i^0}{\sum_{i\in I^{tot}} a_i \Delta s_i^0}$, where $I^{tot} = \bigcup_{I\in\mathcal{I}} I$ denotes the whole system of mutual funds. In the left panel of Figure 4.9 we plot the weighted average $\sum_{I\in\mathcal{I}} w_I \cdot SLR_I$. Equivalently, this is the ratio between total endogenous spillover losses computed separately for each subset of funds and the total exogenous initial losses. The difference between the SLR of the whole system (right panel of Figure 4.9) and the weighted average of SLRs (left panel of Figure 4.9) measures the cross-impact between different mutual fund sectors. Evidently, in 2016, with a high sensitivity of flows to performance, the SLR for the entire system is significantly greater than the weighted average of individual SLRs. Conversely, in 2020, the SLR remains substantial even without considering the effects of cross-sector interactions, which play a lesser role in contributing to overall spillover losses.

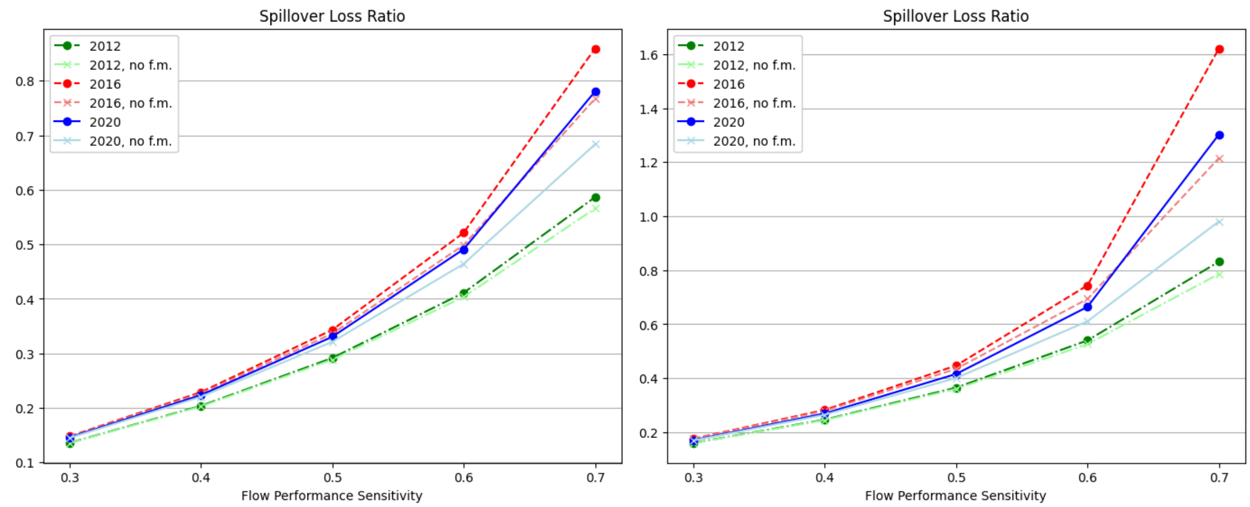

Figure 4.9: Spillover Loss Ratio for different values of flow-to-performance sensitivity and different years with/without first movers. The left panel shows the sum of the SLRs for the subsets of mutual funds detailed in Figure 4.8 weighted by their respective initial exogenous losses. The right panel shows the SLR of the whole system of mutual funds. We apply initial shocks of -5% times the corresponding realized assets' relative volatilities. We use asset holdings data from each year's first quarter.

5 Model Variations

In this section, we consider variations of the baseline framework introduced in Section 3. For each of these variations, we analyze first-mover incentives, aggregate spillover losses, and the overall impact on mutual fund vulnerability.

5.1 Fund Size Reduction

In the baseline specification of the model, the quantity of shares redeemed is proportional to the initial size of the fund. This approach ensures that the volume of redemptions triggered by second movers following a share-price decline Δs is equivalent to that following two smaller, consecutive declines, each of magnitude $\frac{\Delta s}{2}$. Since we interpret the sequence of endogenous share-price changes as a single systemic event, we choose this as the main specification of our model.

Next, we explore a model variant where the number of redemptions by second movers declines as the fund sells assets. Specifically, the quantity of shares from fund i that second movers redeem in the j-th round is represented as $\Delta w_i^{j+1,sm} = -a_i^j(1-\pi_i)b_i\Delta s_i^{j,sm}$, with a_i^j reflecting the size of fund i, after adjusting for the j-th round of asset liquidation. This adaptation introduces a feedback loop that potentially decreases overall redemption volumes and thereby lessens spillover losses: diminished need for asset liquidation to satisfy second movers' redemptions reduces first movers' incentive to redeem, leading to fewer subsequent redemptions by second movers. As the size of the fund size is reduced at each round of second-mover redemptions, the model without first movers is no longer linear in the size of the initial exogenous shock. The plot in Figure 5.1

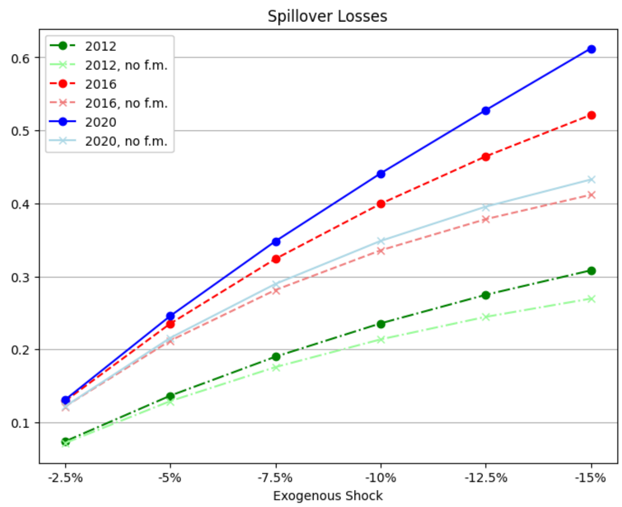

Figure 5.1: Spillover losses (in trillions of dollars) for different initial exogenous shocks with/without first movers using model variation sketched in Section 5.1. Flow-to-performance sensitivity is set to 60%. We use asset holdings data from each year's first quarter.

illustrates the comparative spillover losses with and without first movers. Relative to the baseline model, we observe a notable reduction in spillover losses (compare to Figure 4.5). Furthermore, the graph underscores how the presence of first movers amplifies spillover losses, significantly impacting mutual fund fragility.

5.2 Limiting Redemption Volumes

Funds facing severe redemption surges may halt withdrawals to shield investors from the costs associated with fire sales. Our model can be adapted to evaluate how spillover losses might decrease if redemptions were paused after reaching a predefined limit. In our analysis, depicted in Figure 5.2, we consider scenarios with a cap on the percentage of fund shares that can be redeemed. Even under an ex-ante fragile scenario (using data from Q4 2019 prior to the Covid-19 shock) with a large exogenous shock and a 50% redemption cap, spillover losses do not diminish. Only when a very stringent cap on redemptions is applied do we observe a reduction in spillover losses.

Matta and Perotti (2023) develop a theoretical model to characterize the optimal timing of redemption suspensions, or gates. While our model does not directly incorporate the behavioral response to gates, it is important to highlight that the potential for halting redemptions could lead investors to preemptively initiate withdrawals in response to a financial shock and trigger a market panic event. This phenomenon is documented empirically in Li et al. (2021). Citing similar concerns, the SEC removed gate provisions for money market funds in 2023.

5.3 Heterogeneous Flow-To-Performance Sensitivity

The sensitivity of investor flow to performance varies across different types of funds. Liquid funds often show a convex flow-to-performance relationship, indicating increased investor inflows with

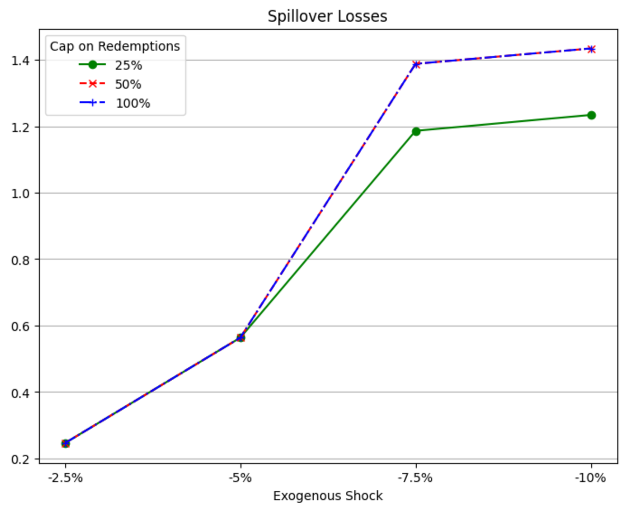

Figure 5.2: Spillover losses (in trillions of dollars) for different caps on the amount of redemptions. Flow-to-performance sensitivity is set equal to 55%. We examine three levels of redemption caps, with the most restrictive cap set at 25%, resulting in the suspension of redemptions once the share of redeemed shares surpasses 25%. Conversely, the least restrictive cap is set at 100%, representing the baseline scenario where redemptions are allowed to continue without interruption. We use asset holdings from Q4 2019.

better performance at an accelerating rate. Conversely, illiquid funds typically exhibit a concave flow-to-performance relationship (as in Chevalier and Ellison (1997), Chen et al. (2010), and Goldstein et al. (2017)). This observed behavior can be explained through the first-mover advantage: as the impact of redemptions on fund performance is more pronounced in illiquid funds, investors in such funds have a stronger incentive to withdraw early which leads to a higher sensitivity to negative performance that to positive performance.

In our framework, the flow-to-performance parameter b quantifies the sensitivity of outflows to negative performance, in the absence of first-mover incentives. The effective outflows implied by our model also depend on the proportion of first movers and the liquidity level of each fund, and are therefore larger in illiquid funds.

We explore two scenarios with varying flow-to-performance sensitivities. In one scenario, we assign specific sensitivities $b_{eq} := 0.419$ for equity, government fixed income, and mixed funds, and $b_{corp} := 0.859$ for other fund types, based on Goldstein et al. (2017)'s estimates for equity and corporate bond funds. Another scenario adjusts each fund's sensitivity based on its portfolio liquidity, setting a linear relationship where a fund entirely invested in stocks has sensitivity b_{eq}, and one fully invested in corporate bonds has b_{corp}.[20] Table 5.1 shows the average flow-to-performance

[20] Let $l^{(i)} := \sum_{k=1}^{K} l_k m_{ik}$ be fund i's illiquidity level, then we set $b_i = b^* + 10^{13} a^* \cdot l^{(i)}$, where a^* and b^* are calibrated such that the flow-to-performance sensitivity of a fund fully invested in stocks is b_{eq} and that of a fund fully invested in corporate bonds is b_{corp}. We obtain $a^* \approx 0.0898$ and $b^* \approx 16.76\%$. We cap the flow-to-performance sensitivity at $b_{max} := b_{corp} - (b_{corp} - b_{eq}) = 129.9\%$. This is because the flow-to-performance sensitivity for municipal bond funds implied by our linear rule would be such that each round's endogenous shock on municipal bond funds is larger than the shock in the previous round. Such a large flow-to-performance sensitivity is unrealistic, because even small negative initial shocks would lead to fund failures.

Fund Type	Average Flow-to-Performance Sensitivity
Domestic Equity	0.407
Foreign Equity	0.405
FI Corporate	0.632
FI Foreign	0.406
FI Government	0.235
FI Muni	1.299
FI Other	0.484
Mixed FI & Equity	0.430
Other	0.286

Table 5.1: Weighted average of flow-to-performance sensitivities by fund type. We use balance sheet data from Q1 2020.

sensitivity of a fund of each type using data from Q1 2020.[21] As it can be seen from Figure 5.3, accounting for a richer dependence between flow-to-performance sensitivity and fund liquidity can amplify the incentive to redeem for first movers, compared to the case where we assign all funds either b_{corp} or b_{eq}.

5.4 Alternative Specification of First Movers

In the main model specification, we identify first movers with institutional investors. This choice stems from our examination of municipal mutual funds detailed in Section 2, as well as insights drawn from the empirical study of money market funds conducted by Schmidt et al. (2016) and the analysis of swing pricing by Jin et al. (2022). Goldstein et al. (2017) show that the first-mover incentive is lower in institutional-oriented funds, which are defined as those with a proportion of institutional share classes of over 80%. This is because large investors tend to internalize the costs generated by their redemptions. On the other hand, institutional investors are more attentive and therefore also respond more strongly to past performance.[22] We consider an alternative specification of first movers to address this consideration.

In Figure 4.5, flow-to-performance is set to 60% and the proportion of first movers is proxied by each fund's fraction of institutional share classes. We alter the specification of the model to be consistent with the findings in Goldstein et al. (2017). For institutional-oriented funds[23] we set the proportion of first movers to 0 and multiply the flow-to-performance sensitivity by a factor of

[21] The average fund of type FI Corporate has flow-to-performance sensitivity significantly lower than b_{corp}. This is because such a fund holds a sizable position in government bonds and therefore its portfolio is more liquid than that of an ideal fund that holds exclusively corporate bonds.

[22] Quoting Goldstein et al. (2017): "Institutional investors react more strongly to past performance because they monitor more, but their reaction to past performance is less affected by the illiquidity of the assets because they are less affected by strategic complementarities."

[23] Defined as those with more than 80% of assets held through institutional share classes, as in Goldstein et al. (2017).

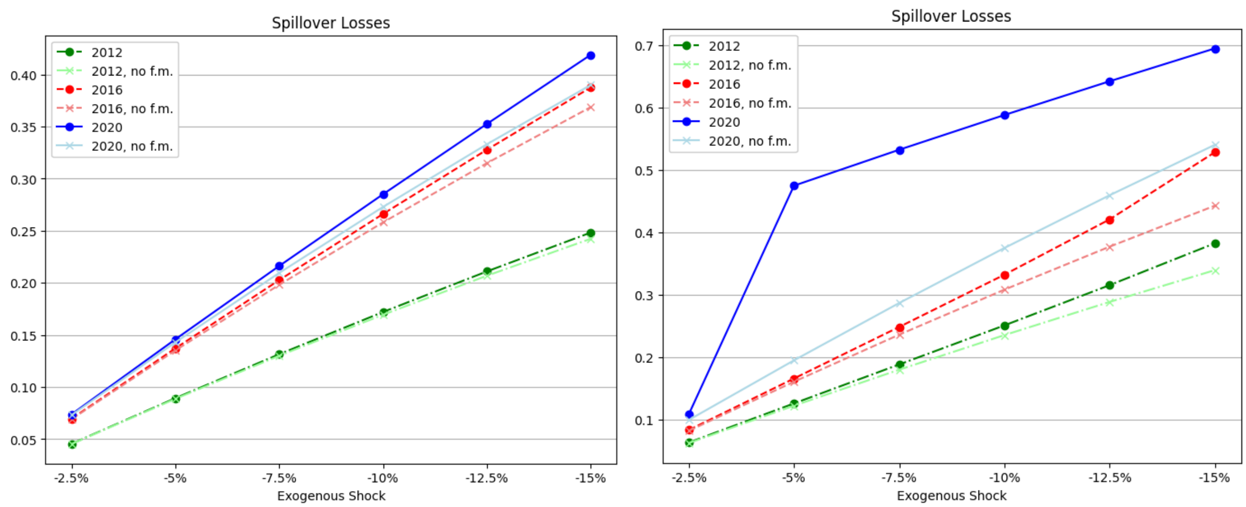

Figure 5.3: Spillover losses (in trillions of dollars) for different initial exogenous shocks with/without first movers with heterogeneous flow-to-performance sensitivities. Left panel: we assign sensitivities $b_{eq} := 0.419$ for equity, government fixed income, and mixed funds, and $b_{corp} := 0.859$ for other fund types, based on Goldstein et al. (2017)'s estimates for equity and corporate bond funds. Right panel: funds' sensitivities are based on a linear interpolation between a fund entirely invested in stocks with sensitivity b_{eq}, and one fully invested in corporate bonds with sensitivity b_{corp}. We use asset holdings data from each year's first quarter.

2 (the coefficient on *Alpha* in Table 9A in Goldstein et al. (2017) is twice as large for institutional-oriented funds as for retail-oriented funds). For all other funds we do not make any adjustment on the proportion of first movers and the flow-to-performance sensitivity. Figure 5.4 shows spillover losses with this model specification. Losses are slightly larger but comparable to those in the main specification shown in Figure 4.5.

6 Policy Assessment

In this section, we employ the stress testing framework outlined in Section 3 to measure the effectiveness and implications of policies aiming for financial stability.

6.1 Swing Pricing

In November 2022, the Securities and Exchange Commission (SEC) proposed a rule mandating that U.S. open-end mutual funds implement *swing pricing*. This rule requires funds to adjust ("swing") their daily NAV to account for the estimated future costs of liquidations stemming from net outflows. The goal of this regulation is to shift the burden of these liquidation costs from the remaining investors to those who are redeeming their shares, thereby mitigating NAV dilution and diminishing the advantage that early redeemers may have. Since 2018, U.S. funds have had the option to apply swing pricing, allowing them to adjust their NAV by up to 2% to cover the anticipated costs related to current redemptions.

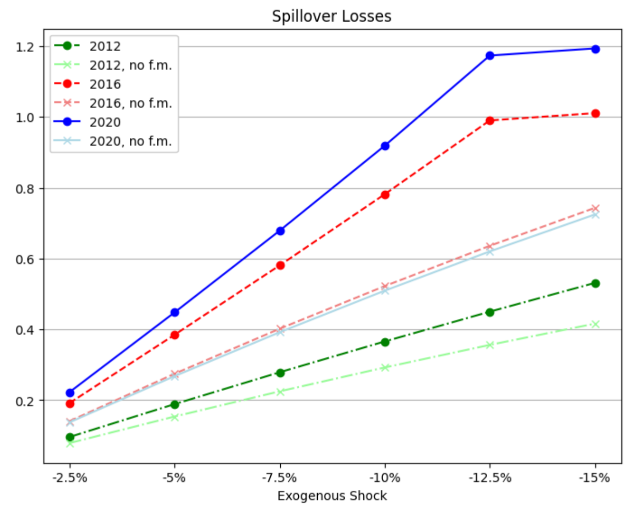

Figure 5.4: Spillover losses (in trillions of dollars) for different initial exogenous shocks with/without first movers. Institutional-oriented funds have no first movers, but larger flow-to-performance sensitivity. We use asset holdings data from each year's first quarter. See Section 5.4 for the details.

The optimal swing pricing strategy ensures that first movers fully internalize the liquidation costs associated with their redemptions, resulting in spillover losses equal to those in a system without first movers (see Capponi et al. (2020)). In the context of our stress testing framework, first movers would receive their payouts at an adjusted NAV $s_i^0 + \Delta s_i^0 + \Delta s_i^{s.p.}$, with the optimal swing pricing adjustment given by $\Delta s_i^{s.p.} := -\sum_{k=1}^{K} m_{ik} l_k \sum_{j=1}^{N} m_{jk} \Delta q_j^{fm}$. This adjustment offsets the first-mover incentive. Under a capped swing pricing policy, the adjustment would instead be equal to $\Delta s_i^{s.p.} := \max\{-\sum_{k=1}^{K} m_{ik} l_k \sum_{j=1}^{N} m_{jk} \Delta q_j^{fm}, -2\%\}$.

In Figure 6.1, we compare spillover losses in the absence of swing pricing, with an optimal swing pricing policy, and with a swing pricing adjustment capped at 2%. As is evident from the figure, implementing a policy with a cap on swing pricing adjustments would have a minimal impact on mitigating overall spillover losses during periods of financial stress, such as those experienced in 2016 and 2020. This observation argues against imposing an ad hoc cap on swing pricing adjustments.

An effective swing pricing rule would fully transfers liquidation costs to redeeming investors. As highlighted by Capponi et al. (2020), in the absence of a liquidity mismatch first movers are disincentivized from redeeming early, aligning spillover losses with those in a system without first movers. Consequently, full adoption of swing pricing across the mutual fund industry would reduce spillover losses, especially during times of reduced liquidity. Instead, partial adoption of swing pricing, with only some funds integrating this policy, may alter the distribution of first movers without enhancing the system's overall resilience to financial shocks. To illustrate this point, we analyze two scenarios: one where no fund employs swing pricing and first movers constitute 50% of investors across all funds, and a hypothetical scenario where swing pricing is adopted by half of the mutual funds while the rest do not, leading first movers to prefer funds without swing pricing

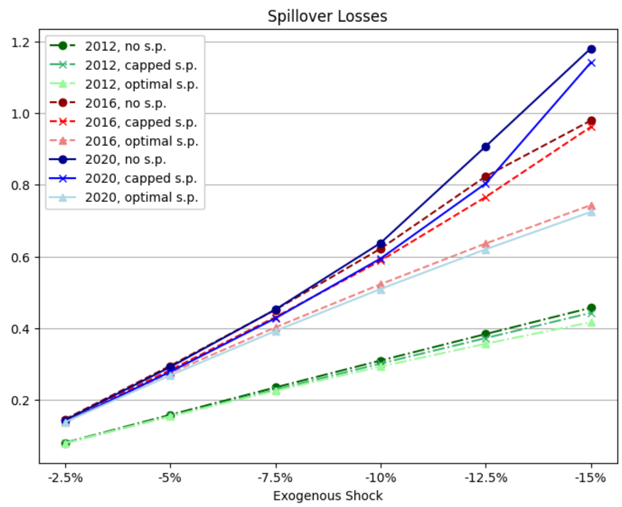

Figure 6.1: Spillover losses (in trillions of dollars) for different initial exogenous shocks with the optimal swing pricing rule, with a swing pricing adjustment capped at 2% and without swing pricing. Flow-to-performance sensitivity is set equal to 60%. We use asset holdings data from each year's first quarter.

to avoid bearing liquidation costs due to their redemptions.[24]

The graph in Figure 6.2 demonstrates that spillover losses are markedly higher in a scenario where first movers concentrate in funds that do not use swing pricing.[25] These findings underscore that while swing pricing can mitigate fire-sale losses at the level of individual funds and, if broadly applied, across the entire system, its effectiveness may be compromised or even prove counterproductive when adopted by only a subset of funds.

Next, we quantify analytically how the distribution of first movers across funds impacts the total change in NAV. In Proposition 2, we show that a higher concentration exacerbates the feedback loop between fund redemptions and asset sales, and imposes a higher downward impact on the NAV. Technical details about the mathematical set-up and the proof of the proposition are relegated to Appendix D.

Proposition 2. *Consider two funds holding identical portfolios, both subject to an initial negative shock* Δs^0. *Let* $\frac{\bar{\pi}}{2} \in (0, \frac{1}{2}]$ *be the proportion of first movers in the system, and let* $\pi \in \left(\frac{\bar{\pi}}{2}, \bar{\pi}\right)$ *be the proportion of first movers in the first fund. The proportion of first movers in the second fund is* $\bar{\pi} - \pi$. *If the price impact is sufficiently small, then for all* π *there exists a fixed point* $\Delta S^*(\pi) = (\Delta s_1^*(\pi), \Delta s_2^*(\pi))^\top$ *of the mapping* $\Delta S^* \to \Delta S^\infty(\Delta S^*)$ *such that* $\Delta s_1^*(\pi) + \Delta s_2^*(\pi)$ *is decreasing in* π. *Since* $\Delta s^0 < 0$, *this implies that the spillover loss ratio is increasing in* π.

[24]In a fund that does not implement swing pricing, first movers' only realized losses after redemption are given by the initial exogenous shock. Instead, in funds that implement swing pricing, their realized losses also include liquidation costs.

[25]This comparison considers the extreme case where all first mover migrate to non-adopting funds, although in reality, some first movers might opt for funds with swing pricing to benefit from reduced fire-sale losses if they do not redeem their shares. The overall impact of uneven swing pricing adoption hinges on the eventual distribution of first movers across funds.

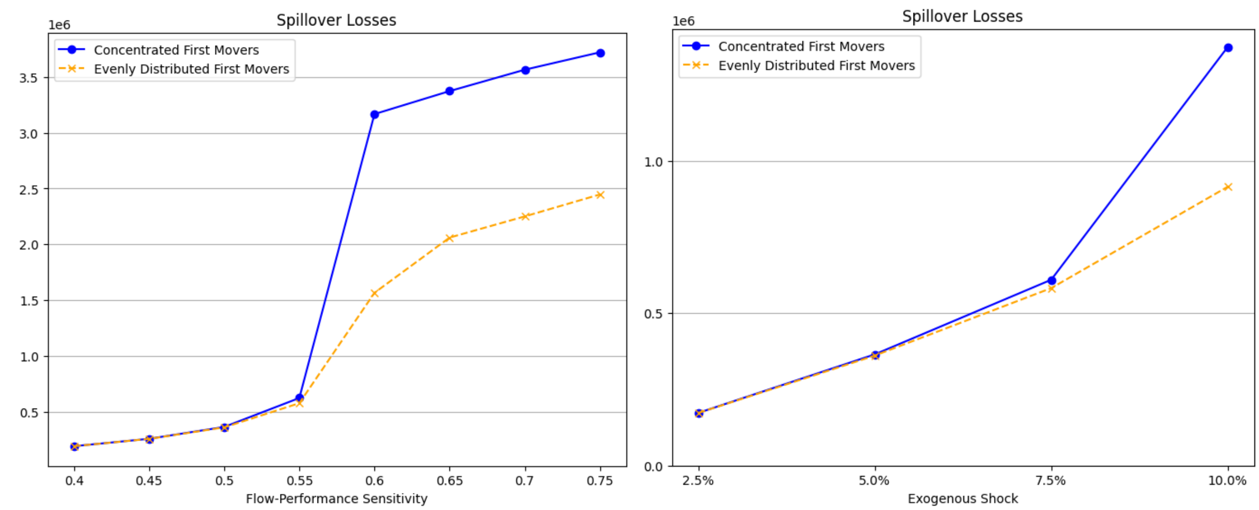

Figure 6.2: Spillover losses when first movers are concentrated in half of the funds (solid line) and when first movers are evenly distributed across funds (dashed line). In the left panel, the exogenous shock applied to each asset is equal to a price change of -5% multiplied by its relative volatility specified in Table 4.2. In the right panel, flow-to-performance sensitivity is set to 50% and the exogenous shock to each asset is obtained by multiplying the price change on the x-axis by each asset's relative volatility. We use asset holdings data from Q4 2019.

As stated in the proposition, the aggregate exposure of the funds to redemption and fire sales is minimized if first movers are evenly distributed between the two funds. This result has implications for policies aimed at mitigating first-mover externalities. It warns that a regulatory intervention that unintentionally alters the distribution of first movers across funds could adversely affect financial stability.

6.2 Preemptive Cash Buffers for Liquidity Management

In the baseline specification of the model, mutual funds are assumed to liquidate assets in proportion to their initial allocations when facing redemptions. However, an alternative strategy involves funds first using their cash reserves to satisfy redemption requests before liquidating less liquid assets. Employing cash buffers as a strategy for liquidity management can effectively reduce the risk of fire sales triggered by outflows and diminish the incentive for investors to redeem shares early.[26]

We introduce a variation to the baseline model to explore the impact of utilizing cash buffers before liquidating other assets. In this adjusted model, funds exhaust their cash reserves prior to selling off assets, which are then liquidated in proportion to their non-cash allocation in the portfolio. This adjustment allows us to assess the extent to which cash buffers can diminish the advantage of redeeming early. Our findings indicate that utilizing cash in this manner lowers spillover losses both in scenarios with and without first movers, as evidenced by comparing the plot

[26] It is worth noting that the empirical evidence regarding the use of cash to meet redemptions is mixed. Some funds are observed to increase their cash holdings in response to outflows, liquidating more assets than necessary to fulfill redemption requests and thereby augmenting their cash positions (e.g., Morris et al. (2017), Shek et al. (2018)).

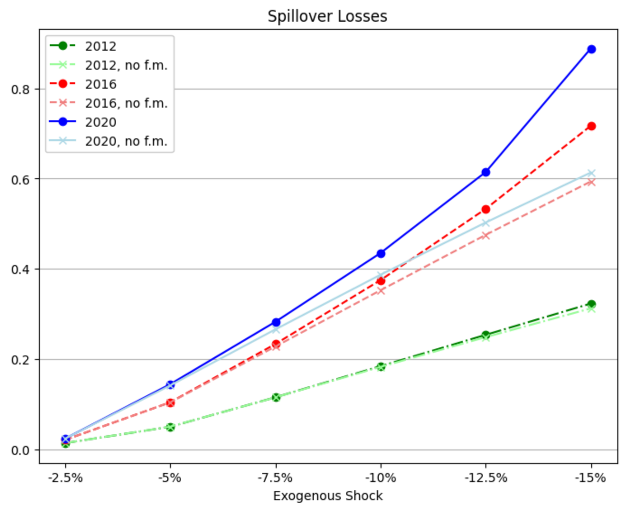

Figure 6.3: Spillover losses (in trillions of dollars) for different initial exogenous shocks with/without first movers when funds deplete their cash buffers first before liquidating other assets. Flow-to-performance sensitivity is set to 60%. We use asset holdings data from each year's first quarter.

in Figure 6.3 with that in Figure 4.5. An alternative liquidation strategy would be for the fund to liquidate more-liquid assets first. We analyze a model variation with a pecking order liquidation strategy in Appendix C.

7 Theoretical Underpinnings and Investor Behavior

The framework of Section 3.2 builds on exogenously specified rules for the behavior of investors and mutual funds. Our stress testing approach therefore does not capture potential equilibrium effects from the strategic responses of market participants. In this section, we discuss the potential for endogenizing the mechanisms in our framework and the potential limitations of our reduced-form approach. We focus on three features of our model: (i) the sensitivity of fund flows to performance; (ii) the behavior of first movers; and (iii) the distribution of first movers across different types of funds.

(i) Flow-performance sensitivity. Our framework assumes that poor returns lead some proportion of investors to exit a fund. The sensitivity of mutual fund flows to fund performance has been studied extensively in the empirical literature; see the survey of Christoffersen et al. (2014) and, in the context of swing pricing, Jin et al. (2022) and Lewrick and Schanz (2023). Berk and Green (2004) develop a model that explains the flow-performance link as the response of rational investors to skilled managers who face decreasing returns to scale in their investment performance. The model of Berk and Green (2004) does not specifically consider the consequences of a market-wide shock, and our model does not differentiate between fund managers; but it is reasonable to assert that the assumption that flows respond to performance is well-established both empirically and theoretically.

(ii) Investor behavior. Building on the assumption that flows respond to performance, our stress test begins with an asset price shock that leads to fund outflows. This shock contrasts with the mechanisms in models of mutual funds that build on the framework of Diamond and Dybvig (1983), particularly Allen and Walther (2021), Lewrick and Schanz (2017), Matta and Perotti (2023), and Ma et al. (2022*a*). The Diamond and Dybvig (1983) approach endogenizes the response of investors, but it requires an exogenous liquidity shock that causes some fraction of investors to withdraw funds early. Our framework could easily be modified to include a liquidity shock in which some fraction of investors exit each fund. For market-wide stress testing, an asset shock proves easier to interpret and calibrate to a realistic size.

Chen et al. (2010) develop a global-games model in which investor flows respond to fund performance and in which the response is stronger for funds holding less-liquid assets. Although this feature is not part of our main framework, we incorporated it in a reduced-form way in Section 5.3 by using different flow sensitivity parameters for different types of funds. Additionally, the model by Chen et al. (2010) predicts that the impact of asset liquidity on flow sensitivity diminishes in funds dominated by large investors, a feature we incorporated in Section 5.4.

Matta and Perotti (2023) utilize a global-games framework to study the optimal timing of gates on redemptions. We incorporated a simplified version of this feature in Section 5.2, where we examined redemption caps.

Our stress testing framework builds on the model in Capponi et al. (2020) and its characterization of first movers. The first movers in Capponi et al. (2020) have rational expectations, in the sense that they respond to anticipated liquidation costs that are subsequently realized. The model does not provide an equilibrium setting to fully endogenize the behavior of first and second movers; such a setting could potentially lead to more refined predictions.

Our first movers "run" on the mutual fund in anticipation of fire-sale losses. In the framework of Diamond and Dybvig (1983), a run corresponds to a bad equilibrium in which investors who are not subject to a liquidity shock nevertheless liquidate their investments early. In the version of the model of Ma et al. (2022*a*) without swing pricing, all investors liquidate early in the run equilibrium. In contrast, our stress testing framework enables the adjustment of the proportion of first movers. This flexibility allows for a nuanced exploration of the run's intensity, offering insights into how varying levels of preemptive liquidation can influence the overall severity of a financial run.

(iii) Distribution of first movers. We have shown in Section 6.1 that the distribution of first movers across funds can have important consequences for spillover losses. However, we have not endogenized the distribution of first movers across funds or the liquidity management choices of different funds. These decisions could have important implications for financial stability, but to the best of our knowledge there are no models that characterize them. Funds can vary their use of cash buffers, asset liquidity, and use of swing pricing. What combinations of these tools should we expect to see in a market equilibrium? How should we expect different types of investors to distribute themselves across funds that use different liquidity management tools? Answers to these

questions (which were also raised in Capponi et al. (2023)) should ideally inform stress testing of the fund industry.

Some interesting empirical evidence relevant to these questions can be found in Jin et al. (2022). They find that funds become less likely to adopt alternative pricing rules (such as swing pricing) as the number of funds adopting them increases: the positive externalities of funds' liquidity management reduces the incentive for other funds to adopt alternative pricing for liquidity management. These findings would be important elements to try to capture in a theoretical framework that explains funds' and investors' preferences for liquidity management tools.

Notably, even within the extensively explored Diamond-Dybvig framework, there appears to be a gap in the literature regarding the endogenization of the distribution of investors with varying liquidity needs across different financial institutions, characterized by their asset liquidity levels. Bridging this gap could significantly enhance our understanding of the dynamics at play in liquidity management practices.

8 Conclusion

We have developed a framework to quantify the vulnerability of the mutual fund sector to fire sales triggered by fund redemptions. The distinguishing feature of our framework is that it accounts for the liquidity mismatch that arises when mutual funds hold illiquid assets but provide same-day liquidity to their investors. We have constructed measures that quantify the mutual fund sector's vulnerability and its sensitivity to key parameters such as the distribution of first movers, shock size, and flow-to-performance sensitivity. We have evaluated these measures using mutual fund holdings data during stressed market conditions. Our framework can serve as a tool to test the impact of policies aimed at reducing spillover losses due to fund runs and common portfolio holdings.

We have shown that the first-mover incentive introduces a nonlinear dependence between spillover losses and the size of initial asset shocks. This nonlinearity can severely exacerbate the aggregate vulnerability of the system for large, yet plausible, sizes of initial shocks if first movers are concentrated in fewer funds or if the investor base of illiquid funds includes a high proportion of first movers.

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A Cross-Sectional Regression of Municipal Bond Fund Flows

To check the robustness of the results in Section 2, we run the following cross-sectional regression using data in the CRSP database for Q1 2020:

$$Flow_i = \alpha + \beta_{inst} InstReturn_i + \beta_{retail} RetailReturn_i + \gamma Controls_i + \varepsilon_i,$$

where $Flow_i$ is the flow for fund i, defined as $\frac{TNA_i^{end} - TNA_i(1+Return_i)}{TNA_i}$, where TNA_i^{end} is the total net asset value of fund share class i at the end of the quarter, TNA_i is the total net asset value at the beginning of the quarter, and $Return_i$ is the fund share class's return. For institutional fund share classes, $InstReturn$ is the fund share class's return, and $RetailReturn$ is set to 0. For retail fund share classes, $RetailReturn$ is the fund share class's return, and we set $InstReturn$ to 0. We control for lagged flow (the flow over the previous quarter), Log(TNA) (the logarithm of total net assets held by the fund at the beginning of the quarter), and Log(age) (the logarithm of the fund's age at the beginning of the quarter, expressed in years). We regress flows against contemporaneous returns (after fees), and not against returns over the previous quarter. Our specification is designed to capture the relation between the Covid-19 market shock and flows within the same quarter. In Table A.2, we report the summary statistics for the fund share classes in our sample. Table A.3 reports the results of the regression. The relation between flows and returns is statistically significant at the 1% level for both institutional and retail fund share classes. Returns are associated with outflows that are larger for institutional fund share classes (0.620) compared to retail fund share classes (0.381), consistent with the view that institutional investors react more strongly to negative returns than retail investors. The difference is on the borderline of the conventional standard for significance: an F-test of the hypothesis that $\beta_{inst} = \beta_{retail}$ has a p-value of 0.060.

Table A.1: Summary statistics for characteristics of fund share classes in the sample for the panel regression in Section 2. We report the mean, median, standard deviation, 5th percentile (P5), 95th percentile (P95) and total number of observations (N).

Institutional Fund Share Classes						
	Mean	Median	Std dev	P5	P95	N
Flow	-0.0097	0.0037	0.4609	-0.6488	0.5352	19,960
Return	-0.0128	0.0100	0.8457	-1.4400	0.5905	19,960
Log(TNA)	4.2125	4.4426	2.5541	-0.8647	7.6385	19,960
Retail Fund Share Classes						
	Mean	Median	Std dev	P5	P95	N
Flow	-0.0106	-0.0009	0.4094	-0.5283	0.4468	37,379
Return	-0.0167	0.0100	0.9152	-1.5400	0.6200	37,379
Log(TNA)	3.8717	4.1005	2.4409	-0.4274	7.4593	37,379

Table A.2: Summary statistics of fund share classes' characteristics in our sample, used for the cross-sectional regression in Appendix A. We report the mean, median, standard deviation, 5th percentile (P5), 95th percentile (P95) and total number of observations (N).

	Mean	Median	Std dev	P5	P95	N
Flow	-0.0089	-0.0144	0.0747	-0.1114	0.1155	1436
InstReturn	-0.0238	-0.0182	0.0213	-0.0711	-0.0001	544
RetailReturn	-0.0238	-0.0191	0.0199	-0.0702	-0.0016	892
Lagged Flow	0.0340	0.0217	0.0858	-0.0810	0.1902	1436
Log(TNA)	4.7140	4.6250	1.7058	2.1604	7.6902	1436
Log(age)	2.5697	2.8396	0.8726	0.8823	3.5440	1436

Table A.3: Relation between flows and returns in municipal bond funds. We source data from the CRSP database for Q1 2020. Flow is the proportional fund share class flow over Q1 2020. InstReturn is the return over Q1 2020 if the fund share class is institutional and 0 otherwise. RetailReturn is the return over Q1 2020 if the fund share class is retail and 0 otherwise. Lagged Flow is the flow over Q4 2019. Log(TNA) is the natural logarithm of total net assets at the beginning of Q1 2020. Log(age) is the natural logarithm of the fund share class age (expressed in years) at the beginning of Q1 2020. We removed index funds, ETFs, ETNs, fund share classes with TNA lower than 5 million dollars, and fund share classes less than one year old. Flows are winsorized at the 1st and 99th percentiles.

	Dependent Variable:
	Flow
Constant	0.041***
	(0.01)
InstReturn	0.620***
	(0.12)
RetailReturn	0.381***
	(0.10)
Lagged Flow	0.270***
	(0.02)
Log(TNA)	-0.001
	(0.00)
Log(age)	-0.016***
	(0.00)
N	1436
Adj. R^2	0.170

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

B Existence and Uniqueness of the Fixed Point

In this section, we show that the procedure described in Section 3.2 has a unique fixed point. Before stating the main result, we state and prove a technical lemma which will be used in the proof of Proposition 1.

Lemma B.1. *Suppose that $f(x, y)$ is continuous in $(x, y) \in X \times Y$, and strictly monotone in x for each y, where $X \subset \mathbb{R}$ and $Y \subset \mathbb{R}^d$ are compact. Then for any sequence $(x_n, y_n) \in X \times Y$ with $\lim_{n\to\infty} y_n = y_0$ and $f(x_n, y_n) = 0$ for all n, there is an $x_0 \in X$ for which*

$$\lim_{n\to\infty} x_n = x_0, \quad f(x_0, y_0) = 0.$$

Proof. Since X is compact, the sequence x_n has at least one limit point, and any limit point must be in X. Let $x_0 \in X$ be a limit point and let x_{n_k} be a subsequence through which $x_{n_k} \to x_0$. Then $(x_{n_k}, y_{n_k}) \to (x_0, y_0)$, and the continuity of f implies that

$$0 = \lim_{k\to\infty} f(x_{n_k}, y_{n_k}) = f(x_0, y_0).$$

Since $f(x, y)$ is strictly monotone in x for each y, x_0 is uniquely determined by y_0. Thus, x_n has just one limit point x_0, and we conclude that $x_n \to x_0$. $\square$

Proof of Proposition 1. Using first the expressions for n_i^{fm} and q_i^{fm} in (8) and then the expression for Δw_i^{fm} in (4), the ratios $U_i^{fm} = q_i^{fm}/n_i^{fm}$ in (9) become

$$U_i^{fm} = \frac{q_i^0 - \Delta q_i^{fm}}{n_i^0 - \Delta w_i^{fm}} = \frac{q_i^0 - \Delta q_i^{fm}}{n_i^0 + a_i \cdot \pi_i \cdot b_i \cdot \Delta s_i^*}. \tag{B.1}$$

The denominator is strictly positive because $n_i^0 = a_i$, $b_i < 1$ by hypothesis, $\pi_i \leq 1$, and $\Delta s_i^* \in [-1, 0]$. Substituting (B.1) into (10) and also substituting the expression for Δp_i^{fm} in (5) into (10), we find that the NAV change of fund i due to the exogenous shock and first movers' redemptions is given by

$$\Delta s_i^{fm} = \frac{n_i^0 - \Delta q_i^{fm}}{n_i^0 + a_i \cdot \pi_i \cdot b_i \cdot \Delta s_i^*} \cdot \left(p_i^0 + \Delta p_i^0 - \sum_{k=1}^{K} \sum_{j=1}^{N} m_{ik} l_k m_{jk} \Delta q_j^{fm} \right) - s_i^0, \tag{B.2}$$

for $i = 1, \ldots, N$. We can similarly use (4) and (5) to write (6) as

$$\begin{aligned} \Delta q_i^{fm} \cdot \left(p_i^0 + \Delta p_i^0 - \sum_{k=1}^{K} \sum_{j=1}^{N} m_{ik} l_k m_{jk} \Delta q_j^{fm} \right) &= \Delta w_i^{fm} \cdot (s_i^0 + \Delta s_i^0) \\ &= -a_i \cdot \pi_i \cdot b_i \cdot \Delta s_i^* \cdot (s_i^0 + \Delta s_i^0). \end{aligned} \tag{B.3}$$

We will use (B.2) and (B.3) to show the existence and uniqueness of a fixed point of the mapping

$\Delta S^* \mapsto \Delta S^\infty(\Delta S^*)$ defined by (11).

<u>Proof of Existence</u>. We will analyze the mapping from $(\Delta s_1^*, \ldots, \Delta s_N^*)$ to $(\Delta q_1^{fm}, \ldots, \Delta q_N^{fm})$ implicitly defined by (B.3), and then utilize it in the mapping from $(\Delta s_1^*, \ldots, \Delta s_N^*)$ to $(\Delta s_1^{fm}, \ldots, \Delta s_N^{fm})$ defined by (B.2).

We will apply Brouwer's fixed point theorem to show the existence of a fixed point of the mapping $\Delta S^* \mapsto \Delta S^\infty(\Delta S^*)$ defined by (11). This boils down to proving the following two statements:

(i) The function in (11) is continuous w.r.t. the input $\Delta S^* \in [-1, 0]^N$.

(ii) For each input $\Delta S^* \in [-1, 0]^N$, the output of the function in (11) is also in $[-1, 0]^N$.

We next state and prove the following lemma:

Lemma B.2. *For sufficiently small $l_1, \ldots, l_K$, there exists a continuous mapping*

$$\begin{aligned} \Phi :\ [-1,0]^N &\ \to\ [0, n_1^0] \times \cdots \times [0, n_N^0] \\ \Phi(\Delta s_1^*, \Delta s_2^*, \ldots, \Delta s_N^*) &= (\Delta q_1^{fm}, \Delta q_2^{fm}, \ldots, \Delta q_N^{fm})^\top, \end{aligned}$$

such that $(\Delta q_1^{fm}, \Delta q_2^{fm}, \ldots, \Delta q_N^{fm})^\top$ solves (B.3).

Proof. The system (B.3) can be regarded as a system of N quadratic equations, which can be solved sequentially.

Fix $\Delta s_1^* \in [-1, 0]$ and $(\Delta q_2, \ldots, \Delta q_N) \in [0, n_2^0] \times \cdots \times [0, n_N^0]$. Then, for $i = 1$, equation (B.3) is a quadratic equation in the variable Δq_1^{fm}. One solution of this equation is given by

$$\Delta q_1^{fm} = \Delta q_1^{fm}(\Delta s_1^*, \Delta q_2, \ldots, \Delta q_N) = \frac{-\beta_1 + \sqrt{\beta_1^2 - 4\alpha_1\gamma_1}}{2\alpha_1}, \tag{B.4}$$

where

$$\alpha_1 = -\sum_{k=1}^{K} m_{1k} l_k m_{1k}, \quad \beta_1 = \Delta p_1^0 + p_1^0 - \sum_{k=1}^{K}\sum_{j=2}^{N} m_{1k} l_k m_{jk} \Delta q_j,$$

and

$$\gamma_1 = a_1 \cdot \pi_1 \cdot b_1 \cdot \Delta s_1^* \cdot (s_1^0 + \Delta s_1^0).$$

Notice that, for sufficiently small $(l_k)_{k=1}^K$, the quantity $\beta_1^2 - 4\alpha_1\gamma_1$ is strictly positive, and hence the function $\Delta q_1^{fm}(\cdot)$ as defined in (B.4) takes only real values. Moreover, it is positive because both the numerator and denominator of $\Delta q_1^{fm}(\cdot)$ are negative quantities. We claim that $\Delta q_1^{fm} \in [0, n_1^0]$. To see why, evaluate (B.3) at the endpoints of this interval. At $\Delta q_1^{fm} = 0$, the left side of (B.3) is zero and thus less than or equal to the right side of (B.3), which is nonnegative because $\Delta s_i^* \leq 0$

and $\Delta s_i^0 > -s_i^0$. At $\Delta q_1^{fm} = n_1^0$, the left side of (B.3) satisfies

$$\begin{aligned}
& n_1^0 \cdot \left(\Delta p_1^0 + p_1^0 - \sum_{k=1}^{K} m_{1k} l_k m_{1k} n_1^0 - \sum_{k=1}^{K} \sum_{j=2}^{N} m_{1k} l_k m_{jk} \Delta q_j \right) \\
& \geq n_1^0 \cdot \left(\Delta p_1^0 + p_1^0 - \sum_{k=1}^{K} \sum_{j=1}^{N} m_{1k} l_k m_{jk} n_j^0 \right) \\
& \geq a_1 \cdot \pi_1 \cdot b_1 \cdot (s_1^0 + \Delta s_1^0) \\
& \geq -a_1 \cdot \pi_1 \cdot b_1 \cdot \Delta s_1^* \cdot (s_1^0 + \Delta s_1^0),
\end{aligned}$$

where the first inequality holds because $\Delta q_j \leq n_j^0$. The second inequality holds for sufficiently small $(l_k)_k$'s because $n_1^0 = a_1$, $\pi_1 \leq 1, b_1 < 1$. The last inequality follows from $\Delta s_1^* \in [-1, 0]$. Hence, by the intermediate value theorem, one of the two roots of (B.3) belongs to the interval $[0, n_1^0]$. Because the root $\frac{-\beta_1 - \sqrt{\beta_1^2 - 4\alpha_1\gamma_1}}{2\alpha_1}$ can be arbitrarily large for sufficiently small $(l_k)_{k=1}^K$, we conclude that $\Delta q_1^{fm}(\cdot)$ as defined in (B.4) takes values in $[0, n_1^0]$.

Next, we show that there exists a constant P_1, independent of $l_1, \ldots, l_K$, such that the following uniform bound holds:

$$P_1 \geq \sup_{\Delta s_1^* \in [-1,0], \Delta q_2 \in [0, n_2^0], \ldots, \Delta q_N \in [0, n_N^0]} \max \left\{ \left| \frac{\partial \Delta q_1^{fm}}{\partial \Delta s_1^*} \right|, \left| \frac{\partial \Delta q_1^{fm}}{\partial \Delta q_2} \right|, \ldots, \left| \frac{\partial \Delta q_1^{fm}}{\partial \Delta q_N} \right| \right\}. \tag{B.5}$$

To see this, set $i = 1$ in (B.3), and rewrite the corresponding equation by treating Δq_1^{fm} as a function of $(\Delta s_1^*, \Delta q_2, \ldots, \Delta q_N)$. This yields

$$\begin{aligned}
& \Delta q_1^{fm}(\Delta s_1^*, \Delta q_2, \ldots, \Delta q_N) \cdot \left(\Delta p_1^0 + p_1^0 - \sum_{k=1}^{K} m_{1k} l_k m_{1k} \Delta q_1^{fm}(\Delta s_1^*, \Delta q_2, \ldots, \Delta q_N) \right. \\
& \left. - \sum_{k=1}^{K} \sum_{j=2}^{N} m_{1k} l_k m_{jk} \Delta q_j \right) = -a_1 \cdot \pi_1 \cdot b_1 \cdot \Delta s_1^* \cdot (s_1^0 + \Delta s_1^0).
\end{aligned}$$

Differentiating the expression above with respect to Δq_2 on both sides leads to

$$\frac{\partial \Delta q_1^{fm}}{\partial \Delta q_2} \cdot \left(\Delta p_1^0 + p_1^0 - 2 \sum_{k=1}^{K} m_{1k} l_k m_{1k} \Delta q_1^{fm} - \sum_{k=1}^{K} \sum_{j=2}^{N} m_{1k} l_k m_{jk} \Delta q_j \right) - \Delta q_1^{fm} \sum_{k=1}^{K} m_{1k} l_k m_{2k} = 0.$$

Because we have previously shown that Δq_1^{fm} takes values in $[0, n_1^0]$, the equality above implies that $\frac{\partial \Delta q_1^{fm}}{\partial \Delta q_2}$ is uniformly bounded for sufficiently small $l_1, \ldots, l_K$ as assumed in this proposition. The other derivatives appearing on the right-hand side of (B.5) can be estimated similarly, and we can thus conclude the existence of a uniform bound P_1 in (B.5).

Next, set $i = 2$ in (B.3). We want to show that, for any $(\Delta s_1^*, \Delta s_2^*, \Delta q_3, \ldots, \Delta q_N) \in [-1, 0]^2 \times [0, n_3^0] \times \cdots \times [0, n_N^0]$, there exists a function $\Delta q_2^{fm}(\Delta s_1^*, \Delta s_2^*, \Delta q_3, \ldots, \Delta q_N)$ in the interval $[0, n_2^0]$.

Rewriting equation (B.3) for $i = 2$, we get

$$\Delta q_2^{fm} \cdot \Bigg(\Delta p_2^0 + p_2^0 - \sum_{k=1}^{K} m_{2k} l_k m_{1k} \Delta q_1^{fm}(\Delta s_1^*, \Delta q_2^{fm}, \Delta q_3, \ldots, \Delta q_N) - \sum_{k=1}^{K} m_{2k} l_k m_{2k} \Delta q_2^{fm} - \sum_{k=1}^{K} \sum_{j=3}^{N} m_{2k} l_k m_{jk} \Delta q_j \Bigg) = -a_2 \cdot \pi_2 \cdot b_2 \cdot \Delta s_2^* \cdot (s_2^0 + \Delta s_2^0). \tag{B.6}$$

We will show that $\Delta q_2^{fm} \in [0, n_2^0]$ by considering the values at the endpoints of this interval. If we set $\Delta q_2^{fm} = 0$ on the left side of (B.6), then the left side evaluates to zero, and it is less than or equal to the right side which is nonnegative. If we set $\Delta q_2^{fm} = n_2^0$ on the left side, then using similar arguments as for the case $\Delta q_1^{fm} = n_1^0$, we obtain

$$\begin{aligned}
& n_2^0 \cdot \Bigg(\Delta p_2^0 + p_2^0 - \sum_{k=1}^{K} m_{2k} l_k m_{1k} \Delta q_1^{fm}(\Delta s_1^*, n_2^0, \Delta q_3, \ldots, \Delta q_N) - \sum_{k=1}^{K} m_{2k} l_k m_{2k} n_2^0 \\
& \qquad - \sum_{k=1}^{K} \sum_{j=3}^{N} m_{2k} l_k m_{jk} \Delta q_j \Bigg) \\
& \geq n_2^0 \cdot \Bigg(\Delta p_2^0 + p_2^0 - \sum_{k=1}^{K} \sum_{j=1}^{N} m_{2k} l_k m_{jk} n_j^0 \Bigg) \\
& \geq a_2 \cdot \pi_2 \cdot b_2 \cdot (s_2^0 + \Delta s_2^0) \\
& \geq -a_2 \cdot \pi_2 \cdot b_2 \cdot \Delta s_2^* \cdot (s_2^0 + \Delta s_2^0),
\end{aligned}$$

where in the first inequality we have used the previously established fact that $\Delta q_1^{fm} \in [0, n_1^0]$, and for the second inequality we have used that $a_2 = n_2^0$, $\pi_2 \leq 1$, and $b_2 < 1$.

Next, we show that the left side of equation (B.3) is an increasing function of Δq_2^{fm} by showing that its derivative with respect to Δq_2^{fm} is positive. Using the chain rule of differentiation, we find that the derivative of the left side of (B.3) with respect to Δq_2^{fm} is given by

$$\begin{aligned}
& \Delta p_2^0 + p_2^0 - \sum_{k=1}^{K} \sum_{j=1}^{2} m_{2k} l_k m_{jk} \Delta q_j^{fm} - \sum_{k=1}^{K} \sum_{j=3}^{N} m_{2k} l_k m_{jk} \Delta q_j \\
& \qquad - \Delta q_2^{fm} \cdot \Bigg(\sum_{k=1}^{K} m_{2k} l_k m_{1k} \frac{\partial \Delta q_1^{fm}}{\partial \Delta q_2^{fm}} + \sum_{k=1}^{K} m_{2k} l_k m_{2k} \Bigg) \\
\geq \quad & \Bigg(\Delta p_2^0 + p_2^0 - \sum_{k=1}^{K} \sum_{j=1}^{N} m_{2k} l_k m_{jk} n_j^0 \Bigg) - n_2^0 \cdot \Bigg(\sum_{k=1}^{K} m_{2k} l_k m_{1k} P_1 + \sum_{k=1}^{K} m_{2k} l_k m_{2k} \Bigg) > 0,
\end{aligned}$$

where the first inequality holds because we have shown that the functions Δq_1^{fm} and Δq_2^{fm} satisfy $\Delta q_1^{fm} \in [0, n_1^0]$ and $\Delta q_2^{fm} \in [0, n_2^0]$, and because each input variable Δq_j is in the interval $[0, n_j^0]$, for $j \geq 3$. The last inequality holds for sufficiently small $(l_k)_k$. Since we have shown that the left side of (B.3) is strictly increasing in Δq_2^{fm} (with Δs_1^* fixed), it follows that (B.3) defines a unique implicit function $\Delta q_2^{fm} = \Delta q_2^{fm}(\Delta s_1^*, \Delta s_2^*, \Delta q_3, \ldots, \Delta q_N) \in [0, n_2^0]$.

The continuity of the function $\Delta q_2^{fm}(\Delta s_1^*, \Delta s_2^*, \Delta q_3, \ldots, \Delta q_N)$ follows by applying Lemma B.1 with $x = \Delta q_2^{fm}(\Delta s_1^*, \Delta s_2^*, \Delta q_3, \ldots, \Delta q_N)$, $y = (\Delta s_1^*, \Delta s_2^*, \Delta q_3, \ldots, \Delta q_N)$, and $f(x, y)$ as the difference between the left and right sides of (B.6). By replacing the input Δq_2 of Δq_1^{fm} with the function Δq_2^{fm}, we can write

$$\begin{aligned}
\Delta q_1 &= \Delta q_1^{fm}(\Delta s_1^*, \Delta q_2^{fm}(\Delta s_1^*, \Delta s_2^*, \Delta q_3, \ldots, \Delta q_N), \Delta q_3, \ldots, \Delta q_N), \\
\Delta q_2 &= \Delta q_2^{fm}(\Delta s_1^*, \Delta s_2^*, \Delta q_3, \ldots, \Delta q_N).
\end{aligned}$$

Then equation (B.3) holds for $i = 1, 2$ simultaneously. Using the boundedness of Δq_1^{fm} and Δq_2^{fm} together with the bound (B.5) (which is needed to bound the derivatives appearing in the chain rule of differentiation), we can show the existence of a uniform bound P_2 such that

$$P_2 \geq \sup_{\Delta s_1^*, \Delta s_2^* \in [-1,0], \Delta q_3 \in [0, n_2^0], \ldots, \Delta q_N \in [0, n_N^0]} \max \left\{ \left| \frac{\partial \Delta q_2^{fm}}{\partial \Delta s_1^*} \right|, \left| \frac{\partial \Delta q_2^{fm}}{\partial \Delta s_2^*} \right|, \left| \frac{\partial \Delta q_2^{fm}}{\partial \Delta q_3} \right|, \ldots, \left| \frac{\partial \Delta q_2^{fm}}{\partial \Delta q_N} \right| \right\}, \tag{B.7}$$

using a similar method to that used to show the existence of P_1 in (B.5), and under the assumption that $l_1, \ldots, l_K$ are sufficiently small.

Repeating the steps above for $i = 3, \ldots, N$, and again assuming $l_1, \ldots, l_K$ are sufficiently small, we obtain N continuous functions

$$\begin{aligned}
&\Delta q_1^{fm}(\Delta s_1^*, \Delta q_2, \ldots, \Delta q_N): \ [-1, 0] \times [0, n_2^0] \times \cdots \times [0, n_N^0] \to [0, n_1^0]; \\
&\Delta q_2^{fm}(\Delta s_1^*, \Delta s_2^*, \Delta q_3, \ldots, \Delta q_N): \ [-1, 0]^2 \times [0, n_3^0] \times \cdots \times [0, n_N^0] \to [0, n_2^0]; \\
&\vdots \\
&\Delta q_{N-1}^{fm}(\Delta s_1^*, \Delta s_2^*, \ldots, \Delta s_{N-1}^*, \Delta q_N): \ [-1, 0]^{N-1} \times [0, n_N^0] \to [0, n_{N-1}^0]; \\
&\Delta q_N^{fm}(\Delta s_1^*, \Delta s_2^*, , \ldots, \Delta s_N^*): \ [-1, 0]^N \to [0, n_N^0].
\end{aligned}$$

If we replace each input variable Δq_j with the function Δq_j^{fm}, all of these functions become functions of $\Delta s_1^*, \ldots, \Delta s_N^*$. We have thus constructed a function $\Phi: \ [-1, 0]^N \ \to \ [0, n_1^0] \times \cdots \times [0, n_N^0]$ for which

$$\Phi(\Delta s_1^*, \Delta s_2^*, \ldots, \Delta s_N^*) = (\Delta q_1^{fm}, \Delta q_2^{fm}, \ldots, \Delta q_N^{fm})^\top$$

solves (B.3). This ends the proof of the claim. □

Next, we show that for each input $\Delta S^* \in [-1, 0]^N$, the corresponding output given by (11) is still in $[-1, 0]^N$. Towards this goal, we will bound U^{fm}, and we begin by showing that

$$-a_i \cdot \pi_i \cdot b_i \cdot \Delta s_i^* \leq \Delta q_i^{fm}. \tag{B.8}$$

Since $\Delta q_j^{fm} \le n_j^0$, we have

$$p_i^0 + \Delta p_i^0 - \sum_{k=1}^{K}\sum_{j=1}^{N} m_{ik} l_k m_{jk} \Delta q_j^{fm} \ge p_i^0 + \Delta p_i^0 - \sum_{k=1}^{K}\sum_{j=1}^{N} m_{ik} l_k m_{jk} \Delta n_j^0 > 0,$$

where the last inequality holds for sufficiently small $(l_k)_k$. Hence, if $\Delta s_i^* = 0$, then $\Delta q_i^{fm} = 0$ and (B.8) is satisfied. Assume now that $\Delta s_i^* < 0$. Then (B.3) yields

$$\begin{aligned} -a_i \cdot \pi_i \cdot b_i \cdot \Delta s_i^* \cdot (s_i^0 + \Delta s_i^0) &= \Delta q_i^{fm} \cdot \left(p_i^0 + \Delta p_i^0 - \sum_{k=1}^{K}\sum_{j=1}^{N} m_{ik} l_k m_{jk} \Delta q_j^{fm} \right) \\ &\le \Delta q_i^{fm} \cdot \left(p_i^0 + \Delta p_i^0 \right). \end{aligned}$$

Hence, using $s_i^0 + \Delta s_i^0 = p_i^0 + \Delta p_i^0 > 0$ we find that (B.8) is again satisfied.

Applying (B.8) in (B.1) shows that $U_i^{fm} \le 1$. In view of the definition in (9), we have that $U_i^{fm} \ge 0$. If $U_i^{fm} = 0$, it follows from (B.2) that $\Delta s_i^{fm} = -s_i^0 < 0$. If $0 < U_i^{fm} \le 1$, then combining (B.1) and (B.2) yields

$$\begin{aligned} \Delta s_i^{fm} &= \frac{n_i^0 - \Delta q_i^{fm}}{n_i^0 + a_i \cdot \pi_i \cdot b_i \cdot \Delta s_i^*} \cdot \left(p_i^0 + \Delta p_i^0 - \sum_{k=1}^{K}\sum_{j=1}^{N} m_{ik} l_k m_{jk} \Delta q_j^{fm} \right) - s_i^0 \qquad \text{(B.9)} \\ &\le p_i^0 + \Delta p_i^0 - \sum_{k=1}^{K}\sum_{j=1}^{N} m_{ik} l_k m_{jk} \Delta q_j^{fm} - s_i^0 \\ &= \Delta p_i^0 - \sum_{k=1}^{K}\sum_{j=1}^{N} m_{ik} l_k m_{jk} \Delta q_j^{fm} \le 0, \end{aligned}$$

where the last inequality follows from the fact that $\Delta p_i^0 < 0$ and $\Delta q_j^{fm} \in [0, n_j^0]$. Hence,

$$\|\Delta S^{fm}\|_\infty \le \max_{i=1,\ldots,K} \left\{ -\Delta p_i^0 + \sum_{k=1}^{K}\sum_{j=1}^{N} m_{ik} l_k m_{jk} n_j^0 \right\},$$

where for a matrix $D = (d_{ij})_{m \times n} \in \mathbb{R}^{m \times n}$ we have defined

$$\|D\|_\infty := \max_{i=1,\ldots,m} \sum_{j=1}^{n} |d_{ij}|.$$

Since $\|U^{fm}\|_\infty \le 1$ as shown above, and because the matrix of portfolio weights $M = [m_{ik}]$ has norm $\|M\|_\infty \leq 1$, then

$$\begin{aligned}
&\|U^{fm} M L M^\top U^{fm} A(I-\Pi)B\|_\infty \\
&\leq \|U^{fm}\|_\infty \|M\|_\infty \|L M^\top\|_\infty \|U^{fm}\|_\infty \|A(I-\Pi)B\|_\infty \\
&\leq \max_{k=1,\dots,K} \left\{ \sum_{j=1}^N l_k m_{jk} \right\} \cdot \max_{i=1,\dots,K} a_i b_i
\end{aligned}$$

Then (11) yields

$$\begin{aligned}
&\left\| \sum_{n=0}^\infty \Delta S^{n,sm} \right\|_\infty \leq \frac{\|\Delta S^{fm}\|_\infty}{1 - \|U^{fm} M L M^\top U^{fm} A(I-\Pi)B\|_\infty} \qquad \text{(B.10)} \\
&\leq \max_{i=1,\dots,K} \left\{ \sum_{k=1}^K \sum_{j=1}^N m_{ik} l_k m_{jk} n_j^0 - \Delta p_i^0 \right\} \Bigg/ \left(1 - \max_{k=1,\dots,K} \left\{ \sum_{j=1}^N l_k m_{jk} \right\} \cdot \max_{i=1,\dots,K} a_i b_i \right) \\
&\leq 1,
\end{aligned}$$

where the last inequality holds by the assumption that $\max_k l_k$ is sufficiently small.

We know from (B.9) that $\Delta s_i^{fm} \leq 0$. Moreover, for each $n \geq 1$,

$$\left(U^{fm} M L M^\top U^{fm} A(I-\Pi)B \right)^n \qquad \text{(B.11)}$$

is the product of matrices with nonnegative entries and therefore has nonnegative entries. Hence,

$$\Delta S^{n,sm} = \left(U^{fm} M L M^\top U^{fm} A(I-\Pi)B \right)^n \Delta S^{fm} \in (-\infty, 0]^N,$$

and thus $\sum_{n=0}^\infty \Delta S^{n,sm} \in [-\infty, 0]^N$. Together with (B.10), we obtain that $\sum_{n=0}^\infty \Delta S^{n,sm} \in [-1, 0]^N$.

To establish the existence of a fixed point of (11) using Brouwer's fixed point theorem, it remains to show that the output in (11) is continuous in ΔS^*. In the Claim above, we established $(\Delta q_1^{fm}, \Delta q_2^{fm}, \dots, \Delta q_N^{fm})$ is a continuous function of ΔS^*. Let $\Delta S^*_{(k)} \in [-1, 0]^N$ be such that

$$\lim_{k \to +\infty} \Delta S^*_{(k)} = \Delta S^*.$$

For $\Delta S^*_{(k)}$, $k = 1, 2, \dots$, denote by $U^{fm}_{(k)}$, $\Delta S^{fm}_{(k)}$ the corresponding terms in (11). Because of the continuous dependence of $(\Delta q_1^{fm}, \Delta q_2^{fm}, \dots, \Delta q_N^{fm})$ on ΔS^*, it holds that

$$\lim_{k \to +\infty} U^{fm}_{(k)} = U^{fm}, \quad \lim_{k \to +\infty} \Delta S^{fm}_{(k)} = \Delta S^{fm}.$$

To see why $\lim_{k\to+\infty} U^{fm}_{(k)} = U^{fm}$, recall that $U^{fm}_{(k),i} = \frac{q^{fm}_{(k),i}}{n^{fm}_{(k),i}}$. Moreover, observe that

$$\lim_{k\to+\infty} n^{fm}_{(k),i} = n^0_i + a_i \cdot \pi_i \cdot b_i \cdot \lim_{k\to+\infty} \Delta s^*_{(k),i} = n^0_i + a_i \cdot \pi_i \cdot b_i \cdot \Delta s^*_i > 0.$$

Because the output $(\Delta q^{fm}_1, \Delta q^{fm}_2, \ldots, \Delta q^{fm}_N)$ depends continuously on the input ΔS^*, we have

$$\lim_{k\to+\infty} q^{fm}_{(k),i} = q^0_i - \lim_{k\to+\infty} \Delta q^{fm}_{(k),i} = q^0_i - \Delta q^{fm}_i = q^{fm}_i.$$

By combining the two limits above, we confirm that $\lim_{k\to+\infty} U^{fm}_{(k)} = U^{fm}$.

Notice also that

$$\left\| \left(U^{fm}_{(k)} MLM^\top U^{fm}_{(k)} A(I-\Pi)B \right)^n \right\|_\infty \leq \left(\max_{k=1,\ldots,K} \left\{ \sum_{j=1}^N l_k m_{jk} \right\} \cdot \max_{i=1,\ldots,K} a_i b_i \right)^n,$$

$$\|\Delta S^{fm}_{(k)}\|_\infty \leq \max_{i=1,\ldots,K} \left\{ \sum_{k=1}^K \sum_{j=1}^N m_{ik} l_k m_{jk} n^0_j - \Delta p^0_i \right\}.$$

Therefore the dominated convergence theorem implies that

$$\lim_{k\to+\infty} \sum_{n=0}^{+\infty} \left(U^{fm}_{(k)} MLM^\top U^{fm}_{(k)} A(I-\Pi)B \right)^n \Delta S^{fm}_{(k)} = \sum_{n=0}^{+\infty} \left(U^{fm} MLM^\top U^{fm} A(I-\Pi)B \right)^n \Delta S^{fm},$$

which shows the continuity of the output of (11) with respect to the input ΔS^*.

In sum, the mapping (11) is continuous, and it maps $[-1,0]^N$ into itself. According to Brouwer's theorem, it follows that the mapping has a fixed point.

Proof of Uniqueness. Define the smooth mapping

$$\begin{aligned} \Psi:\ & [0, n^0_1] \times \cdots \times [0, n^0_N] \ \to\ \mathbb{R}^N \\ & \Psi(\Delta q^{fm}_1, \Delta q^{fm}_2, \ldots, \Delta q^{fm}_N) = (-\Delta s^*_1, -\Delta s^*_2, \ldots, -\Delta s^*_N)^\top, \end{aligned} \tag{B.12}$$

so that

$$-\Delta s^*_i = \frac{1}{(s^0_i + \Delta s^0_i) a_i \pi_i b_i} \cdot \Delta q^{fm}_i \cdot \left(\Delta p^0_i + p^0_i - \sum_{k=1}^K \sum_{j=1}^N m_{ik} l_k m_{jk} \Delta q^{fm}_j \right), \quad i = 1, \ldots, N.$$

Using the notations above, we may restate Lemma B.2 as follows:

$$[0,1]^N \subset \Psi\big([0, n^0_1] \times \cdots \times [0, n^0_N]\big),$$

i.e., for each $(-\Delta s^*_1, \ldots, -\Delta s^*_N) \in [0,1]^N$ we can find an input $(\Delta q^{fm}_1, \ldots, \Delta q^{fm}_N)$ such that

$(\Delta s_1^*, \ldots, \Delta s_N^*)$ is the corresponding output:

$$\Psi(\Delta q_1^{fm}, \Delta q_2^{fm}, \ldots, \Delta q_N^{fm}) = (-\Delta s_1^*, -\Delta s_2^*, \ldots, -\Delta s_N^*)^\top.$$

Showing the uniqueness of solutions to (B.3) is now equivalent to showing that Ψ is injective. Viewing $l_1, \ldots, l_K$ as parameters and denoting by $J(\Delta q_1^{fm}, \ldots, \Delta q_N^{fm}, l_1, \ldots, l_K)$ the Jacobian of Ψ, we can easily show by direct calculation that, for any $(\Delta q_1^{fm}, \ldots, \Delta q_N^{fm})^\top \in [0, n_1^0] \times \cdots \times [0, n_N^0]$,

$$J(\Delta q_1^{fm}, \ldots, \Delta q_N^{fm}, 0, \ldots, 0) \geq \delta I_N, \tag{B.13}$$

where

$$\delta := \min_{i=1,\ldots,N} \left\{ \frac{\Delta p_i^0 + p_i^0}{(s_i^0 + \Delta s_i^0) a_i \pi_i b_i} \right\} > 0,$$

and the operator $A \geq B$ means that the matrix $A - B$ is positive semidefinite. Notice that $[0, n_1^0] \times \cdots \times [0, n_N^0]$ is compact and a direct calculation reveals that the function J is smooth in $(\Delta q_1^{fm}, \ldots, \Delta q_N^{fm}, l_1, \ldots, l_K)$. Because J is continuous function defined on a finite-dimensional compact domain, it is uniformly continuous. In view of the uniform continuity of J, there exists a neighborhood L of $(0, \ldots, 0)^\top$ such that for $(\Delta q_1^{fm}, \ldots, \Delta q_N^{fm}, l_1, \ldots, l_K)^\top \in [0, n_1^0] \times \cdots \times [0, n_N^0] \times L$,

$$\begin{aligned} &J(\Delta q_1^{fm}, \ldots, \Delta q_N^{fm}, l_1, \ldots, l_K) \\ &= J(\Delta q_1^{fm}, \ldots, \Delta q_N^{fm}, 0, \ldots, 0) + R(\Delta q_1^{fm}, \ldots, \Delta q_N^{fm}, l_1, \ldots, l_K), \end{aligned} \tag{B.14}$$

where $\|R(\Delta q_1^{fm}, \ldots, \Delta q_N^{fm}, l_1, \ldots, l_K)\|_2 < \frac{\delta}{2}$, i.e., for any $q \in \mathbb{R}^N$, $q \neq 0$,

$$|R(\Delta q_1^{fm}, \ldots, \Delta q_N^{fm}, l_1, \ldots, l_K) q| \leq \frac{\delta}{2} \|q\|_2. \tag{B.15}$$

Next, we fix $(l_1, \ldots, l_K) \in L$. Let $q_1, q_2 \in [0, n_1^0] \times \cdots \times [0, n_N^0]$, and denote by

$$g(t) := \langle \Psi\big(q_1 + t(q_2 - q_1)\big), q_2 - q_1 \rangle, \quad t \in [0, 1].$$

Then

$$\langle \Psi(q_2) - \Psi(q_1), q_2 - q_1 \rangle = g(1) - g(0) = \int_0^1 g'(t) dt.$$

Observe that $q_1 + t(q_2 - q_1) \in [0, n_1^0] \times \cdots \times [0, n_N^0]$. Using (B.13), (B.14) and (B.15),

$$\begin{aligned}\int_0^1 g'(t)dt &= \int_0^1 \langle J\big(q_1 + t(q_2 - q_1), l_1, \ldots, l_K\big)(q_2 - q_1), q_2 - q_1\rangle dt \\ &= \int_0^1 \langle J\big(q_1 + t(q_2 - q_1), 0, \ldots, 0\big)(q_2 - q_1), q_2 - q_1\rangle dt \\ &\quad + \int_0^1 \langle R\big(q_1 + t(q_2 - q_1), l_1, \ldots, l_K\big)(q_2 - q_1), q_2 - q_1\rangle dt \\ &\geq \delta |q_2 - q_1|^2 - \frac{\delta}{2}|q_2 - q_1|^2 = \frac{\delta}{2}|q_2 - q_1|^2.\end{aligned}$$

The above inequality can be equivalently restated as

$$\langle \Psi(q_2) - \Psi(q_1), q_2 - q_1\rangle \geq \frac{\delta}{2}|q_2 - q_1|^2,$$

Hence, if $\Psi(q_2) = \Psi(q_1)$, then

$$0 \leq \frac{\delta}{2}|q_2 - q_1|^2 \leq \langle \Psi(q_2) - \Psi(q_1), q_2 - q_1\rangle = 0,$$

i.e., $q_1 = q_2$. This proves that Ψ is injective.

□

C Spillover Losses and Pecking Order of Liquidation

In this section, we quantify the dependence of aggregate vulnerability to flow-to-performance sensitivity and first movers under a different asset liquidation strategy followed by funds. Specifically, we assume that funds follow a pecking order of liquidation, meaning that they sequentially liquidate assets in increasing order of price impact parameters. First, funds use cash, then they liquidate government bonds, and then sequentially the other assets. We assume that the assets labeled as "Other Securities," "Other Equities," and "Other Fixed-Income Securities" are the last ones to be liquidated because we do not have granular information on those assets.[27]

If all funds follow the pecking order liquidation strategy and the flow-to-performance sensitivity is low, aggregate spillover losses are significantly lower compared to the case of proportional liquidation. This is due to two compounding effects. First, the use of cash and the sale of liquid assets to repay redeeming investors reduces the downward impact on asset prices caused by redemptions and asset liquidation. Second, fire sales are concentrated in fewer (and more liquid) assets, which reduces asset price contagion. However, if the sensitivity of flow to performance is large, the first-mover advantage substantially increases the aggregate vulnerability of the mutual fund system.

[27] For computational reasons, we aggregate funds within each of the nine types. Unlike the proportion liquidation strategy, we cannot use matrix algebra to compute the quantities needed to estimate the SLR.

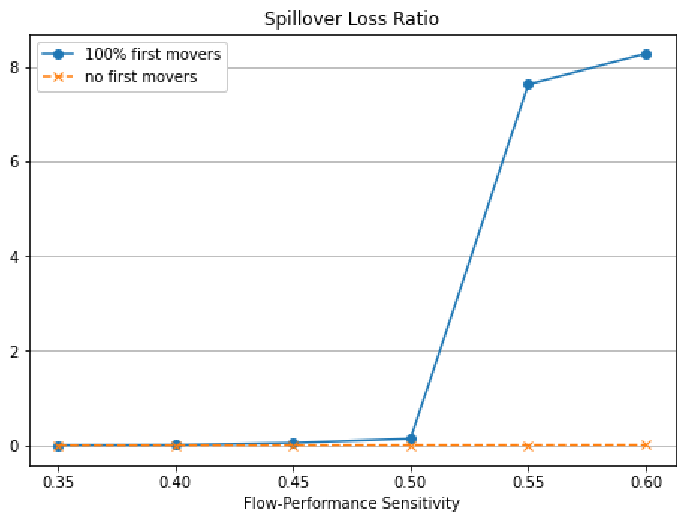

Figure C.1: The figure shows the Spillover Loss Ratio for different values of flow-to-performance ratio when funds follow a pecking order liquidation strategy both in the absence of first movers (dashed line) and if all investors are first movers (solid line). For each asset, we apply a shock equal to -5% price change times its relative volatility. We use asset holdings data from Q1 2020.

In Figure C.1, we compare the SLR in the two polar cases with no first movers, or all first-mover investors. Spillover losses are orders of magnitude larger in the system with only first movers compared to the system without first movers. The reason is that all funds first liquidate the same assets, severely impacting their prices and precipitating the spiral of redemptions and fire sales. This scenario is reminiscent of the disruption of Treasury markets during the Covid-19 crisis: as discussed in Ma et al. (2022b), concentrated sales of their most liquid assets by fixed-income mutual funds led to a significant increase in Treasury yields.

D Analytical Results on First Mover Concentration

Consider a system with two funds holding identical portfolios. Let $\frac{\bar{\pi}}{2}$ be the proportion of first movers in the whole system, and let $\pi \in \left(\frac{\bar{\pi}}{2}, \bar{\pi}\right)$ be the proportion of first movers in the first fund. The proportion of first movers in the second fund is $\bar{\pi} - \pi$. Let $\ell := \sum_{k=1}^{K} m_{ik}^2 l_k$. Both funds are subject to an initial identical portfolio shock Δs^0. Let $\Delta Q^{fm} = x = (x_1, x_2)$ be the number of portfolio units each fund sells to repay first movers, and let $\Delta S^* = y = (y_1, y_2)$ be the aggregate shock to each fund's NAV. The amounts x_1 and x_2 are the solutions to the system

$$
\begin{aligned}
x_1(s^0 + \Delta s^0 - \ell(x_1 + x_2)) &= -ab(s^0 + \Delta s^0)\pi y_1, \\
x_2(s^0 + \Delta s^0 - \ell(x_1 + x_2)) &= -ab(s^0 + \Delta s^0)(\bar{\pi} - \pi) y_2,
\end{aligned}
$$

where we assume without loss of generality that $p_0 = s_0$. More explicitly,

$$x_1 = f_{x,1}(y_1, y_2) := \pi y_1 \frac{s^0 + \Delta s^0 - \sqrt{(s^0 + \Delta s^0)(s^0 + \Delta s^0 + 4ab\ell(\pi y_1 + (\bar{\pi} - \pi)y_2))}}{2\ell(\pi y_1 + (\bar{\pi} - \pi)y_2)},$$
$$x_2 = f_{x,2}(y_1, y_2) := (\bar{\pi} - \pi)y_2 \frac{s^0 + \Delta s^0 - \sqrt{(s^0 + \Delta s^0)(s^0 + \Delta s^0 + 4ab\ell(\pi y_1 + (\bar{\pi} - \pi)y_2))}}{2\ell(\pi y_1 + (\bar{\pi} - \pi)y_2)},$$

where we have chosen the smallest roots, i.e., the ones corresponding to the least amount of assets funds would have to liquidate to meet first movers' redemptions. The NAV change of fund 1 observed by second movers is

$$\Delta s_1^{fm} = \frac{a - x_1}{a + ab\pi y_1}(s^0 + \Delta s^0 - \ell(x_1 + x_2)) - s^0.$$

We may rewrite the above expression, and obtain that each fund's NAV change observed by second movers is equal to

$$\Delta S^{fm} = (\Delta s^0 - \ell(x_1 + x_2)) \begin{pmatrix} 1 \\ 1 \end{pmatrix} - (s^0 + \Delta s^0 - \ell(x_1 + x_2)) \begin{pmatrix} \frac{x_1 + ab\pi y_1}{a + ab\pi y_1} \\ \frac{x_2 + ab(\bar{\pi} - \pi)y_2}{a + ab(\bar{\pi} - \pi)y_2} \end{pmatrix}.$$

Define $v_1 := \frac{a - x_1}{a + ab\pi y_1}$, $v_2 := \frac{a - x_2}{a + ab(\bar{\pi} - \pi)y_2}$. The matrix U^{fm} defined in Section 3.2 is then given by

$$U^{fm} = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix},$$

and in each round of second movers' redemptions the NAV change is multiplied by the matrix $T := U^{fm} M L M^\top U^{fm} A(I - \Pi)B$. An explicit calculation yields

$$T = ab\ell \begin{pmatrix} (1 - \pi)v_1^2 & (1 - (\bar{\pi} - \pi))v_1 v_2 \\ (1 - \pi)v_1 v_2 & (1 - (\bar{\pi} - \pi))v_2^2 \end{pmatrix}.$$

For sufficiently small ℓ, the matrix $I - T$ is invertible and the aggregate impact on each fund's NAV is then given by

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} f_{y,1}(x, y) \\ f_{y,2}(x, y) \end{pmatrix} := \sum_{n=0}^{\infty} T^n \Delta S^{fm} = (I - T)^{-1} \Delta S^{fm}. \tag{D.16}$$

Therefore, to find the aggregate NAV change we need to solve the fixed point of the system $x = f_x(y)$ and $y = f_y(x, y)$. The component $y = (y_1, y_2)$ gives the aggregate NAV changes for funds 1 and 2, respectively.

Next, we restate Proposition 2 using the notation introduced in this section. In particular, the fixed point y^* plays the role of ΔS^* in Proposition 2. We make the dependence of the functions f_x and f_y on π explicit by writing f_x^π and f_y^π.

Proposition 2'. *Assume $\Delta s^0 \in (-1, 0)$ and $b \cdot \Delta s^0 \in (-1, 0)$. For sufficiently small ℓ, there exists* *a fixed point $y^*(\pi)$ for $f_y^\pi(f_x^\pi(y), y)$, where $\pi \in (\frac{\bar{\pi}}{2}, \bar{\pi})$ is the proportion of first movers in the first fund. Define $g(\pi) := \lim_{\ell \downarrow 0} \frac{1}{\ell}(y_1^*(\pi) + y_2^*(\pi) - 2\Delta s^0)$, i.e., $y_1^*(\pi) + y_2^*(\pi) = 2\Delta s^0 + \ell \cdot g(\pi) + o(\ell)$. The function $g(\pi)$ is decreasing in π.*

Proof. Fix $\pi \in (\frac{\bar{\pi}}{2}, \bar{\pi})$ and let $y^\pi := y^*(\pi)$ be the vector of aggregate NAV changes if the proportion of first movers for each fund is, respectively, π and $\bar{\pi} - \pi$. For $\ell = 0$, asset liquidation does not move prices, and therefore $y^\pi = (\Delta s^0, \Delta s^0)$. By continuity, a fixed point y^π exists for sufficiently small ℓ. Assume $y^\pi = (\Delta s^0, \Delta s^0) + \ell \cdot y^{\pi,1} + o(\ell^2)$, where $y^{\pi,1}$ is independent of ℓ. The first order expansion of $(x_1, x_2) = f_x^\pi(y^\pi)$ yields

$$
\begin{aligned}
x_1 &= -ab\pi\Delta s^0 + \ell ab\pi \frac{ab\bar{\pi}(\Delta s^0)^2 - y_1^{\pi,1}(s^0 + \Delta s^0)}{s^0 + \Delta s^0} + o(\ell), \\
x_2 &= -ab(\bar{\pi} - \pi)\Delta s^0 + \ell ab(\bar{\pi} - \pi) \frac{ab\bar{\pi}(\Delta s^0)^2 - y_2^{\pi,1}(s^0 + \Delta s^0)}{s^0 + \Delta s^0} + o(\ell).
\end{aligned}
$$

After plugging the expansion for (x_1, x_2) into the right-hand side in equation (D.16), we obtain

$$
(I - T)^{-1}\Delta S^{fm} = \Delta s^0 + \ell \cdot \begin{pmatrix} \frac{ab\Delta s^0(2 + b\pi\Delta s^0(2 - \bar{\pi}))}{1 + b\pi\Delta s^0} \\ \frac{ab\Delta s^0(2 + b(\bar{\pi} - \pi)\Delta s^0(2 - \bar{\pi}))}{1 + b(\bar{\pi} - \pi)\Delta s^0} \end{pmatrix} + o(\ell).
$$

Hence, by comparing the terms of order ℓ in equation (D.16), we get

$$
\begin{aligned}
y_1^{\pi,1} &= ab\Delta s^0 \frac{2 + b\pi\Delta s^0(2 - \bar{\pi})}{1 + b\pi\Delta s^0}, \\
y_2^{\pi,1} &= ab\Delta s^0 \frac{2 + b(\bar{\pi} - \pi)\Delta s^0(2 - \bar{\pi})}{1 + b(\bar{\pi} - \pi)\Delta s^0}.
\end{aligned}
$$

In particular, $y_1^\pi + y_2^\pi = 2\Delta s^0 + \ell \cdot g(\pi) + o(\ell)$, where

$$
g(\pi) = ab\Delta s^0 \left[\frac{2 + b\pi\Delta s^0(2 - \bar{\pi})}{1 + b\pi\Delta s^0} + \frac{2 + b(\bar{\pi} - \pi)\Delta s^0(2 - \bar{\pi})}{1 + b(\bar{\pi} - \pi)\Delta s^0} \right].
$$

The first derivative of $g(\pi)$ is

$$
ab^3(\Delta s^0)^3\bar{\pi} \frac{(2\pi - \bar{\pi})(2 + b\bar{\pi}\Delta s^0)}{(1 + b\pi\Delta s^0)^2(1 + b(\bar{\pi} - \pi)\Delta s^0)^2},
$$

which is negative because $2\pi > \bar{\pi}$, $\Delta s^0 < 0$, and $b \cdot \Delta s^0 > -1$. This concludes the proof. □

An increase in first mover concentration has a twofold effect on each fund's NAV. First, additional first-mover redemptions at the first fund negatively impact its NAV (and, conversely, fewer first mover redemptions at the second fund increase its NAV). This effect is symmetric across the two funds. Second, an increase in the proportion of first movers reduces the number of investors that bear the cost of first movers' redemptions, while this externality is spread over more investors for the fund with fewer first movers. This effect is asymmetric, because it exacerbates the impact of first movers on the first fund's NAV and reduces the benefit of having fewer first movers. Hence, the aggregate effect of first mover concentration on the system is negative.

References

Allen, F., Babus, A. and Carletti, E. (2009), 'Financial crises: theory and evidence', *Annual Review of Financial Economics* **1**, 97—-116.

Allen, F. and Walther, A. (2021), 'Financial architecture and financial stability', *Annual Review of Financial Economics* **13**, 129–151.

Berk, J. B. and Green, R. C. (2004), 'Mutual fund flows and performance in rational markets', *Journal of political economy* **112**(6), 1269–1295.

Bouveret, A. and Yu, J. (2021), 'Risks and vulnerabilities in the US bond mutual fund industry', *IMF Working Papers* **2021**(109).

Capponi, A., Glasserman, P. and Weber, M. (2020), 'Swing pricing for mutual funds: Breaking the feedback loop between fire sales and fund redemptions', *Management Science* **66**(8), 3581–3602.

Capponi, A., Glasserman, P. and Weber, M. (2023), 'Swing pricing: Theory and evidence', *Annual Review of Financial Economics* **15**, 617–640.

Capponi, A. and Larsson, M. (2015), 'Price contagion through balance sheet linkages', *Review of Asset Pricing Studies* **5**(2), 227–253.

Cetorelli, N., Duarte, F. and Eisenbach, T. (2016), 'Are asset managers vulnerable to fire sales?', *Liberty Street Economics* .

Chen, Q., Goldstein, I. and Jiang, W. (2010), 'Payoff complementarities and financial fragility: Evidence from mutual fund outflows', *Journal of Financial Economics* **97**(2), 239–262.

Chernenko, S. and Sunderam, A. (2020), Measuring the perceived liquidity of the corporate bond market, Technical report, National Bureau of Economic Research.

Chevalier, J. and Ellison, G. (1997), 'Risk taking by mutual funds as a response to incentives', *Journal of Political Economy* **105**(6), 1167–1200.

Choi, J., Hoseinzade, S., Shin, S. S. and Tehranian, H. (2020), 'Corporate bond mutual funds and asset fire sales', *Journal of Financial Economics* **138**(2), 432–457.

Christoffersen, S. E., Musto, D. K. and Wermers, R. (2014), 'Investor flows to asset managers: Causes and consequences', *Annual Review of Financial Economics* **6**(1), 289–310.

Diamond, D. W. and Dybvig, P. H. (1983), 'Bank runs, deposit insurance, and liquidity', *Journal of Political Economy* **91**(3), 401–419.

Duarte, F. and Eisenbach, T. M. (2021), 'Fire-sale spillovers and systemic risk', *Journal of Finance* **76**(3), 1251–1294.

Franzoni, F. and Schmalz, M. C. (2017), 'Fund flows and market states', *The Review of Financial Studies* **30**(8), 2621–2673.

Fricke, C. and Fricke, D. (2021), 'Vulnerable asset management? The case of mutual funds', *Journal of Financial Stability* **52**, 100800.

Gennaioli, N., Shleifer, A. and Vishny, R. (2013), 'A model of shadow banking', *Journal of Finance* **68**, 1331—-1363.

Goldstein, I., Jiang, H. and Ng, D. T. (2017), 'Investor flows and fragility in corporate bond funds', *Journal of Financial Economics* **126**(3), 592–613.

Greenwood, R., Landier, A. and Thesmar, D. (2015), 'Vulnerable banks', *Journal of Financial Economics* **115**(3), 471–485.

Jiang, H., Li, D. and Wang, A. (2021), 'Dynamic liquidity management by corporate bond mutual funds', *Journal of Financial and Quantitative Analysis* **56**(5), 1622–1652.

Jiang, H., Li, Y., Sun, Z. and Wang, A. (2022), 'Does mutual fund illiquidity introduce fragility into asset prices? Evidence from the corporate bond market', *Journal of Financial Economics* **143**(1), 277–302.

Jin, D., Kacperczyk, M., Kahraman, B. and Suntheim, F. (2022), 'Swing pricing and fragility in open-end mutual funds', *The Review of Financial Studies* **35**(1), 1–50.

Johnson, W. T. (2004), 'Predictable investment horizons and wealth transfers among mutual fund shareholders', *The Journal of Finance* **59**(5), 1979–2012.

Lewrick, U. and Schanz, J. (2023), 'Towards a macroprudential framework for investment funds: swing pricing and investor redemptions', *International Journal of Central Banking, Forthcoming* .

Lewrick, U. and Schanz, J. F. (2017), 'Liquidity risk in markets with trading frictions: What can swing pricing achieve?'.

Li, L., Li, Y., Macchiavelli, M. and Zhou, X. (2021), 'Liquidity restrictions, runs, and central bank interventions: Evidence from money market funds', *The Review of Financial Studies* **34**(11), 5402–5437.

Li, Y., O'Hara, M., Zhou, X. et al. (2023), 'Mutual fund fragility, dealer liquidity provisions, and the pricing of municipal bonds', *Management Science* .

Ma, Y., Xiao, K. and Zeng, Y. (2022*a*), 'Bank debt, mutual fund equity, and swing pricing in liquidity provision', *Jacobs Levy Equity Management Center for Quantitative Financial Research Paper* .

Ma, Y., Xiao, K. and Zeng, Y. (2022*b*), 'Mutual fund liquidity transformation and reverse flight to liquidity', *The Review of Financial Studies* **35**, 4674–4711.

Matta, R. and Perotti, E. C. (2023), 'Pay, stay or delay? how to settle a run', *Review of Financial Studies, Forthcoming* .

Morris, S., Shim, I. and Shin, H. S. (2017), 'Redemption risk and cash hoarding by asset managers', *Journal of Monetary Economics* **89**, 71–87.

Schmidt, L., Timmermann, A. and Wermers, R. (2016), 'Runs on money market mutual funds', *American Economic Review* **106**(9), 2625–57.

Shek, J., Shim, I. and Shin, H. S. (2018), 'Investor redemptions and fund manager sales of emerging market bonds: how are they related?', *Review of Finance* **22**(1), 207–241.