MATHEMATICAL KNOWLEDGE FOR TEACHING COLLEGE ALGEBRA AT COMMUNITY COLLEGES

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We report on the relationship between community college instructors' performance on an instrument measuring mathematical knowledge for teaching college algebra with their teaching experience and their frequency of using specific tasks of teaching. The findings support the argument that the instrument assesses critical knowledge for teaching college algebra. We propose further work based on these findings.

Studying the relationship between teacher knowledge and what teachers do in their classroom has a long history in mathematics education. Successful efforts to establish this relationship range from small-scale qualitative approaches in multiple settings to large-scale investigations, mainly in K-8 education, which developed instruments to measure both teachers' knowledge and their work in the classroom. One of the major limitations to replicate such findings in other contexts is the lack of reliable instruments that can measure such knowledge, especially in community college settings. Consequently, our project developed an instrument to assess mathematical knowledge for teaching college algebra among instructors who teach at community colleges in the United States. Community colleges are non-university tertiary institutions that enrol 38% of all undergraduate students in the country (American Association of Community Colleges, 2023) and offer opportunities for remediation, vocational training, worker retraining, general education, and transfer to four-year undergraduate institutions (Mesa, 2017). Our main goal is to establish a measure that can be used to determine the impact of professional development programs that target mathematical knowledge for teaching. In this report, we briefly describe the instrument we developed along with information that supports the argument that the instrument assesses mathematical knowledge for teaching college algebra in this context.

RELEVANT LITERATURE

Most research seeks to understand the nature and composition of mathematical knowledge for teaching (MKT). Ball and colleagues (2008) proposed a framework for the MKT construct that accounted for multiple components of that knowledge (e.g., of students, of curriculum, and of mathematical horizon). Their work identified a relationship between teachers' knowledge about mathematics and its teaching, and the quality of their work in the classroom and that the quality of instruction had a positive impact on student learning in K-8 education (Hill et al., 2008). However, attempts to make connections between teachers' mathematical knowledge and their teaching experience measured as years in the profession have yielded mixed results. Hill (2010)

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found no correlation between elementary teachers' overall years of mathematics teaching and their MKT scores. Krauss et al. (2008) found no relationship between years of experience teaching mathematics and performance on pedagogical content knowledge (PCK) and content knowledge (CK) assessments for secondary teachers, but those teachers prepared in pedagogy and mathematics content, when controlling for CK, demonstrated better PCK performance. Their study showed that beliefs rather than years of experience played a larger role in PCK performance. Some studies have found a positive relationship between MKT scores and experience teaching specific courses. For example, Herbst and Kosko (2014) found that high-school teachers who had taught geometry for at least three years had better scores on an MKT instrument designed to measure MKT in geometry compared to those instructors who had not taught such course. A data analysis from high-school algebra teachers who responded to an MKT instrument that assessed knowledge for teaching high-school algebra showed that respondents who had taught courses beyond algebra (e.g., calculus) performed better in the instrument than teachers who had not (Ko et al., 2021). Similar connections were also found by Hill (2007): middle school teachers with more experience teaching higher grades had higher MKT scores.

THEORETICAL UNDERPINNINGS

We assume that the knowledge needed for teaching is better assessed in practice and that it is closely linked to the content that students need to learn. In the community college context, presenting concepts to students via examples is a common instructional practice which demands that instructors engage in at least two distinct tasks of teaching: (1) choosing problems that exemplify mathematical notions and (2) understanding students' work to ascertain whether students have understood the material (Mesa & Herbst, 2011). Once instructors have students' utterances or written work, they engage in a new process that demands selecting a problem that would help students to clear misconceptions or that may create a specific dissonance in their knowledge. Following the theory of didactical situations (Brousseau, 1997), and in particular the work of Herbst and colleagues (e.g., Herbst & Chazan, 2012), we assume that in any instructional situation, instructors must manage the interactions between students and content, and have the dual responsibility of, on one hand, offering students work that will directly relate to learning a piece of mathematics, and on the other, analysing the way in which mathematics is being addressed in students' utterances or in written work produced while learning mathematics. Attending to practices related to these two tasks of teaching (*choosing problems* and *understanding students' work*) can shed light on the connections within mathematical knowledge for teaching.

Since instructors at community colleges typically use examples to anchor the presentation of the material and solve them collaboratively with students, we developed an instrument that assesses MKT in the context of college algebra at community colleges and hypothesized that the knowledge needed to engage in these two tasks is different (Mesa et al., 2023). We also assumed that instructors who would more frequently engage in these two tasks of teaching would have a higher MKT.

Because teaching is a practice, it is also worthwhile to know how teaching experience relates to performance in MKT. As the studies in the literature review section show, there does not seem to be an association between number of years teaching mathematics in general, but with experience gained through teaching advanced courses that can build specific tasks of teaching. Given this background, we investigated the following two questions: What is the relationship between community college instructor performance in the Mathematical Knowledge for Teaching Community College Algebra (MKT-CCA) test and (1) their teaching experience and (2) their reported frequency of use of activities related to the tasks of choosing problems and understanding student work?

METHODS

We recruited college algebra instructors from two-year degree granting institutions in the United States by inspecting community college websites and then sending direct invitations to instructors for participation in this project. Respondents included faculty from 260 different community colleges (~22% of institutions) in 42 states with 50% enrolling a majority of non-White students. Forty-eight percent of the participants identified as male and 46% as female; in terms of race, 76% identified as White, 10% as Asian, 4% as Black, 2% as mixed, and 4% chose Other. Seventy-eight percent of the participants said they held full-time positions; 9% were on tenure track. The average number of years of teaching experience was close to 17 years (mean = 16.76, SD = 9 years; range: 1.5 to 47 years). The majority (63%) held a master's degree in mathematics, mathematics education or another mathematics-related field, and 12% held PhDs (about 5% were in Mathematics Education).

Instruments

The MKT-CCA test consists of 55 items that aim to measure MKT across six hypothesized dimensions (Duranczyk et al., 2023; Mesa et al., 2023). An analysis of the psychometric properties of items using the Two-Parameter Logistic (2PL) Item Response Theory (IRT) recommended the removal of 17 items that had a low discrimination estimate of less than 0.61, resulting in 38 items (Mesa et al., 2024). An exploratory factor analysis with the remaining 38 items was conducted to examine the structure of the item responses. This analysis identified six items suggested to be loaded on different factors than those where all other items were loaded. We excluded them from subsequent analyses because they did not appear to add anything new to the intended MKT construct. We established a unidimensional construct (MKT) with the remaining 32 items, using the weighted least square mean and variance adjusted (WLSMV) estimator. The model suggested a good fit (RMSEA = 0.016, CFI = 0.981, TLI = 0.980) according to the thresholds considered for a good fit (RMSEA < 0.06, CFI > 0.95, TLI > 0.95, Hu & Bentler, 1999). Standardized items loadings were all significant and greater than 0.30, indicating that all the items significantly contribute to the common MKT construct.

The eight items assessing frequency of tasks of teaching use a 5-point Likert scale (1- *never*, *almost never, sometimes, often*, 5- *very often*) and are statements in the form of activities related either to choosing problems (e.g., Modifying problems from a textbook or from colleagues) or to understanding student work (e.g., Noticing that a student's mathematical approach to a problem is valid even though it is not standard in a college algebra course). An exploratory factor analysis with the eight items revealed a two-factor model. Three items reflecting the task of choosing problems loaded on one factor (C1: Modifying problems from a textbook or from colleagues, C2: Evaluating how well a problem meets your instructional goal, C3: Assisting a student in a small way, such as by giving a mathematical hint or asking a question, without solving the problem for them) and three items reflecting the task of understanding student work loaded on another factor (U1: Trying to understand how students came up with their answer to a problem, U2: Reading students' work to figure out their thinking process, U3: Noticing that a student's mathematical approach to a problem is valid even though it is not standard in a college algebra course). The remaining two items had loadings of less than 0.3 on either factor. After examining these items, we excluded them from further analysis as they deviated from either the task of *understanding student work* or *choosing problems*.

Participants' teaching experience was assessed using two items, one that asked the number of years of full-time-equivalent teaching experience in mathematics and another that asked for the number of times the participants had taught post-college algebra courses such as Calculus 1, 2, 3, Business Calculus, Differential Equations, Linear Algebra, etc. This item was assessed on a 4-point scale (0 - *Never taught*, *Taught less than 5 times*, *Taught at least 5 times but less than 10*, 3 - *Taught 10 or more times*).

Analysis

To answer the first research question, we predicted the MKT construct by the two variables of teaching experience. To answer the second research question, we predicted the MKT score by the frequencies of performing tasks that require understanding student work or choosing problems. All analyses were conducted in Mplus (Muthén & Muthén, 1998-2023). Latent constructs were identified by setting factor means to 0 and factor variances to 1.

RESULTS

We began conducting a regression analysis predicting the MKT from the frequency of two tasks of teaching. In the model, one of the items (task_C3 in Figure 1) asking the frequency of choosing problems was re-coded to prevent zero-frequency cells when using the WLSMV estimator. The model fit the data well (RMSEA = 0.018, CFI = 0.978, TLI = 0.977) and it showed a significant effect of the task frequency of choosing problems on MKT, but not with understanding student work. This significant effect indicates that instructors who frequently do the task of choosing problems tend to have higher MKT-CCA scores. Specifically, a one unit increase in the factor of task frequency is associated with 0.4 increase in MKT-CCA score (mean = 0, SD = 1) (B = 0.383, SE = 0.134, p = 0.004).

Second, we incorporated two additional predictors—the number of years teaching mathematics and the number of times instructors taught courses beyond college algebra—into the previous model. In this model, MKT-CCA is predicted by the two tasks of teaching frequency factors, the number of years teaching mathematics, and the number of times teaching courses post-college algebra. Additionally, the two task frequency factors were set to be predicted by the number of times teaching courses post-college algebra. The estimated structural model diagram is presented in Figure 1.

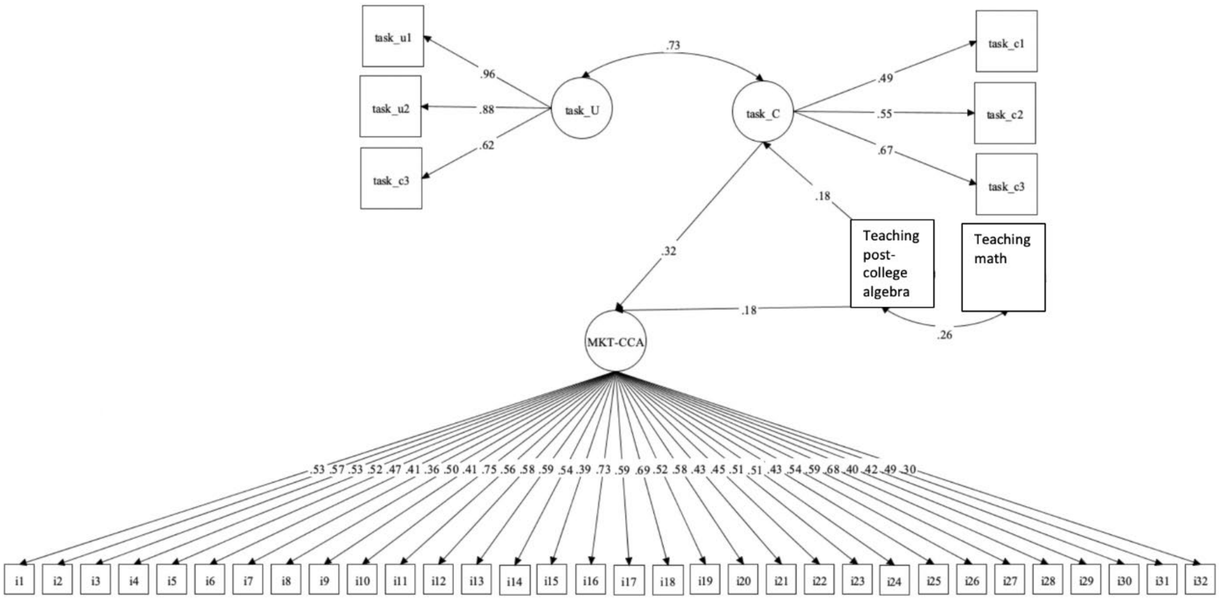

Figure 1. Structural model diagram showing only significant effects (standardized; $p < .05$)

The model fits the data well (RMSEA = 0.018, CFI = 0.974, TLI = 0.972). The result suggested a significant association between MKT-CCA and both the frequency of choosing problems (B = 0.323, SE = 0.138, p = 0.019) and the number of times teaching post-college algebra courses (B = 0.178, SE = 0.055, p = 0.001). When controlling for the number of times teaching post-college algebra courses, the effect of the task of choosing problems decreased in size and significance yet remained significant. This could be due to the relationship between the frequency of doing the task of choosing problems and the frequency of teaching post-college algebra courses, which is consistent with the significant effect of the number of times teaching post-college algebra courses on the frequency of doing the task of choosing problems (B = 0.179, SE = 0.066, p = 0.007). In contrast to the significant effect of the number of times teaching post-college algebra courses, there was no significant association between the MKT-CCA and the number of years teaching mathematics. This suggests that our MKT-CCA instrument measures a construct related to instructors' MKT associated with experience teaching post-college algebra courses.

DISCUSSION

Our findings regarding the relationship between MKT-CCA and teaching experience, confirm prior findings (Ko et al., 2021) about the connection between MKT scores and teaching courses beyond college algebra. In that study, we found that with an instrument designed to assess knowledge for teaching 9th grade Algebra 1, community college instructors who had taught advanced courses scored better than those who had taught only college algebra. We believe that this finding speaks to a connection between the items assessed in this instrument and the knowledge that is acquired while teaching more advanced mathematics courses; such experiences allow teachers to reflect on the foundations that students will need to understand about mathematics that they will encounter later (the mathematics in the horizon, Ball et al., 2008). If instructors notice calculus students struggling with difference quotients, for example, they may realize the importance of bringing clarity to the notion of slope. We found that years of full-time experience teaching mathematics in general is not associated with MKT-CCA scores; this departs from prior finding, such as Hill (2010), who found that "more experienced [elementary] teachers—and particularly those with over 20 years of experience—have more MKT, and this overall relationship looks approximately linear" (p. 533). The discrepancy may rely on the nature of mathematics taught in elementary versus college courses.

For the second question we found a significant and positive relationship between the reported frequency of engaging in choosing problems and MKT-CCA performance. This suggests that the MKT construct that is being assessed by the instrument, might be associated with a task of teaching. Instructors with higher MKT might be more comfortable exercising choice for problems that might be more suitable to meet instructional goals, including those that address specific students' misunderstandings. A higher MKT-CCA score allows for identifying instructors who have a more nuanced set of instructional goals that will demand more attention when choosing problems. It may be that the instrument as designed might be capturing the knowledge needed to choose problems and thus, instructors who engage in that task more frequently will answer more items correctly. The model also showed that the frequency of use of the two tasks of teaching, *choosing problems* and *understanding student work*, are highly correlated. We believe that this is because as instructors spend time interpreting students' work, they might be using the opportunities to think about possible features of tasks that either led to student responses or think about tasks that could help students answer differently next time. But the reverse could also be true; as instructors choose problems, they might be generating hypotheses of the ways which students will answer them; once they receive responses, they will need to engage in understanding the work to decide whether their hypotheses were correct. Future studies will need to examine the mediating role of frequency of choosing problems and teaching courses post-algebra in relation to the MKT-CCA score, which was not possible to assess in this study because of the size of the analytical sample. Finally, we also found that the frequency of understanding student work did not predict the score in the MKT-CCA.

We think that this might be explained by the opportunities that community college instructors may have to interpret student work. As the number of students in these courses increase, colleges have been relying more on automated grading done by textbook companies (e.g., Pearson) that offer placement and homework systems (e.g., ALEKS) that eliminate the task of collecting and grading homework. It might be that instructors routinely have to understand and evaluate student work, making this practice similarly frequent across different teachers, which in turn does not significantly impact MKT. On the other hand, choosing problems, which is known to be less frequent, might potentially influence MKT due to the differing levels of experience among teachers. Alternatively, it could be possible that our instrument is less robust in assessing MKT used in understanding student work in the absence of choosing problems.

CONCLUSION/IMPLICATIONS

This paper presents some preliminary results of an instrument designed to measure mathematical knowledge for teaching college algebra at the community college. The MKT-CCA links instructors teaching practices, particularly the reported frequency of choosing problems, with higher performance on the test. The MKT-CCA score also seems to increase based on the number of times instructors teach courses that build on college algebra skills. These findings, and further analyses, show promise for identifying instructor experiences that can be explored in professional development settings and help raise awareness of teaching practices that enhance mathematical knowledge for teaching.

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