An Algorithm for the Assignment Game Beyond Additive Valuations

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The assignment game, introduced by Shapley and Shubik [1971], is a classic model for two-sided matching markets between buyers and sellers. In the original assignment game, it is assumed that payments lead to transferable utility and that buyers have unit-demand valuations for the items being sold. There has since been substantial work studying various extensions of the assignment game. The first main area of extension is to imperfectly transferable utility, which is when frictions, taxes, or fees impede the transfer of money between agents. The second is with more complex valuation functions, in particular gross substitutes valuations, which describe substitutable goods. Multiple efficient algorithms have been proposed for computing a competitive equilibrium, the standard solution concept in assignment games, in each of these two settings. However, these lines of work have been mostly independent, with no algorithmic results combining the two.

Our main result is an efficient algorithm for computing competitive equilibria in a setting encompassing both those generalizations. We assume that sellers have multiple copies of each items. A buyer i's quasi-linear utility is given by their gross substitute valuation for the bundle S of items they are assigned to, minus the sum of the payments $q_{ij}(p_j)$ for each item $j \in S$, where p_j is the price of item j and q_{ij} is piecewise linear, strictly increasing. Our algorithm combines procedures for matroid intersection problems with augmenting forest techniques from matching theory.

We also show that in a mild generalization of our model without quasilinear utilities, computing a competitive equilibrium is NP-hard.

The full version of the paper can be found at https://arxiv.org/abs/2406.13620.

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