# Enhancing Image Reconstruction via Phase-Constrained Data in an Iterative Process

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**Abstract.** This paper proposes an innovative approach to improve residual artifacts in image-based parallel magnetic resonance imaging (MRI) reconstruction. Despite its superior signal-to-noise ratio (SNR) over the conventional Sensitivity Encoding (SENSE) method, SENSE is hindered by persisting residual artifacts, causing it to be less effective in image-based parallel MRI reconstruction. We propose a joint estimation of actual and virtual coil sensitivity maps, along with the reconstructed image. Inspired by the principles of the Joint Sensitivity Encoding (JSENSE) method, the proposed approach employs an iterative optimization process via phase-constrained data of virtual conjugate coils, progressively refining these integral components to achieve superior image quality. Experimental results show that the proposed method not only enhances MRI image quality by suppressing residual artifacts but also paves the way for future research into the potential of virtual conjugate coils in image-based MRI reconstruction. Different from the phase-constrained data for enhancing kspace-based parallel MRI, the method shows that the phase-constrained data also improve image-based parallel MRI reconstruction.

**Keywords:** Magnetic Resonance Imaging, Virtual Conjugate Coil, Phase-Constrained Reconstruction

### 1 Introduction

Virtual Conjugate Coil (VCC) [1,2] has undeniably brought about significant improvements in the quality of parallel Magnetic Resonance Imaging (MRI) reconstructions and deep network-based methods. Renowned techniques such as VCC-GRAPPA [2], VCC-LORAKS [3], and VCC-ESPIRIT [4] along with newer approaches like virtual coil augmentation for MR coil extrapolation via deep learning [11], have showcased their efficacy in suppressing residual artifacts in reconstructed images and enhancing the Signal-to-Noise Ratio (SNR).

Virtual Conjugate Coil Sensitivity Encoding (VCC-SENSE) [2], a technique that has gained significant attention in the field, has demonstrated a superior SNR compared to the conventional Sensitivity Encoding (SENSE) [5] approach. However, this approach is not without its flaws. Residual artifacts, lingering remnants from the re-

construction process that interfere with the interpretation of the final image, continue to persist in VCC-SENSE reconstructions. This shortcoming has led to VCC being less studied in the realm of image-based parallel MRI reconstruction compared to k-space-based reconstruction [2,3,4].

In this context, it becomes imperative to devise innovative strategies that focus on the reduction and potential elimination of these residual artifacts. In this paper, our primary goal is to improve these residuals in SENSE reconstructions. Our approach involves solving an iterative optimization problem, a strategy that has proven successful in various computational tasks. Motivated by the principles of Joint Sensitivity Encoding (JSENSE) [8], we propose a joint estimation of actual coil sensitivity maps, virtual coil sensitivity maps, and the reconstructed image. This is not a straightforward approach, as it involves iteratively enhancing and refining these three integral components to achieve superior image quality. The coil sensitivity maps – both actual and virtual – along with the image to be reconstructed, undergo a series of improvements, progressively refining the final output.

The iterative optimization process offers a systematic and guided approach to improve the quality of the reconstructed image. The idea is to start with an initial estimation for each of the three components and iteratively refine them, with each iteration offering an improvement over the last. The process continues until an optimal or near-optimal solution is found. The joint estimation approach provides a mechanism for the system to learn from the residuals, enabling it to correct and suppress these artifacts. The suppression of these residuals can significantly improve the quality of the reconstructed images. By addressing one of the major shortcomings of the SENSE-related techniques, we aim to bring VCC to image-based MRI reconstruction and provide an improved approach that balances both SNR and the minimization of residual artifacts. In this paper, the first and the section sections of this paper present an introduction and background. The proposed method is given in the third part. Experimental results and conclusions are provided in the fourth and fifth sections.

# 2 Background

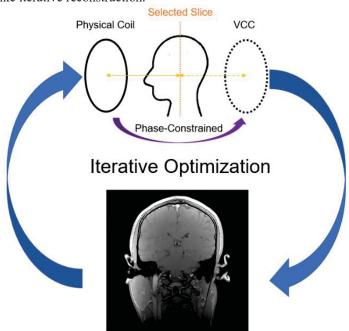
Joint Sensitivity Encoding (JSENSE) [8] is a magnetic resonance imaging (MRI) technique designed to overcome certain limitations of conventional SENSE [5] method, which rely heavily on precise estimations of coil sensitivity maps for image reconstruction. The conventional SENSE method can suffer from inaccuracies in these initial estimates, leading to degraded image quality. JSENSE adopts an iterative approach optimizing both coil sensitivity maps and the image concurrently. This innovative strategy allows for the refinement of the actual coil image sensitivity profile during the image reconstruction process, thus potentially yielding higher-quality images. Despite the computational demands of this iterative process making JSENSE more resource-intensive than conventional SENSE methods, ongoing research including the application of deep learning methods is focused on enhancing the performance and efficiency of JSENSE, particularly in situations where initial coil sensitivity profiles are inaccurate or change during MRI scan.

Efforts to improve calibration accuracy in MRI reconstruction have necessitated mining valuable data within the restricted auto-calibration signal (ACS) lines, where a notable strategy involves the application of the VCC concept. The VCC enhances encoding power, effectively bolstering the reconstruction performance of numerous methodologies such as SENSE [2], GRAPPA [2], ESPIRIT [4], KerNL [6], iterative RAKI [9], nonlinear GRAPPA [7], even multi-contrast data [10], and PROPELLER [13]. Additionally, the VCC method introduces extra equations into the inverse reconstruction matrix by incorporating additional phase information, augmenting the precision of the reconstructed images. In the context of machine learning, VCC serves as an effective data augmentation technique, contributing to the enhanced performance of learning models [6]. However, while VCC improves the reconstruction quality, it results in increased computational costs due to the doubling of channels in the k-space data used in the process. For instance, a dataset involving a 32-coil k-space would necessitate a total of 64 coils for reconstruction, including the original 32 and an added 32 virtual coils, making the procedure more computationally demanding.

# 3 Proposed Method

## 3.1 The Proposed Framework

The proposed methodology framework is illustrated in Fig.1. Phase-constrained data are generated as VCC signals. Both physical coil data and VCC data are used in JSENSE-like iterative reconstruction.



**Fig. 1.** Framework of the proposed method. Phase-constrained data are incorporated into the iterative reconstruction process, which is supported by JSENSE-like method.

#### 3.2 Generating Phase-Constrained Data

In parallel MRI, one coil's k-space data denotes the Fourier Transform (FT) of the distribution of the spatial spins combined with the coil sensitivities. Furthermore, in practical imaging, background phase effects caused by B0 field inhomogeneity, flow, and pulse sequence also exist in the effective coil sensitivities. The coil k-space data can be represented as [1]

$$S_j(\mathbf{p}) = \mathrm{FT}\left(\rho(\mathbf{x}) \cdot e^{i\varphi(\mathbf{x})} \cdot C_j(\mathbf{x})\right),\tag{1}$$

where  $\rho(\mathbf{x})$  represents the spin distribution,  $e^{i\varphi(\mathbf{x})}$  denotes background phase,  $C_j(\mathbf{x})$  is the coil sensitivities of the  $j^{th}$  coil,  $\mathbf{p}$  is the k-space data vector, and  $\mathbf{x}$  represents the vector in the image domain. The symmetric complex-conjugate k-space data can be represented as [1]

$$S_j^*(\mathbf{p}) = \text{FT}[\rho(\mathbf{x}) \cdot e^{-i\varphi(\mathbf{x})} \cdot C_j^*(\mathbf{x})], \qquad (2)$$

where \* is the complex-conjugate operator. Additional phase information is provided in the VCC, although the magnitude sensitivities are the same between actual and virtual coils.

Additional equations are added in the VCC-based reconstruction, and reconstructed image quality is improved due to the additional encoding power from VCC. The explicit knowledge of the background phase information is not required when VCC is combined with GRAPPA reconstruction [1] for improving the quality. Encoding power is significantly improved by using the phase variations in the complex coil sensitivities. On the other hand, insufficient phase variations and inaccurate knowledge of spatial phase information cause artifacts [1].

# 3.3 Iterative JSENSE Reconstruction Using VCC Data

The SENSE technique takes advantage of the spatial sensitivity variations of multiple surface receiver coils. It's a parallel imaging method that helps reduce scan times by acquiring less k-space data, thus speeding up the imaging process. SENSE uses an array of multiple receiver coils. Each of these coils has a different spatial sensitivity pattern, which means they pick up signals with varying strength depending on their position relative to the body. As a result, each coil can provide a unique "view" of the body, which contains spatial encoding information in addition to the signal data. The SENSE technique then uses these unique views to fill in the gaps in the undersampled k-space data, thus allowing for a reduction in the number of phase-encoding steps and consequently faster image acquisition. It is important to note that the accuracy of the reconstructed image depends on the correct estimation of the coil sensitivity profiles. Misestimation could lead to errors known as aliasing artifacts. For the SENSE imaging formulation,

$$\mathbf{Ef} = \mathbf{d},\tag{3}$$

(6)

where **d** represents the acquired k-space data from all actual coils, the encoding matrix **E** contains the product of Fourier encoding with undersampled k-space and coilspecific sensitivity modulation over the image, and **f** is the unknown image to be reconstructed.

For JSENSE, the imaging equation becomes

$$\mathbf{E}(\mathbf{a})\mathbf{f} = \mathbf{d},\tag{4}$$

where **a** represents unknown actual coil sensitivities. In our study, we are introducing a method where we propose a simultaneous estimation of actual coil sensitivity maps, virtual coil sensitivity maps, and the reconstructed image. The traditional approach usually treats these aspects independently, but our method acknowledges their interconnected nature and leverages this relationship for a more accurate and effective reconstruction process. By jointly estimating these factors, we can address challenges in MRI reconstruction and improve the quality of the resultant image. In particular, the actual coil sensitivity maps are key to accounting for the distinct signal reception profiles of different coils. On the other hand, the virtual coil sensitivity maps introduce extra equations to the inverse reconstruction matrix by incorporating additional phase information. Furthermore, the reconstructed image integrates these considerations to result in improved final output. Overall, our proposed method aims to enhance MRI image reconstruction by comprehensively considering all the key contributing factors in a unified estimation process. So, the image equation is

$$\mathbf{E}(\mathbf{a}, \mathbf{a}')\mathbf{f} = \mathbf{d},\tag{5}$$

where a' denotes the unknown virtual coil sensitivities. We apply iterative optimization to solve the equation (5). In phase-constrained reconstruction, actual and virtual coils are harmoniously combined. They are fed into the reconstruction procedures without separating them, resulting in a unified input. This integrated approach is also maintained during the iterative optimization process, where both coil types are jointly involved. Therefore, the actual and virtual coils coexist throughout the iterations, contributing to the overall solution.

Specifically, the cost function is alternatively minimized for 
$$\{[a,a'],f\} = \arg\min_{[a,a'],f} U([a,a'],f).$$

In each computational cycle of the reconstruction process, the image that is being restructured is integrated with actual and virtual coil sensitivity maps — computational representations of the coil's sensitivities. This integration results in two unique images, an actual coil image and a virtual coil image, which facilitate a more accurate image reconstruction. The coil sensitivity maps are initially generated through a self-calibration process, establishing a foundation for the iterations to improve the image. After all the iterations are completed, a final reconstructed image is produced. This image is then assessed for its clarity, detail, and fidelity compared to the original image. This evaluation process gauges the effectiveness of the reconstruction process and identifies potential enhancements for future iterations, with the final image demonstrating the success of the applied method.

# 4 Experimental Results

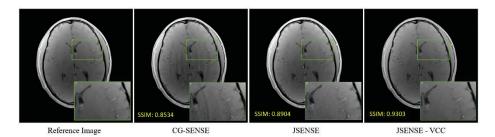
#### 4.1 Datasets and Evaluation Metrics

Two datasets are used to evaluate the reconstruction performance of the proposed random feature method. The first dataset of axial brain images was acquired on a 3T scanner (SIEMENS AG, Erlangen, German) with a 32-channel head coil using a 2D gradient echo sequence (TE/TR = 2.29/100 ms, flip angle = 25°, matrix size = 256 × 256, slice thickness = 3 mm, and FOV = 24 × 24 cm²). The second set of coronary brain data was acquired using a 2D gradient echo sequence (slice thickness = 3.0 mm, matrix size = 256 × 256, FOV = 24 × 24 cm², and TE/TR = 2.29/100 ms). The k-space data was subject to undersampling by a reduction factor, with the count of the ACS lines is set as 32. The reconstruction algorithm was executed in MATLAB, a high-level programming language developed by MathWorks based in Natick, Massachusetts. All image reconstruction was carried out on a laptop equipped with an i7 processor and 32GB of RAM. Given that the proposed technique does not apply deep learning, there was no requirement for a graphics processing unit (GPU).

In addition to the subjective evaluation, the suggested technique is also benchmarked against alternative methods employing two quantifiable evaluation standards. These standards encompass the normalized mean square error (NMSE), which measures the magnitude of error, and the structural similarity index measurement (SSIM), a method that gauges image quality by comparing changes in structural information.

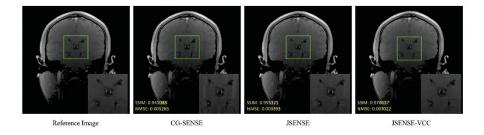
#### 4.2 Reconstruction Results

For the first dataset of axial brain, k-space is undersampled with 32 ACS lines and the outer reduction factor of 4. The fully sampled k-spaced data are inversely Fourier transformed to image space and all coil images are combined to generate the final image. Missing k-space data are recovered by CG-SENSE [12], JSENSE [8], and the proposed JSENSE-VCC methods. A region-of-interest (ROI) is extracted for comparing the details of reconstructed images. In Fig.2, it is seen that the proposed method can suppress aliasing artifacts and noise in the reconstructed image. In contrast, CG-SENSE image has typical aliasing artifacts and JSENSE also have artifacts and noise. The proposed method has the closest appearance of image content to the reference image. In addition, SSIM values are presented in Fig.2. It is seen that the proposed JSENSE-VCC method has the highest SSIM value 0.9303 in all three images reconstructed from undersampled k-space data.



**Fig. 2.** For the axial brain data, reconstruction performance comparison among the reference image, CG-SENSE, JSENSE, and the proposed JSENSE-VCC method. The reference image is fully sampled. The proposed method can suppress aliasing artifacts in comparison to CG-SENSE. In comparison to the conventional JSENSE, JSENSE-VCC can restore more details. The proposed method has the highest SSIM value.

For the second dataset of the coronary brain, 32 ACS lines and the outer reduction factors of 2, 4, and 8 are used to undersample k-space data, respectively. The fully sampled k-spaced data are inversely Fourier transformed to image space and all coil images are combined to generate the final image. Missing k-space data are recovered by CG-SENSE, JSENSE, and the proposed JSENSE-VCC methods. A ROI is extracted for comparing the details of reconstructed images. In Fig.3, it is seen that the proposed method can suppress aliasing artifacts and noise in the reconstructed image. CG-SENSE image has typical aliasing artifacts and noise, and JSENSE reconstruction also has noise. The proposed method has the closest appearance of image content to the reference image. In addition, SSIM and NMSE values are presented in Fig.3. It is seen that the proposed JSENSE-VCC method has the highest SSIM value 0.9706 and the lowest NMSE value 0.003022 in all three images reconstructed from undersampled k-space data. Besides the JSENSE-VCC, feature selection-based reconstruction [14], dual-interpolator-based reconstruction [16], and broad learning reconstruction [15] may also be combined with VCC concept for further improvement of performance.



**Fig. 3.** For the coronary brain data, reconstruction performance comparison (outer reduction factor 4) among the reference image, CG-SENSE, JSENSE, and the proposed JSENSE-VCC method. The reference image is fully sampled. The proposed method can suppress aliasing artifacts in comparison to CG-SENSE. In comparison to the conventional JSENSE, JSENSE-VCC can restore more details. The proposed method has the highest SSIM and the lowest NMSE values.

To quantitatively evaluate the reconstruction performance, the undersampled k-space data of the coronary brain are reconstructed by CG-SENSE, JSENSE, and JSENSE-VCC, respectively. Quantitative results are shown in Table 1. It is seen that the proposed JSENSE-VCC method has the highest SSIM values for images reconstructed at the outer reduction factor 2, 4, and 8, respectively.

**Table 1.** Quantitative Metric Values for Evaluating Reconstruction Performance of Coronary Brain Data.

	R2		R4		R8	
	SSIM	NMSE	SSIM	NMSE	SSIM	NMS
						E
CG-	0.943532	0.002597	0.941088	0.005265	0.794037	0.021442
SENSE						
JSENSE	0.963015	0.003143	0.955321	0.003393	0.829628	0.015490
JSENSE-	0.971696	0.002107	0.970637	0.003022	0.921765	0.007200
VCC						

#### 5 Conclusion

In conclusion, the study presented an innovative approach to improve the quality of image-based MRI reconstruction. Our methodology leverages an iterative optimization process that jointly estimates actual and virtual coil sensitivity maps, along with the image to be reconstructed. Each iteration refines these three components and enhances the final output. The proposed method notably elevates the quality of reconstructed images through suppressing residual artifacts. The results not only bolster the promise of image-based MRI reconstruction but also highlight the potential of this approach in improving both SNR and minimizing residual artifacts.

# Acknowledgment

This work was supported by the National Science Foundation under Grant No. 2050972.

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