Where Is the Referent Unit When Teachers Attending Fraction Multiplication Problems?

Kun Wang, Rethink Learning Labs, kun.wang@rethinklearning.com
Chandra Orrill, Rethink Learning Labs, chandra.orrill@rethinklearning.com
Rachael Brown, Penn State Abington, reb37@psu.edu

Abstract: In this study, we investigated teachers' flexibility when they attend identifying referent units on their understanding of fractions and fraction operations. We conducted five pilot interviews with middle school math teachers and analyzed the data to identify different levels of mastery in identifying referent units. The results showed that teachers attended to referent units differently, and their level of mastery was related to how they viewed the whole through their thinking.

Background

Many studies have investigated the understanding of fractions among both in-service and pre-service teachers (Lee et al., 2011; Orrill et al., 2008; Orrill & Brown, 2012; Copur-Gencturk & Ölmez, 2022; Izsák et al., 2019). Given that fractions play an important role for upper elementary and middle school grades, including topics such as using visual models to understand fraction multiplication and division, a robust teaching of this content is necessary. Recent studies have reported that teachers have constraints on comprehension of fraction operations and on facilitating fractions across different representations (e.g., Orrill et al., 2008; Orrill & Brown, 2012).

Recent research has noted teachers' struggle with referent unit (e.g., Philipp & Hawthorne, 2015). Referent unit is different from unit in a way that referent unit may change according to the situational needs. Majority research demonstrated that teachers could solve fraction multiplication/division problems by algorithm, but more importantly, they need to make sense of it both for themselves and thus for their students. Specifically, in the case of referent unit, they need to know how to facilitate the fractions operations in different visual representations, so that they can help with future students conceptual understanding.

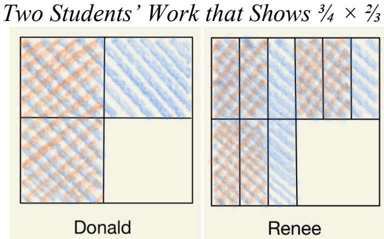
Perspective

According to Shulman (1986), it requires not only teachers' content knowledge but also their pedagogical content knowledge, since PCK includes knowledge of students and making sense of different representations to students. To explore teachers' knowledge through this lens, we are driven by this research question: to what extent teachers attend to referent unit when interpreting drawn representation?

Methods

In this study, five middle school mathematics teachers, two of whom (Kevin and Hunter) were currently practicing and three who were recently retired (Laura, Beth, and Wendy), were interviewed over Zoom. They are all pseudonymous. They were asked to work on tasks on Jamboard, containing 19 pages of questions related to nine different situations, with one to two questions per screen. The focus of the study was on only one situation with area model representation, form where there are four different students' work. Before they responded, they were informed to consider the fraction multiplication problem $\frac{3}{4} \times \frac{2}{3}$, and students' work was to solve it using an area model, students colored in the model in different ways as shown below (see Figure 1). Following the students' drawing, we asked them to solve task one with two separate questions: Where do you see $\frac{3}{4}$ in Donald's drawing? How about 2/3? And what about Renee? The teachers were asked to identify the different wholes in the representation and their responses were analyzed qualitatively by three researchers analyzing independently.

Figure 1
Two Students' Work that Shows $\frac{3}{4} \times \frac{2}{3}$



Results

All five teachers were able to identify that the blue-shaded parts in both Donald's and Renee's drawings represented 3/4 of the whole. However, only two teachers (Kevin and Laura) were able to identify that the two orange parts in Donald's drawing represented 2/3 of the three blue parts.

Kevin explicitly noted "Blue = 3/4 of whole", while Laura explained that "the two-thirds are the orange lines going this way, and the two-thirds doesn't include this one [the top right blue part], so it is 2/3 of the blue lines." The other three teachers expressed confusion when trying to identify 2/3 in Donald's work, with Wendy and Hunter wanting to partition the entire rectangle into fourths or into thirds. Beth gave an alternative way of partitioning the entire rectangle into twelfths, but she was trying to find both 3/4 and 2/3 in the original rectangle, rather than attending to the whole to which each fraction referred. In contrast, Kevin and Laura were smoothly and explicitly transitioning from the entire rectangle as a whole into the blue-shaded part as a whole. Thus, it was concluded that only two of the five teachers showed some flexibility of referent unit as they successfully identified what the unit is based on the situational needs.

However, with Renee's work is very similar to Donald's work but just with more partitioning on each of the blue box, teachers presented seemingly comfortable with identifying 2/3 on Renee's work. For example, Kevin said "Yeah, I can see for each of the squares that were shaded in blue, she cut them in the third, and then shaded in two thirds of those pieces. So each of the two orange shaded in represent two-thirds of the three-fourths parts." Wendy, who had challenge with identifying 2/3 in Donald's work, easily found out the "new whole" for Renee's work with indicating "the blue is the 9/12, so [it is] 3/4. Then the two-thirds, if you counted just the blues, because that is your new whole…[counting] so they did 6 out of 9, which is two-thirds." Similarly, Hunter and Beth adopted the same way of seeing 2/3 in Renee's work.

Discussion

This study's finding is consistent with previous research about the issue as the teacher not attending to two levels of units: the whole rectangle and the new "whole" comprised of the three blue boxes. The study also highlights the importance of considering whether teachers attended to referent units in a drawn representation situated in a fraction multiplication problem. This study provides valuable insights that can help in designing effective teaching strategies that promote a deeper understanding of mathematical concepts. Moreover, identifying the different of extend of how they attend the content knowledge (Shulman, 1986) will inform future PD development.

Interestingly, researchers had anticipated that Donald's drawing would be relatively clear for teachers. However, it turned out that Renee's work was easier for teachers to understand. Further analysis is needed to determine the extent to which the teachers attended to the shifting unit in the other drawn representations.

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Acknowledgments

This material is based upon work supported by the National Science Foundation under Grants No. DRL-1125621, DRL-1621290, and 2201125. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

