A Broadcast Channel Framework for Joint Communications and Sensing-Part II: Superposition Coding

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***Abstract*—The technology of joint communications and sensing (JCS) integrates both functions in the same waveform and thus the same frequency band. It is expected to be a distinguishing feature in 6G wireless networks. A major challenge to JCS is how to seamlessly integrate the two historically distinct functions of communications and radar sensing. In the second part of this paper, a framework of superposition coding, motivated by the similarity to broadcast channels, is proposed for the functional multiplexing in JCS, which is motivated by the studies on broadcast channels in data communications. In this framework, communications and sensing are considered as genuine and virtual users, respectively. Sensing is considered as the bottom user in the layered structure of superposition coding; thus a sensing waveform is generated according to a certain criterion of sensing, which plays the role of cloud in superposition coding. Then, the communication message is superimposed on top of the cloud. Different superposition schemes are proposed, each corresponding to one type of mathematical operation on vectors in linear spaces. Moreover, the waveform diversity recently proposed in the radar community, which prepares a set of waveforms for handling the variance of environment, is taken into account. The cases of sensing waveform known/unknown to the communication receiver are discussed. The performance of the proposed JCS schemes is demonstrated using numerical simulations.**

I. Introduction

Joint communications and sensing (JCS) is expected to be a promising signaling technique in 6G communication systems, due to its efficient spectrum utilization and wide applications in various cyber physical systems (CPSs), such as autonomous driving and unmanned aerial vehicle (UAV) networks. In JCS, both functions of communications and sensing are accomplished in a single round of transmission of electromagnetic (EM) wave: the data message is delivered in the forward propagation, while the information of sensed target is fetched back by the reflected EM wave. Since JCS uses the same waveform for both functions, the spectral efficiency is substantially improved, thus helping to solve the challenge of spectrum congestion.

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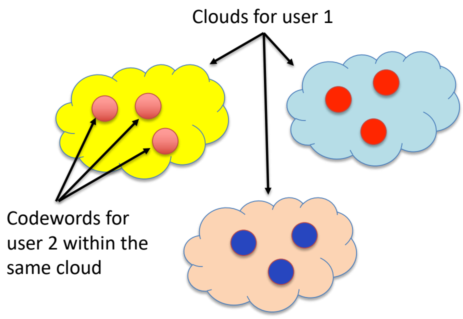

Fig. 1: Clouds in superposition coding.

The major challenge in JCS is how to integrate both functions of communications and sensing in the same waveform. One effective framework is to leverage the studies on broadcast channels (BCs) in information theory and downlink cellular networks, since sensing can be considered as a 'virtual' user sharing the bandwidth with the genuine communication user. The corresponding feasible performance region, using the duality between broadcast and multiaccess channels, has been studied in the first part of this paper, while concrete waveform synthesis schemes, motivated by the broadcast channel framework, have been studied by the author in terms of frequency division multiplexing (FDM) [1] and dirty paper coding (DPC) [2]. In this paper, we study the waveform synthesis from the aspect of channel coding, which is motivated by the broadcast channel framework and unifies the sensing waveform synthesis (in particular the waveform diversity) and communication channel coding.

As illustrated in Fig. 1, broadcast channel coding can be implemented using the superposition coding [3]–[6], in which the messages of user 1 can be represented by the 'clouds' while the messages of user 2 are encoded into each cloud. For decoding, user 1 simply determines which cloud is transmitted, while user 2 first determines the cloud and then estimates the codeword within the cloud, in an onion-peeling manner. Similarly, in the context of JCS, we can consider sensing and communications as users 1 and 2, respectively. For user 1 (sensing), we prepare multiple waveforms as the clouds. Note that a common important issue in both communications and sensing is the adaptation to the environment (e.g., the type of clutters and the frequency selectivity of the target echoes).

We consider two possible situations, which are both reasonable in practice: (a) Waveform Available at Receiver (WAR): The communication receiver has received notifications from the JCS transceiver and knows the sensing waveform (cloud) used by the JCS transceiver. This is reasonable when the environment changes slowly, such that the optimal waveform does not change rapidly. (b) Waveform Unavailable at Receiver (WUR): The communication receiver does not know the exact sensing waveform (cloud), while it knows the set of possible waveforms. In the context of superposition coding, the communication receiver does not know the selected codeword of the virtual user of sensing, while it knows the corresponding codebook.

In summary, the novel contribution of this paper is to leverage the framework of superposition coding in broadcast channels to naturally integrate the functions and communications and sensing and incorporate several concrete JCS schemes as special examples. Although superposition coding has been proposed in the communication community for decades, this paper is the first to employ it as a unified framework in the context JCS, to our best knowledge.

In this paper, we mainly focus on the oretical framework and study the detailed coding schemes. The remainder of the paper is organized as follows. The related work and signal model are introduced in Sections II and III, respectively. Then, the superposition coding schemes for WAR and WUR are discussed in Sections IV and V, respectively. Finally, the numerical results and conclusions are provided in Sections VI and VII, respectively.

II. Related Works

Comprehensive surveys on JCS can be found in [7], [8]. For data communications, BC has been intensively studied in early 2000s. While the BC channel capacity region has been identified for degraded channels [9], it is still an open problem for the multiple-input-multiple-output (MIMO) case. A major breakthrough is the introduction of DPC [10], disclosed by [11]. Based on DPC, the duality between multiple-access (MAC) and BC is identified in [12]. Superposition coding has been studied for both broadcast and interference channels, mainly from the viewpoint of information theory. Different types of theoretical superposition coding schemes have been compared in [3]. More practical coding schemes have been proposed in [4]. A survey on superposition code can be found in [6]. Particularly, successive interference cancellation is considered in [13] for the purpose of JCS. However, it has not formed a unified framework for JCS.

III. System Model

In this section, we introduce the system model for JCS.

A. Signal Model

We consider a single JCS transceiver, a communication receiver, a desired radar target and background reflectors generating clutters. The available transmit power and bandwidth are denoted by P_t and W, respectively. For simplicity, we consider discrete-time signals, in which each chip is denoted by $x[n]$, $n = 0, ..., N-1$, where N is the number of chips within each symbol (pulse) period, and the chip period is given by $\frac{1}{W}$. The time-domain signal can also be represented by the frequency spectrum $X(j\omega)$ or the samples $X[n]$, $n = 0, ..., N-1$, which is the discrete Fourier transform (DFT) of $x[n]$. For simplicity, we consider single-antenna systems and do not consider the Doppler shift of the target. The extension to MIMO case and Doppler shift (thus realizing a space-time filtering for mitigating clutters) has been studied by the author in [14]..

Different from most recent studies on JCS, we consider the mitigation of clutters, which is of critical importance in radar sensing. The clutters are signal-dependent and can be characterized by the impulse response h_c (or transfer function H_c) of the undesired reflectors. Therefore, the received signal at the JCS receiver is given by

$$\left\{ \begin{array}{l} y_s[n] = x * (h_t[n] + h_c[n]) + w_s[n] \\ Y(j\omega) = X(j\omega)(H_t(j\omega) + H_c(j\omega)) + W_s(j\omega) \end{array} \right. , \qquad (1)$$

where w_s and W_s are the thermal noise in the time and frequency domains, h_t and h_c are the impulse responses of the target and clutters, and H_t and H_c are the frequency responses of the target and clutter. Then, the goal of the waveform design, for the purpose of sensing, is to mitigate the clutter and enhance the reflection from the desired target.

The channel between the JCS transceiver and the communication receiver is characterized by the impulse response $h_d[n]$ and the corresponding communication channel gain $H_d(j\omega)$ in the frequency domain. Therefore, the received signal at the communication receiver is given by

$$\left\{ \begin{array}{l} y_c[n] = x * h_d[n] + w_c[n] \\ Y_c(j\omega) = X(j\omega)H_d(j\omega) + W_c(j\omega) \end{array} \right. , \qquad (2)$$

where w_c (h_d) and W_c (H_d) are the time- and frequency-domain noise (channel gain) at the communication receiver. Different from the function of sensing, the reflections at all reflectors are taken into account for communications.

B. Performance Metrics

We introduce the performance metrics for the functions of communications and sensing in JCS:

- Data rate for communications: When the detailed modulation scheme is not specified, we consider the channel capacity, where N_0 is the noise power density (DSP) and $P_x(j\omega) = E\left[|X(j\omega)|^2\right]$:

$$C = \int \log_2 \left(1 + \frac{P_x(j\omega)\left|H_d(j\omega)\right|^2}{N_0}\right) d\omega. \quad (3)$$

When detailed modulation is proposed, we can further calculate the data rate of reliable transmission.

- Signal-to-clutter-and-noise ratio (SCNR) for sensing: The SCNR γ characterizing the prominence of the received reflection from the desired target is given by

$$\gamma = \frac{\int P_x(j\omega)\left|H_t(j\omega)\right|^2 d\omega}{\int P_x(j\omega)\left|H_c(j\omega)\right|^2 d\omega + N_0 W}. \quad (4)$$

C. Sensing Waveform Optimization

We follow the methodology proposed in [15] for the waveform synthesis. The goal is to maximize the SCNR. According to the direct band approach proposed in Chapter 3.3 of [15], the solution is given by

$$\tilde{P}_x(j\omega) = \begin{cases} \frac{\sqrt{N_0}(\lambda|H_t(j\omega)| - \sqrt{N_0})}{|H_c(j\omega)|^2}, & \omega \in \Omega_s, \\ 0, & \omega \notin \Omega_s \end{cases}, \quad (5)$$

where $\lambda = \max_\omega \frac{\sqrt{N_0}}{\sqrt{P_x(j\omega)}}$, and Ω_s is the support of the signal power spectrum, namely the frequency band in which the signal power is positive. Then, the optimized waveform for sensing is an N-vector $\mathbf{x}_s$, which is to be refined for communication data modulation and the final transmitted signal $\mathbf{x}$.

IV. Superposition Coding: WAR Case

In this section, we consider the case of WAR in which the the waveform has been optimized corresponding to the environment and the communication receiver has been informed the selection of this waveform. In the context of superposition coding, this means that the communication receiver has been informed the identity of the cloud, which significantly reduces the decoding complexity.

A. Manifold of Optimal Waveforms

We notice that the sensing waveform $\mathbf{x}_s$ obtained from (5) is not unique. The phases of the waveform at different frequencies are not specified, since they are known to the JCS receiver and can be compensated. The optimization of $\mathbf{x}_s$ is focused on the power spectrum, such that the reflection of target is enhanced while the clutters are mitigated. Therefore, due to the ambiguity in the phase, the set of optimal waveforms forms a manifold $[0, 2\pi]^N$, which provides the degrees of freedom for data communications.

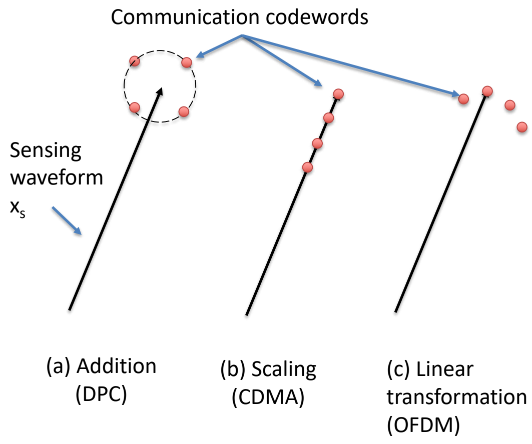

Fig. 2: Schemes of superposition.

B. Superposition Schemes

Given the optimized sensing waveform $\mathbf{x}_s$ which is known to the communication receiver, several possible superposition coding schemes are illustrated in Fig. 2. They correspond to different operations on vectors in linear spaces.

1) Addition (DPC): The final waveform is obtained by adding a vector $\mathbf{x}_c$, determined by both the communication message and the sensing waveform $\mathbf{x}_s$, to $\mathbf{x}_s$, namely

$$\mathbf{x} = \mathbf{x}_s + \mathbf{x}_c. \quad (6)$$

One can apply the celebrated Tomlinson-Harashima coding scheme [16], as the symbol-level DPC, such that the impact of $\mathbf{x}_s$, which is considered as the known interference appended to the communication signal, on $\mathbf{x}_c$ can be significantly eliminated. Therefore, the function of communications is almost decoupled from sensing; meanwhile, the final waveform $\mathbf{x}$ becomes sub-optimal for sensing due to the deviation from $\mathbf{x}_s$ by $\mathbf{x}_c$.

2) Scaling (Spectrum Spreading): The communication message is modulated to a scalar symbol x_c (e.g., a quadratic amplitude modulation (QAM)). Then, the signal sent by the JCS transceiver is given by the scalar multiplication of vectors:

$$\mathbf{x} = x_c \mathbf{x}_s. \quad (7)$$

Such a scheme is very similar to spread spectrum communications such as code-division-multiple-access (CDMA), in which the bandwidth of communication signal is expanded by a spreading code (namely $\mathbf{x}_s$ in the context of JCS).

3) Linear transformation (OFDM): A more flexible operation on a vector is the linear transformation. In this case, the communication is modulated by the selection of a matrix $\mathbf{X}_c$ such that the signal sent by the JCS transceiver is given by

$$\mathbf{x} = \mathbf{X}_c \mathbf{x}_s. \quad (8)$$

The selection of $\mathbf{X}_c$ should make $\mathbf{x}$ within or close to the optimal waveform manifold. In the context of this paper, we set

$$\mathbf{X}_c = \mathbf{F}^{-1}\mathbf{\Lambda}\mathbf{F}, \tag{9}$$

where $\mathbf{F}$ is the $N \times N$ DFT matrix, $\mathbf{F}^{-1}$ is the inverse DFT (IDFT) matrix and $\mathbf{\Lambda}$ is a diagonal matrix $\mathbf{\Lambda} = (\lambda_1, ..., \lambda_N)$. Then, in (8), $\hat{\mathbf{x}} = \mathbf{F}\mathbf{x}_s$ is the N subcarriers of the sensing waveform (in the terminology of orthogonal frequency division multiplexing (OFDM)). The subcarriers are scaled by the elements $\{\lambda_n\}_{n=1,...,N}$. To assure no performance loss for sensing, we adopt phase shift keying (PSK) for subcarriers of significant power, such that the power spectrum of $\mathbf{x}_s$ is unchanged:

$$\lambda_n = \begin{cases} e^{j\theta_n}, & |\hat{x}_n| \geq \bar{x}, \\ 1, & |\hat{x}_n| < \bar{x} \end{cases}, \tag{10}$$

where $\bar{x}$ is a predetermined threshold, and θ_n is the angle selected by the PSK modulation. Then, the IDFT $\mathbf{F}^{-1}\hat{\mathbf{x}}$ converts the frequency domain back to the time domain.

Then, the major challenge is to determine the order of PSK in each subcarrier. We consider the following two schemes:

- Fixed M-PSK: We fix the order of the PSK to M. The advantage is that the communication receiver does not need to know the sensing waveform $\mathbf{x}_s$ for the information of modulation order M; meanwhile, the fixed modulation order M may waste bandwidth at subcarriers with large power while incurring unreliability of transmission at subcarriers with small power allocation.
- Adaptive modulation: To handle the disparity of power allocation in P_x, we can consider the modulation scheme as an adaptive modulation with respect to the power spectrum P_x as well as the channel gains $|H_d(j\omega)|$. To this end, we consider the schemes of $M = 2^m$-PSK, $m = 1, 2, ...$ For each subcarrier n, when using the 2^m-PSK, the error rate of each bit is denoted by $P_e^m(P_x[n])$, which is a function of the power P_x at subcarrier n. The function for PSK error rate can be obtained by numerical integral (p.194, [17]). Then, the estimation is obtained as

$$m^* \quad = \quad \arg\max_m m\left(1 - H\left(P_e^m\left(P_x[n]\right)\right)\right), \tag{11}$$

  which means to select the modulation scheme yielding the maximum reliable throughput. The disadvantage of the adaptive modulation is that, when the sensing waveform $\mathbf{x}_s$ is unknown to the communication receiver, it needs to be estimated by the communication receiver in order to know the modulation order M.

There could be methodologies beyond the above operations in the linear space; e.g., using the lattice for modulation and waveform shaping. We will exploit possible nonlinear methodologies in our future research.

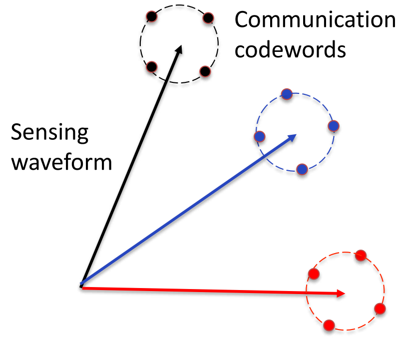

Fig. 3: Waveform diversity and superposition.

V. Superposition Coding: WUR Case

In this section, we consider the case in which the sensing waveform $\mathbf{x}_s$ is unknown to the communication receiver, but confined to a set of possible waveforms (similarly to a codebook in superposition code). Then, the major challenge is that, when the adaptive modulation introduced above is used, the communication receiver needs to identify the selected sensing waveform $\mathbf{x}_s$ (namely the cloud in Fig. 1) and then decode the communication message within the cloud. For simplicity, we assume that the communication channel gain $H_c(j\omega)$ is known to the communication receiver.

As illustrated in Fig. 3, the JCS transceiver prepares a set of sensing waveforms $\{\mathbf{x}_{sk}\}_{k=1,...,K}$ (respect to the corresponding power spectra $\{P_{xk}\}_{k=1,...,K}$) for K distinct sensing scenarios. We assume that the JCS transceiver sends a selected sensing waveform $\mathbf{x}_s$ for L rounds. For the received L consecutive power spectra $\{Q_{xl}\}_{l=1,...,L}$, the communication receiver needs to decode the communication message without the knowledge of $\{\mathbf{x}_{sk}\}_{k=1,...,K}$.

To handle the unknown sensing waveform (or the cloud in the superposition coding), there are two methodologies corresponding to the three superposition schemes:

- Joint Waveform Estimation and Decoding: For the superposition schemes of addition (DPC) and scaling (spread spectrum), the sensing waveform is only perturbed by the communication message. Therefore, we can carry out a joint waveform estimation and decoding, and the final output is the decoded communication message.
- Cascaded Waveform Estimation and Decoding: In the linear transform scheme (the OFDM scheme), the original sensing waveform has been substantially distorted (although the power spectrum is remained). Therefore, the sensing waveform $\mathbf{x}_s$ needs to be estimated first.

The details of the estimation and decoding algorithms are given subsequently.

A. Joint Waveform Estimation and Decoding

For the addition (DPC) and scaling (spread spectrum) schemes, the waveform estimation and decoding are carried out jointly:

- Addition scheme: The estimation of the L consecutive communication symbols $\mathbf{m} = (m_1, ..., m_L)$ and the waveform index k is estimated as

$$(\hat{\mathbf{m}}, \hat{k}) = \arg\min_{\mathbf{m},k} \sum_{l=1}^{L} \|\mathbf{y}_l - \mathbf{x}_{k,m_l}\|^2, \tag{12}$$

  where $\mathbf{x}_{k,m_l}$ is the transmitted DPC waveform determined by $\mathbf{x}_{sk}$ and communication message m_l.
- Scaling scheme: The decoding for the scaling scheme is similar to the decoding of CDMA receivers:

$$(\hat{\mathbf{m}}, \hat{k}) = \arg\min_{\mathbf{m},k} \sum_{l=1}^{L} \|\mathbf{y}_l - x_{m_l}\mathbf{x}_k\|^2. \tag{13}$$

When the number of possible sensing waveforms is not large, the above decoding procedure can be carried out by an exhaustive search over all possible sensing waveforms.

B. Cascaded Waveform Estimation and Decoding

For the above scheme of linear transformation (OFDM), if a fixed PSK order is used, the communication receiver can directly demodulate the PSK symbols over the subcarriers after DFT, regardless of the sensing waveform. However, when the PSK is adaptive, the sensing waveform (or equivalently the signal power spectrum) needs to be obtained, such that the PSK modulation order can be derived. As stated above, the original sensing waveform $\mathbf{x}_s$ is substantially distorted by the communication messages. Therefore, we need to first identify the sensing waveform, and then demodulate the communication symbols according to the derived modulation order.

1) Individual Estimation: For each subcarrier n, one can estimate the modulation order from the corresponding received power spectral density (PSD), normalized over all subcarriers, and then estimate the PSK modulation order M.

- Nonparametric estimation: When the distribution of noise is unknown, we simply calculate the average power over each subcarrier n:

$$\hat{P}[n] = \frac{1}{L} \sum_{l=1}^{L} Q_{xl}[n] - \hat{N}_0, \tag{14}$$

  where $Q_{xl}[n]$ is the measured power over the n-th subcarrier of the l-th waveform, and $\hat{N}_0$ is the estimated noise power.
- Maximum likelihood estimation: A reasonable assumption is that the noise is circular symmetric Gaussian distributed with zero expectation and variance N_0. Then, the noise power over each subcarrier is Rayleigh distributed. Therefore, the estimation of $P[n]$ can be obtained from the maximum likelihood estimation:

$$\begin{aligned} \hat{P}[n] &= \arg\max_k \prod_{l=1}^{L} \prod_{n=1}^{N} \left| |P_{xk}[n]| H_c[n]|^2 - Q_{xl}[n] \right| \\ &\times \quad e^{-\frac{|P_{xk}[n]|H_c[n]|^2 - Q_{xl} n[]|^2}{2N_0}}. \end{aligned} \tag{15}$$

2) Joint Estimation: In practice, the frequency selectivity of the target reflection (as well as the clutters) is not radical. Therefore, a joint estimation over the subcarriers, due to their correlations in the power spectrum, may further improve the performance of sensing waveform estimation, at the cost of more computation. The details are omitted due to the limited space.

VI. Numerical Results

In this section, we introduce the numerical results for the proposed superposition coding of JCS.

A. Simulation Setup

We consider the 2.5GHz band and a bandwidth of 400MHz. For simplicity, we assume that the target is 100m or 50m away from the JCS transceiver. The target is assumed to be a ball with radius of 0.3m or 2m. Since the target size is comparable with the wavelength (0.12m at 2.5GHz), the reflection of the target is frequency-selective. For the four combinations of target distance and radius, we generate four sensing waveforms, using (5). Then, the communication signal is superimposed to the sensing waveform, while the overall signal power is increased by an excessive factor η.

B. WAR Decoding

We first carried the numerical simulation for the WAR case, in which the communication receiver has the information of the sensing waveform. We tested the signal-to-noise ratios (SNRs) ranging from -10dB to 10dB, for the three approaches introduced in this paper (denoted by 'DPC', 'CDMA' and 'OFDM'). The parameter η is set to 0.2. The results are shown in Fig. 4. In the upper part of the figure, we plotted the PSDs of the four waveforms, which are significantly different. In the lower part of the figure, we plotted the channel capacity versus the SNR. Note that the capacity is obtained from estimated demodulation bit error rate and binary symmetric channel capacity. We use QPSK for all the three schemes. We observe that, in the low SNR regime, CDMA outperforms the other two approaches, due to the power gain of spectrum spreading; as the SNR increases, the capacities of both the DPC and OFDM increase and surpass that of CDMA. In summary, for the setup of the numerical simulation, unless the SNR is very low, the linear transformation (OFDM) approach is optimal.

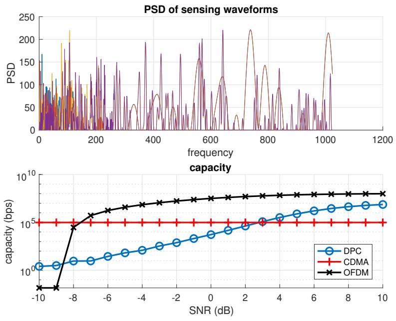

Fig. 4: Capacities when the sensing waveform is known

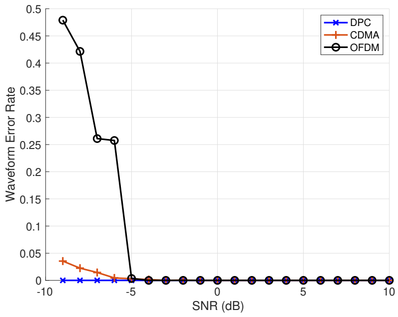

Fig. 5: Waveform detection error probability vs. SNR

C. WUR Decoding

Then, we simulated the case of WUR, in which the communication receiver has to estimate the sensing waveform. Figure 5 shows the sensing waveform detection error rate, when the same sensing waveforms are repeated for $L = 50$ times. The parameter η is set to 0.3. We observe that the DPC scheme achieves almost 0 error rate, while the OFDM has a very bad performance in the low SNR regime. As the SNR increases beyond -5dB, the error rate of OFDM quickly drops to 0. Figure 6 compares the performances in term of capacity for WUR and WAR. In WUR, we assume that the communication receiver uses the most recently estimated sensing waveform. We observe that, except for CDMA scheme in which there is a slight performance degradation for WUR in the low SNR regime, there is no performance distinction between WUR and WAR. This demonstrates the validity of the proposed superposition coding approach.

VII. Conclusions

In this paper, we have proposed the framework of superposition coding for JCS. In particular, we have considered the layered structure in which the waveform for sensing is synthesized first, to which the communication messages are then superimposed. Different operations on vectors in linear space have been considered for the superimposition coding schemes, which correspond to different coding/modulation/spreading schemes in traditional data communications. Numerical simulations have demonstrated the validity of the proposed approaches for JCS.

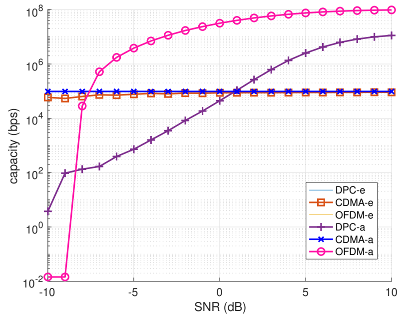

Fig. 6: Performance comparison between WAR and WUR

References

[1] H. Li, “Dirty paper coding for waveform synthesis in integrated sensing and communications: A broadcast channel approach,” in *Proc. IEEE International Conference on Communications (ICC)*, 2022.

[2] ——, “Dual-function multiplexing for waveform design in OFDM-based joint communications and sensing: An Edgeworth Box framework,” in *IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, 2022.

[3] L. Wang, E. Sasoglu, B. Bandemer, and Y. Kim, “A comparison of superposition coding schemes,” in *Proc. of IEEE International Symposium on Information Theory (ISIT)*, 2013.

[4] S. Vanka, S. Srinivasa, Z. Gong, P. Vizi, K. Stamatiou, and M. Haenggi, “Superposition coding strategies: Design and experimental evaluation,” *IEEE Trans. Wireless Commun.*, vol. 11, no. 7, pp. 2628–2639, 2012.

[5] L. Wang, Y. Kim, H. Park, and E. Sasoglu, “Sliding window superposition coding: Two user interference channels,” *IEEE Trans. Inf. Theory*, vol. 66, no. 6, pp. 3293–3316, 2020.

[6] R. Zhang and L. Hanzo, “A unified treatment of superposition coding aided communications: Theory and practice,” *IEEE Commun. Surv. Tutor.*, vol. 13, no. 3, pp. 503–520, 2010.

[7] L. Zheng, M. Lops, Y. C. Eldar, and X. Wang, “Radar and communication co-existence: An overview,” *arXiv:1902.08676*, 2019.

[8] F. Liu, C. Masouros, A. Petropulu, H. Griffiths, and L. Hanzo, “Joint radar and communication design: Applications, state-of-the-art, and the road ahead,” *IEEE Trans. Commun.*, vol. 68, pp. 3834–3862, 2020.

[9] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. Wiley, 2006.

[10] M. H. M. Costa, “Writing on dirty paper,” *IEEE Trans. Inf. Theory*, vol. 29, no. 3, 1983.

[11] G. Caire and S. Shamai, “On the achievable throughput in multiple antenna Gaussian broadcast channel,” *IEEE Trans. Inf. Theory*, vol. 49, no. 7, pp. 1691–1706, 2003.

[12] N. Jindal, S. Vishwanath, and A. Goldsmith, “On the duality of gaussian multiple-access and broadcast channels,” *IEEE Trans. on Info. Theory*, vol. 50, no. 5, pp. 768–783, 2004.

[13] A. Chiriyath, P. Bryan, and D. W. Bliss, “Radar-communication convergence: Coexistence, cooperation and co-design,” *IEEE Trans. on Cognitive Communications and Networking*, vol. 3, pp. 1–7, 2017.

[14] H. Li, “Waveform synthesis for MIMO joint communications and sensing with clutters-part I: Space-time-frequency filtering,” in *Proc. IEEE International Conference on Communications (ICC)*, 2022.

[15] U. Pillai, K. Y. Li, I. Selesnick, and B. Himed, *Waveform Diversity: Theory and Applications*. McGraw Hill, 2011.

[16] D. Tse and P. Vishwanath, *Fundamentals of Wireless Communication*. Cambridge, 2005.

[17] J. G. Proakis and M. Salehi, *Digital Communications*. McGraw Hill, 2007.