TAKING STRUCTURE SERIOUSLY: WHAT DO TEACHERS NOTICE ABOUT INVARIANCE IN FRACTIONS

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Keywords: Rational Numbers & Proportional Reasoning, Teacher Knowledge, Mathematical Knowledge for Teaching

Objective

For decades, there has been a call in mathematics teaching and learning to focus on mathematical structures. This was a pillar of "new math" (Phillips, 2015) and part of the way the National Council of Teachers of Mathematics (NCTM) defined mathematics in both sets of standards (NCTM, 1989, 2000). It is also a Standard for Mathematical Practice in the Common Core (CCSSI, 2010). However, little research considers teachers' knowledge of structural relationships. Certainly, it has been posited that teachers who understand mathematical structure will be more capable of supporting students in learning about them (Mason et al., 2009). This conjecture has neither been tested nor explored in terms of the extent to which teachers are aware of structural relationships.

Here, we report a pilot study aimed at understanding how practicing teachers understand invariance as it relates to fractions. While invariance is often considered for proportional reasoning situations, it is rarely a focus of discussion or instruction for fractions. Thus, we wondered how teachers might make sense of novel tasks that asked them to attend to aspects of invariance with fractions.

Response to Issue

This poster reports an exploratory study undertaken as a pilot of an interview instrument. We interviewed a convenience sample with five current (Kevin and Hunter) or recently retired (Laura, Beth, and Wendy) middle school mathematics teachers. (All names are pseudonyms)

We considered how the teachers responded to a single item contrasting fractions and proportions by considering how two different drawings (one area model and one with shaded dots) might show that 2/3 is equivalent to 8/12. One of the main take-aways was that these participants did not attend to the relationship of the numerator to the denominator in their sensemaking about equivalence. Interestingly, the teachers also had different conceptions of the referent unit as they interpreted the drawings, as well.

Given that this was only one task considered by five participants, we are careful not to overstep our claims. More questions need to be asked of more teachers to understand this issue. However, the work reported here supports our assertion that teachers may not attend to structural aspects of fraction situations when solving for themselves. Further, it suggests that research is warranted in this area.

Acknowledgments

This material is based upon work supported by the National Science Foundation under Grant No. 2201125. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

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