Impact of Learning a Subgoal-Directed Problem-Solving Strategy Within an Intelligent Logic Tutor

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Abstract. Humans adopt various problem-solving strategies depending on their mastery level, problem type, and complexity. Many of these problem-solving strategies have been integrated within intelligent problem-solvers to solve structured and complex problems efficiently. One such strategy is the means-ends analysis which involves comparing the goal and the givens of a problem and iteratively setting up subgoal(s) at each step until the subgoal(s) are straightforward to derive from the givens. However, little is known about the impact of explicitly teaching novices such a strategy for structured problem-solving with tutors. In this study, we teach novices a subgoal-directed problem-solving strategy inspired by means-ends analysis using a problem-based training intervention within an intelligent logic-proof tutor. As we analyzed students' performance and problem-solving approaches after training, we observed that the students who learned the strategy used it more when solving new problems, constructed optimal logic proofs, and outperformed those who did not learn the strategy.

Keywords: Means-ends Analysis · Subgoal · Problem Solving · Intelligent Tutor

1 Introduction

The existing literature frequently mentions three problem-solving strategies in the domain of structured problem-solving: 1) Forward strategy - starts from the givens of a problem and moves towards the goal by applying valid rules and actions in the problem domain [1], 2) Backward strategy - starts from the goal and at each step, the goal is refined to a new goal (also referred to as subgoal) until the givens are reached [1], and 3) Means-ends analysis [12] - carries out problem solving as a search for subgoals at each step to recursively reduce the distance between the goal and givens until the subgoal can be directly derived from the givens [2]. By definition, backward strategy and means-ends analysis involve subgoaling, where subgoaling can be referred to as refining the given goal to a new goal (or subgoal) to reduce the distance between the givens and the goal [8,11,19].

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Researchers have often stated means-ends analysis to be closely aligned with the natural human strategy for complex problem-solving where they work forwards while keeping backward subgoals (identified by comparing the goal and the givens) in mind [12,16]. The comparison carried out between the goal and givens reduces the search space of possible next steps and the type of comparison usually depends on the nature of the problem. Also, the comparison is set out to select the best action or subgoal to optimally reduce the distance between the goal and the givens [6,16]. This general problem-solving strategy rarely leads to dead-ends when there is a specific goal [16]. Thus, researchers have integrated Means-ends analysis in automated problem-solvers for efficient problem-solving (for example, in *General Problem Solver* [12]). Prior research suggests that although novices try to adopt means-ends analysis more due to their low prior knowledge (to reduce search space of possible next steps) [17], experts might be more able to use this strategy than novices [14]. However, methods to train and motivate novices to learn/use this strategy or the impact of learning this strategy on their problem-solving skills have been rarely explored.

Thus, in this study, we integrated problems that enforce the use of a mixed problem-solving strategy (MS) inspired by means-ends analysis within the training session of an intelligent logic-proof tutor. In the MS problems, the first few steps must be subgoaling steps carried out using the backward strategy (BS). After the subgoaling steps, the subgoals may be achieved using forward strategy (FS). To analyze the impact of learning MS, we implemented two training conditions within the tutor: 1) Control: this group was not taught MS, and 2) Treatment: this group was taught MS using examples and practice problems. We evaluated the impact of the mixed strategy training on the basis of the following research questions:

RQ1 (Students' Experience): How difficult is the mixed strategy training for students when integrated with an intelligent tutor? [Note: This question is important since novices often use intelligent tutors in the absence of human tutors. Thus, the difficulty level of a training intervention in a tutor should not be too high so that students can persist and learn successfully].
RQ2 (Impact on Performance): How does learning mixed strategy impact students' performance in new problems?
RQ3 (Impact on Problem-solving approach and skills): How does learning mixed strategy impact students' problem-solving and subgoaling approach and skills?

2 Related Work

The means-ends analysis (MEA) was first introduced by Newell and Simon [12] in an AI problem solver called *General Problem Solver* (GPS). They emphasized that MEA is actually a simulation of natural human thought processes. MEA involves calculating the distance between the goal and givens of a problem at each step using a function or method appropriate for the problem type. Then, the best action that will generate a subgoal to optimally reduce the goal-givens distance is executed. Since the 50s, this method has been used in AI problem solvers for efficient problem-solving. For example, GPS (general problem solver) [12], Prodigy (math problem solver) [20], Multilevel Flow Modeling or MFM (industrial process planner) [9] etc.

Although being a very well-known efficient problem-solving strategy, little research is found on the impact of explicitly MEA learning on students' problem-solving skills. In a recent study, Permatasari and Jauhariyah [13] found that incorporating MEA in problem-solving-based physics learning improved students' learning. Researchers have also recently found this strategy to be effective in improving students' critical thinking skills (skills to analyze and execute decisions) for mathematical [4,7,18] and geometrical [21] problem-solving. However, these results need to be further verified in other domains. Specifically, the integration of problems or processes to have students learn such strategy needs to be explored within problem-based intelligent tutors.

3 Method

3.1 Deep Thought (DT), the Intelligent Logic Tutor

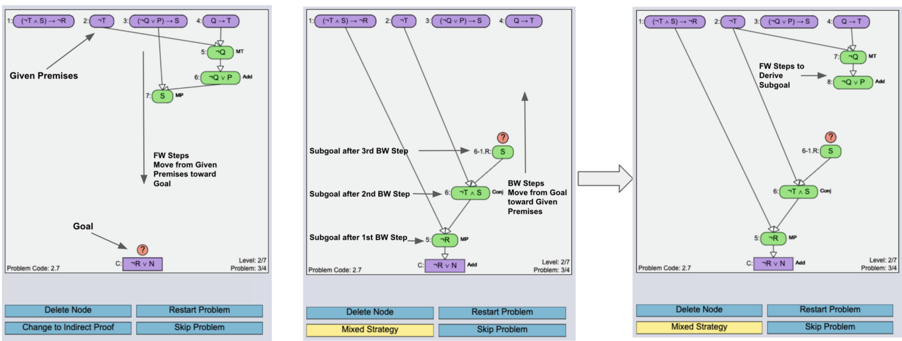
We conducted this study using Deep Thought or DT, an intelligent logic tutor, that teaches students logic-proof construction [10]. Generally, each problem within DT is either a worked example (WE) of logic-proof construction constructed by the tutor or a problem-solving (PS) problem needed to be solved by the students. In each problem, the given premises and goal conclusion are shown as visual nodes Fig. 1a. Valid logic rules from a given rule palette need to be iteratively applied on the nodes to generate new propositions or nodes to complete the proof. DT is organized into one pretest level (level 1), 5 training levels (levels 2–6), and one posttest level (level 7). In the pretest level, students are first shown two sample logic-proof problems (1.1–1.2) to acquaint them with the different features of DT. Then, they solve two pretest problems (1.3–1.4). After the pretest, students go through 5 training levels each containing 4 logic-proof problems (x.1–x.4). In the first three problems in each training level (x.1–x.3), the tutor offers on-demand step-level hints if the problem is of type PS. The last problem in each training level (x.4) is a PS that students need to solve independently without any tutor support. These are called training-level tests. After training, students enter a posttest level containing 6 posttest problems (7.1–7.6). Each student receives a score for each of the pretest, training-level test, and posttest problems. The score is a function of problem-solving time, step count (count of derived nodes), and logic-rule application accuracy and is scaled between 0 and 100. Solutions constructed in less time with fewer steps and fewer incorrect rule applications get higher scores.

3.2 Experiment Design

Problem Types: To facilitate our study, we used 4 types of logic-proof problems implemented within training levels of DT. The problem types are:

1. Problem Solving (PS): These are the default logic-proof construction problems in DT where students need to construct proofs themselves using forward/FW (derivations that move from the given premises towards the goal Fig. 1a), backward/BW (derivations that move from the goal towards given premises Fig. 1b), or both strategies without the tutor requiring them to use a specific strategy.
2. Worked Examples(WE): These are default worked examples available in DT where the tutor demonstrates logic-proof construction using only forward (FW) derivations.
3. Mixed-strategy Problem Solving(MPS): MPSs require the student to construct logic proofs using mixed strategy. Most DT problems require 5–10 steps to solve. If the subgoals derived in the first 2–3 BW steps are correct, the rest of the proof becomes straightforward. Thus, in MPS, students must derive the first three steps backward Fig. 1b and during this time, forward derivations are disabled. Then, they can use the forward strategy if they like Fig. 1c. The purpose of having the students start with backward steps is to explicitly motivate and involve them in subgoaling as in means-ends analysis.
4. Mixed-strategy Worked Examples(MWE): These are worked examples showing the mixed problem-solving strategy. The tutor carries out the first 2–3 steps backward refining the goal into subgoals which are followed by forward steps to derive the subgoal(s).

PS and MPS are visually depicted in Fig. 1. The interface for WE and MWE is the same as PS/MPS. The only difference is that WEs and MWEs are constructed by the tutor step by step as the students click on a 'Next Step' button.



a) PS (can Start with FW or BW Step)

b) MPS (must Start with BW Steps)

c) MPS (BW Steps might be Followed by FW Steps)

Fig. 1. PS (a) and MPS (b and c) Interface in DT.

Training Conditions: Using the 4 types of problems, we implemented two training conditions as described below:

1. Control(C): Students assigned to this condition received only PS and WE.
2. Treatment(T): Students assigned to this condition received MWE and MPS in addition to PS and WE.

The problem organization within the two training conditions is shown in Fig. 2. Notice that the treatment group students receive MWEs in each training level (up to level 5) to remind them of the mixed problem-solving strategy [blue squares in Fig. 2]. However, they receive MPSs only in the first half of the training [green squares in Fig. 2]. This was done to ensure that the students were given an opportunity to explore the strategy independently in the second half of the training. In a prior study [15], such organization of training treatment was found to be effective in having students learn a strictly BW strategy. Also, note that our problem organization ensures that students from both training conditions get an equal amount of examples (WE/MWE) and practice (PS/MPS) during training.

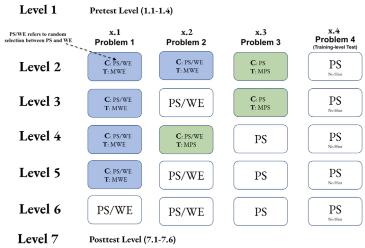


Fig. 2. Problem Organization in the DT Training Levels for the 2 Training groups. (Color figure online)

Data Collection: We deployed DT with the two training conditions in an undergraduate logic course offered at a public research university in the USA. The students taking the course were required to submit a DT assignment that involved completing all levels of DT to receive full credit. Each student after completing the pretest was automatically assigned to one of the training conditions. Our assignment algorithm ensures equal distribution of students between the conditions. It also ensures that the pretest scores of the training conditions come from a similar distribution. At the end of the experiment, 50 students in the C group and 45 students in the T group completed the tutor. We collected students' pretest, training-level test, and posttest scores. Additionally, we collected the time-series log data from DT that detail each step (derivation/deletion of nodes, direction, and time of derivation) of students while they construct proofs within DT. We used this data to carry out statistical and graph-mining-based proof-construction approach analysis to answer our research questions.

4 RQ1 (Students' Experience): Difficulties Across MPS and PS Problems

To understand students' experience with mixed strategy learning training, we compared the difficulty level of training MPS problems against training PS problems in terms of the time students took (i.e. the problem-solving time) to solve each type of problem. The reason for considering PS as a baseline is that PSs are simple proof-construction problems where students are not restricted to using a specific strategy unlike in MPSs. In a prior study [15], we observed that being restrictive about using a strategy could lead to significant difficulties for students. However, in MPS, the strict restriction is applied for only the first 3 steps where students must carry out subgoaling using BW steps which should limit the difficulties associated with MPS. Also, note that we did not consider MWEs and WEs in this analysis, since those are solved by the tutor.

To compare the difficulty level of MPS and PS, we carried out a mixed-effect regression analysis with problem-type (all MPS against all PS problem-solving instances) as fixed-effect, problem ID as random-effect (to eliminate the impact of differences in problems), and problem-solving time as the dependent variable. The analysis gave a p-value of < 0.001 [n(PS) = 410, n(MPS) = 135, problem-solving time(PS) = 14.4 (7.4) min, problem-solving time(MPS) = 18.0 (20.4) min]. The p-value indicates a significant difference in the difficulty levels of MPS and PS problems where MPS seemed to be more difficult in terms of problem-solving time. However, further investigation showed that this difference was introduced by only 25% (above 75th percentile) MPS problem-solving instances (n = 34, mean problem-solving time for these instances = 41.8 min). The rest of the 75% (up to 75th percentile) MPS instances (n = 101, mean problem-solving time for these instances = 10.0 min) had no significant differences from PS problems in terms of problem-solving time as per a Mann-Whitney U test. Recall that during training, the T group students solved three training MPS problems. As we further analyzed each of these problems separately, we observed that 95% of the 34 MPS instances with significantly higher problem-solving time than PS occurred in the first MPS problem that the students received in training level 2. Problem-solving time for the MPS problems in training levels 3 and 4 did not have any significant differences from the problem-solving time of the PS instances. Additionally, we identified three training problems for which we found both PS (n = 67) and MPS (n = 71) problem-solving instances in the collected data. However, interestingly, MPS instances had marginally lower step counts than the PS instances according to Mann-Whitney U test [Step Count(PS, MPS) = 10.1, 8.9, p = 0.08] which indicated adopting mixed-strategy possibly led students to more efficient proof construction.

Overall, our analysis results indicate that in most cases MPS problems are as easy as PS problems. However, students might require more time to solve the first MPS they receive during training, possibly to figure out how to address the strict requirement of using the mixed strategy. However, solving a problem using mixed strategy may help students to solve logic-proof construction problems with higher efficiency.

5 RQ2: Students' Performance After Training

To understand the impact of learning mixed problem-solving strategy on students' performance, we compared students' test scores across the control (C) and treatment (T) groups. We performed two mixed-effect regression analyses: one for the training-level test (4th problem in each training level: 2.4–6.4) and one for the posttest (problems in level 7: 7.1–7.6) problems to understand improvement over the period of training and after training respectively. Note that these problems are solved by students without any tutor help and thus, are good candidates for performance analysis. In both of the analyses, we incorporated training conditions (control (C)/treatment (T)) as the fixed effect, problem IDs as the random effect (to exclude the impact of differences in the problems), and problem scores as the dependent variable. The analysis for the training-level test problems did not show any significant differences across the two training groups: n(50,45), mean(std)[C, T] = 65.11(24.08), 65.14(23.36), $p = 0.99$. However, the analysis for the posttest problems showed that overall the treatment group significantly outperformed the control group: n(50, 45), mean(std) [C, T] = 70.42(22.88), 74.10(21.51), $p = 0.04$ [$p < 0.05$ indicates significant difference]. As we analyzed students' scores for each of the problems separately using Mann-Whitney U tests, we observed for most of the problems T group students had higher scores than the C group. This trend can be visualized in Fig. 3a, where the curve for the T group (solid orange line) mostly lies above the curve for the C group (dashed blue line) starting from problem 5.4. The p-values in the figure show the problems with significantly higher averages for the T group. Also, note that we did not find any problem where the T group received significantly lower scores than the C group.

To further investigate the reason for the difference in problem scores across the training groups, we analyzed their step counts, problem-solving time, and logic rule application accuracy in the training-level test and posttest problems. We did not find any significant differences in the rule application accuracies and problem-solving time (Fig. 3c) across the training groups. However, in a mixed-effect regression analysis similar to the one for problem scores, we found significant differences in step counts across C and T [Step Count (C, T) = 13.4, 10.1, $p = 0.031$]. Also, the T groups had lower step counts than the C group in most of the problems [notice the orange curve for the T group in Fig. 3b starting from problem 5.4].

Overall, the results indicate that learning mixed problem-solving strategy helped students to construct logic-proofs with fewer steps which led to higher scores. In RQ3, we analyzed students' proof construction approach to identify how learning mixed strategy helped to achieve this efficiency.

6 RQ3: Proof-Construction and Subgoaling Approach/Skills After Training

To investigate students' proof-construction and subgoaling approach and skills across the training conditions, we modeled students' proof-construction attempts using approach maps [3] for each of the training-level test and posttest problems that they solved independently without the tutor requiring them to use a specific strategy. Approach maps are high-level graphical representations of students'

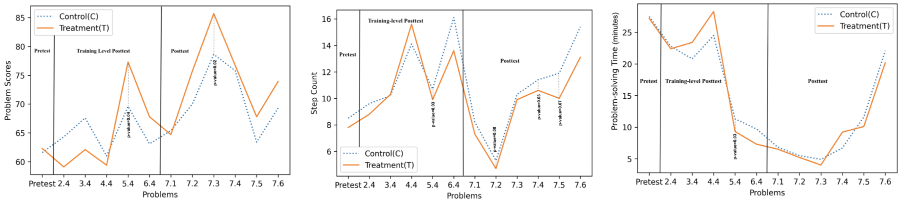


Fig. 3. Problem Score, Step Count, and Problem-solving Time across the Control (C) and Treatment (T) groups over the period of pretest, training, and posttest [p-values < 0.05 (obtained from Mann-Whitney U tests) indicates a significant difference].

proof construction attempts for a given problem that show the propositions they commonly derived during proof construction [Fig. 4]. To generate the approach map for a problem, students' state transitions during the construction of a proof for that problem are presented using a graph called an interaction network. Here, a state contains all nodes or propositions (lexicographically ordered) present on DT interface at a moment during proof construction. Students move from state to state during proof construction by deriving/deleting nodes, i.e. via steps. The Girvan-Newman clustering algorithm [5] is applied on this interaction network to identify clusters of closely connected states. Later, from those clusters, we extracted effective propositions that contributed to students' final solutions. As shown in Fig. 4, finally, approach maps become a graph where the start state (containing only given premises) is connected to the goal via clusters of effective propositions. Each path from start to goal via the clusters is a student approach (A1, A2, etc.). The propositions written in purple were derived using BW derivation (i.e. subgoaling) by students who adopted mixed strategy. Note that for simplicity, we do not detail all effective or unnecessary propositions[1] derived by all students in the approach maps. We only show the commonly derived effective nodes that are sufficient to describe differences in student approaches. However, we recorded counts of all effective/unnecessary propositions derived by each student since both increase step counts. In the next sections, we explain differences in student approaches across the training conditions using approach maps and statistically analyze proof derivation efficiency (using Mann-Whitney U tests). Lower time, fewer unnecessary, and effective propositions indicate higher efficiency. Note that this method to analyze student approaches can be adopted for structured problems from any domain where states and transitions during problem solving can be defined definitely. Effective or unnecessary steps can be identified using the differences between the final problem state and earlier states.

6.1 Student Approaches in Training-Level Test Problems

In the training-level test problems (2.4–6.4), up to 49% of the T group students used mixed strategy, whereas this percentage for C group students is only 24%.

[1] Propositions derived by the students but were not part of their final solutions.

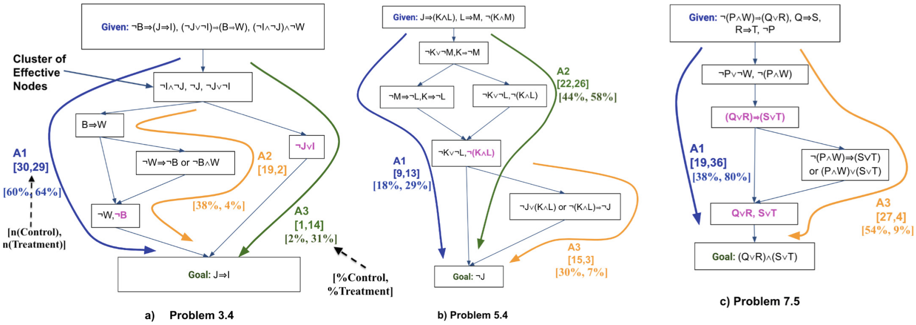


Fig. 4. Approach Maps for Training-level Test Problems a) 3.4, b) 5.4, and c) Posttest Problem 7.5.

The student counts adopting mixed strategy for each of the problems are shown in Fig. 5a. We also observed that, in these problems, T group showed sporadic signs of improved skills (marginally shorter solutions/fewer unnecessary propositions). For example, in problem 2.4, the proof length was marginally lower for the T group than the C group [mean(std) = C: 5.8(5.5), T:4.5(5.4), test statistic = U(50, 45) = 1213.0, p-value = 0.09]. In problem 3.4, T group students had significantly fewer unnecessary propositions in their solution attempt than C students [C: 7.3(10.2), T:5.2(4.7), U(50, 45) = 1398.5, 0.003]. Additionally, T students who adopted mixed strategy or T_{MS} (n(T_{MS}) = 17 in 3.4) had significantly shorter solutions than those of C and marginally shorter solutions than those of T students who did not adopt the strategy [$C = 8.1(2.0)$, ${T_{FS}}^2 = 7.2(3.4)$, $T_{MS} = 5.3(2.8)$, $U_{T_{MS}<C}(50, 17) = 534.0$, 0.01; $U_{T_{MS}<T_{FS}}(28, 17) = 281.5$, 0.08]. The approach map for problem 3.4 is shown in Fig. 4a. As shown in the figure, A3 is the shortest solution for this problem. 13 out of 17 of the T_{MS} students derived optimal subgoal $\neg J \vee I$ using BW derivation which led them to the shortest solution A3. We observed a similar pattern in problem 4.4 as well. From these statistics, we concluded that by the first three training levels, not all T group students achieved an equal level of mastery in using mixed strategy. However, those who *were* able to learn and adopt the strategy (i.e. the T_{MS} students) constructed proofs more efficiently.

In problem 5.4, only 4.4% of T group students adopted mixed strategy. However, overall T group students had significantly smaller solution length than C group students [C: 8.02(1.18), T:7.0(2.3), U(49,45) = 1224.5, 0.007]. The approach map for 5.4 is shown in Fig. 4b. As shown in the figure, A1 and A2 are the optimal solutions for this problem where $\neg(K \wedge L)$ is a subgoal. Although only 2 students from the T group explicitly used mixed strategy and derived $\neg(K \wedge L)$ as a subgoal using BW derivation at the beginning of their proof construction attempt, almost all T students were observed to follow one of the 2 optimal approaches (A1/A2). On the other hand, 15 C group students adopted a lengthier approach, A3, while solving this problem. Moreover, C-group students derived

[2] Treatment group students who only used forward strategy.

significantly more unnecessary propositions [C: 4.1(2.2), T:2.7(1.3), U(49,45) = 1126.5, 0.01]. In problem 6.4, overall, T-group students showed a similar efficiency by identifying subgoals that led to shorter proofs. These results indicate that by the time the students finished the training levels (i.e. level 6), T students were more skilled in proof construction and identifying better subgoals that will lead to shorter proofs. However, we did not observe all T students explicitly use the mixed strategy. Possibly, the T group students identified subgoals using mixed strategy but carried out the steps in forward direction in the tutor (i.e. BW strategy in mind for *implicit subgoaling*, which was not reflected by their derivations in the tutor).

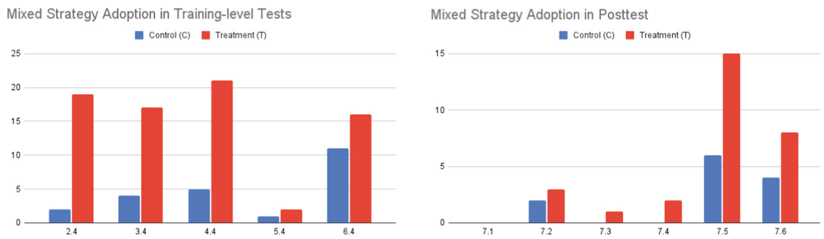


Fig. 5. Students across the control (C) and Treatment (T) groups Adopting Mixed Strategy in a) training level posttest problems, and b) posttest problems.

6.2 Student Approaches in Posttest Problems

In the first 4 posttest problems (7.1–7.4), we observed only a few students from the T and C groups demonstrated the adoption of mixed strategy. However, in problems 7.5–7.6, more students explicitly adopted the mixed strategy (17–33% T students and 8–12% C students). Note that the posttest problems are organized in increasing order of difficulty and Fig. 5b indicates that more students adopted subgoaling-based mixed strategy explicitly in the last two, harder problems.

Problem 7.1 is a trivial problem with a 3-step shortest solution and we did not find much difference for this problem across the training groups. However, T students constructed significantly shorter proofs for problems 7.2–7.5 [For 7.2, C:3.9(1.6), T:3.3(1.3), U(50, 45) = 1315.5, 0.04. For 7.3, C: 8.0(1.4), T:7.4(1.1), U(50,45) = 1183.5, 0.03. For 7.4, C: 8.7(2.0), T:7.5(1.9), U(50, 45) = 1415.0, 0.001. For 7.5, C: 6.4(2.7), T:5.6(2.6), U(50,45) = 1355.5, 0.04]. As a sample, we showed the approach map of Problem 7.5 in Fig. 4c. For this problem, the optimal solution is A1 as shown in the figure. T students who used mixed strategy derived $Q \vee R$ and $S \vee T$ as the first set of subgoals (resulted from their first BW derivation). They derived $(Q \vee R) \Rightarrow (S \vee T)$ as the next subgoal. These subgoaling steps led them to optimal solution A1. The T students who did not demonstrate explicit subgoaling also constructed the same proof using only FW derivations. However, above 50% C group students constructed a longer proof (approach A3 in Fig. 4c). Additionally, in problem 7.6, the T group derived significantly fewer unnecessary propositions than the C group [C: 8.6(5.9), T:5.9(3.7), U(50,45) = 1167.5, 0.005]. The results from our approach map analysis showed that the T-group students who learned mixed strategy with MWE+MPS during training achieved higher efficiency in proof construction and subgoaling. They possibly learned to compare the goal and givens effectively to derive better subgoals leading to shorter proofs, and fewer unnecessary propositions. However, note that our statistical tests did not detect a similar efficiency in the few C students who demonstrated mixed strategy but were not trained beforehand unlike T students.

7 Discussion

Overall, the results of our study showed that learning the mixed strategy with MWE+MPS during training posed a similar level of difficulty in most cases as learning only the forward strategy. Additionally, learning and using this strategy to construct proofs via subgoaling could be beneficial for students to help improve their proof-construction efficiency (i.e. shorter proofs with subgoaling). However, our observations indicate that explicit adoption of mixed strategy can depend on the complexity of the problem or the mastery level of students in using mixed strategy. Some T-group students were observed to explicitly use the strategy in the first few training-level tests (2.4–3.4) and showed improved performance early. On the contrary, some of the students did not use mixed strategy explicitly at all, or only in harder problems, but demonstrated improved performance over time (possibly by using an implicit subgoaling strategy).

8 Conclusion and Future Work

The contributions of this paper are 1) an efficient training strategy for mixed strategy learning within an intelligent tutor using problems that involves subgoaling first and then deriving the subgoal as in means-ends analysis [this training strategy can be adopted within any tutor containing structured problems with specific goals that can be refined into subgoals using actions from a finite set], and 2) an evaluation of the impact of learning this subgoal-directed strategy that showed that it helped to improve students' subgoaling and proof construction skills. Students who learned the strategy achieved higher scores by deriving better subgoals and constructing shorter proofs. This efficiency can be compared to what automated problem-solvers achieve by exploiting means-ends analysis to reduce the search space of possible next steps and select the most efficient one. One limitation of this study is that we observed many treatment group students (trained with MWE+MPS) demonstrated improved performance without explicitly using the strategy which we identified as potential cases of implicit use. Thus, in future studies, students learning the mixed strategy should be interviewed to confirm how they used mixed strategy to identify better subgoals and construct better proofs.

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