Reference Governor for Constraint Enforcement in Air Conditioning Systems

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***Abstract*— Advanced control for heating ventilation and air-conditioning (HVAC) systems has primarily focused on optimizing energy efficiency. However, ensuring the safe and reliable operation of HVAC equipment is another crucial control objective. This paper presents a reference governor (RG) that can augment existing energy-optimized controllers with the capability to enforce constraints to ensure safe and reliable operation. The RG makes minimal adjustments to the reference room temperature set-point to ensure that the refrigerant entering the compressor is super-heated, thereby preventing potential damage. Designing the RG requires synthesizing a constraint admissible positive invariant (CAPI) set which characterizes the subset of states for which the closed-loop system will satisfy the safety constraints. We employ an indirect data-driven method wherein we identify a control-oriented model and uncertainty bounds to synthesize a robust (CAPI) set. We validated the RG in a case study and we observed constraint satisfaction.**

***Index Terms*— Constrained control, Data driven control, LMIs, Predictive control for nonlinear systems**

I. Introduction

Heating ventilation and air-conditioning (HVAC) systems are used to create a comfortable and healthy indoor environment. While prior HVAC research predominantly prioritized energy efficiency [3], [8], this paper shifts focus to the safe operation of HVAC hardware. Energy efficiency has historically been the main concern, thanks to advanced control methods' well-documented ability to cut energy use significantly [9] [12]. Model predictive control (MPC) is a popular choice for HVAC control due to its direct optimization of energy efficiency, see the survey [4] for details. Fuzzy logic has also been employed to encode expert knowledge into control laws that enhance energy efficiency [14]. More recently, machine learning (ML) has proven successful in designing HVAC controllers that effectively reduce energy consumption [6].

Employing a controller designed purely to optimize the energy efficiency of an HVAC system can have unintended consequences. For instance, an efficiency-optimizing controller can increase wear-and-tear on the HVAC hardware or cause damage. One mechanism for damaging the HVAC hardware is by drawing liquid-phase or two-phase refrigerant into a compressor designed for only gas-phase refrigerant. This can be prevented by enforcing a constraint that the refrigerant discharged from the evaporator is super-heated i.e. refrigerant temperature is strictly greater than the evaporation temperature at which the phase transitions from liquid to gas. However, this can reduce efficiency since it requires additional energy to super-heat the refrigerant. Thus, an efficiency-optimizing controller may eliminate this super-heat. This paper considers the problem of designing an add-on controller that enforces a safety constraint on the discharge-temperature super-heat.

This paper presents a reference governor (RG) [5] for ensuring the safe operation of an air conditioner (AC) plant. An RG is an add-on controller that adjusts the reference to a closed-loop system to enforce constraints. Thus, the presented RG will not replace an efficient-energy optimizing controller, but rather augment this controller with the capability to enforce safety constraints. We develop a RG to enforce the constraint on the discharge temperature super-heat to ensure that only gas-phase refrigerant enters the compressor. The presented RG dynamically adjusts the reference indoor air temperature provided to the existing controller so that the closed-loop system satisfies the discharge temperature super-heat constraint. The RG is posed as an optimization problem that minimizes the discrepancy between the desired and implemented reference temperature. Thus, the RG will minimize its interference with the existing energy-optimizing AC controller which has been carefully tuned and validated. In practice, the RG slows the implementation of step-function changes to the reference room temperature set-point to ensure that a rapid transient does not cause two-phase refrigerant to enter the compressor.

Designing an RG requires synthesizing a constraint admissible positive invariant (CAPI) set for the closed-loop dynamics and constraints. Since the CAPI set is constraint admissible, it describes states of the closed-loop system that satisfy safety constraints. Since the CAPI set is positive invariant (PI), the closed-loop dynamics remain inside this safe set. Synthesizing a CAPI set for the AC system is challenging due to the nonlinear and uncertain dynamics. The AC unit has non-trivial nonlinearities due to the refrigerant changing phase throughout the refrigeration cycle. Although the indoor air has simple dynamics, they are uncertain since the HVAC unit is installed in the indoor space by a third party. An energy-optimizing MPC or ML controller will be nonlinear. Furthermore, if this controller is proprietary then it will be highly uncertain. To address these challenges, we use an indirect data-driven approach wherein we identify a control-oriented model from plant data and bound the modeling errors. This model is used to synthesize a robust CAPI set for the closed-loop AC system. The RG uses this CAPI set to modify the reference temperature set-point to enforce the discharge super-heat constraint.

The contributions of this paper are summarized:

- An indirect data-driven method for synthesizing a CAPI set for the closed-loop AC system.

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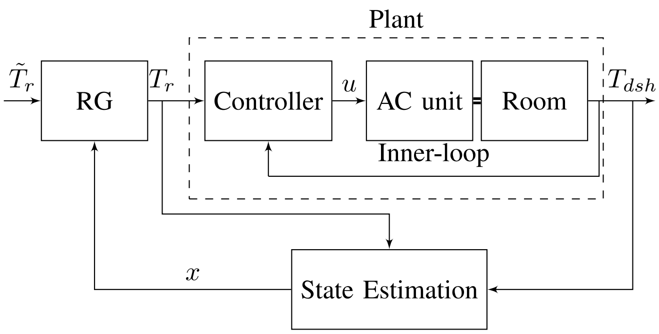

Fig. 1. Reference governor (RG) design for a air conditioner (AC) plant (dashed box) equipped with an inner-controller.

- The design of an RG that uses the CAPI set to adjust the indoor reference temperature set-point to enforce a safety constraint so that only gas-phase refrigerant enters the compressor.
- A computationally efficient method for implementing the RG in real-time.

The remainder of this paper is organized as follows: In Section II we define the AC constraints enforcement problem. In Section III we introduced the model derivation and explained the CAPI synthesis with propositions and proofs that make our approach viable. We validated our approach in Section IV by applying it to a case study.

II. AC Constraint Enforcement Problem

This section describes the AC plant and the constraint to be enforced. In addition, we describe RGs which will be used to enforce the constraint.

A. Air Conditioner Plant

In this section, we describe the operating principles of the AC plant and a constraint for safe operation.

1) Air Conditioner Operation: The AC plant is the closed-loop system consisting of the (i) air conditioner unit, (ii) the indoor space, and (iii) the controller, as shown in the dashed box in Fig. 1. The purpose of the AC unit is to move heat from the indoor space to the outdoor space. There are four components of the AC unit; the compressor, condenser, evaporator, and expansion valve as shown in Fig. 2. In the air conditioning cycle, the refrigerant travels from one component to the other and undergoes phase changes in the process. The compressor circulates refrigerant throughout the AC unit. It compresses the low-pressure gas-phase refrigerant into a high-pressure state, increasing its temperature. The high-temperature refrigerant then enters the condenser where it releases heat into the outdoor air. This heat transfer is expedited by an outdoor fan that moves outdoor air across the condenser heat-exchanger. As a result, the refrigerant cools and transitions into a high-pressure liquid-phase. Next, the high-pressure liquid-phase refrigerant enters the expansion valve where its pressure and temperature drop. Finally, the refrigerant enters the evaporator where it absorbs heat from the indoor air. This heat transfer is expedited by an indoor fan which moves the indoor air across the evaporator heat-exchanger [11]. The refrigerant discharged

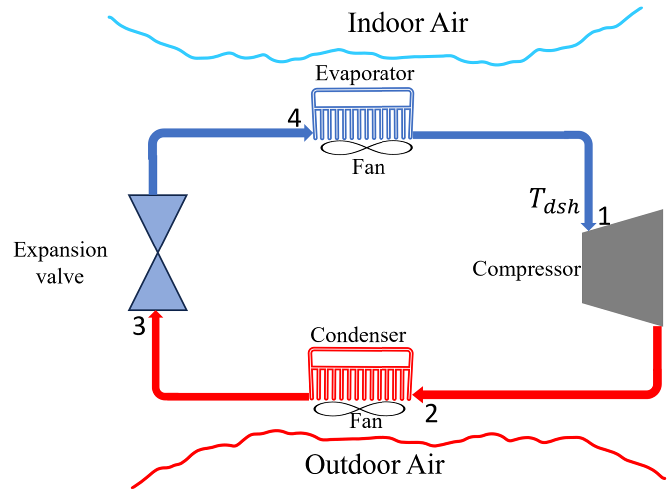

Fig. 2. Schematic of a vapor compression cycle in an air conditioner (AC) unit showing components and refrigerant flow direction

from the evaporator should be super-heated vapor. The temperature of the refrigerant above its evaporation temperature is called the discharge super-heat temperature T_{dsh}.

In addition to the AC unit, the closed-loop AC plant includes the indoor space and the controller. The controller manipulates the compressor speed, the indoor and outdoor fan speeds, and the expansion valve to regulate the temperature of the indoor air to a desired reference temperature i.e. $T_k \to T_r$ as $k \to \infty$. The RG will manipulate this reference T_r to ensure constraint enforcement. The controller is an proprietary controller with unmodeled and complex dynamics. The indoor air is a thermal mass of unknown volume and specific heat. The temperature T_k of the indoor air changes based on the removal of heat by the AC unit and the addition of heat from disturbance sources.

The challenges addressed in this paper arise from the complex and unmodeled dynamics of the closed-loop AC plant. While the dynamics of the AC unit are well-modeled [13], they have complex nonlinearities due to the phase changes the refrigerant undergoes as it moves through the cycle. In contrast, the indoor air has simple but, unmodeled thermal dynamics since the AC unit is installed by a third party. The controller has both unmodeled and complex dynamics since HVAC uses advanced control techniques which are typically nonlinear and propriety. Our objective is to augment, rather than replace, this controller to enforce constraints.

2) Air Conditioner Safety Constraint: The compressor is designed to operate with gas-phase refrigerant. Liquid-phase or two-phase refrigerant entering the compressor can cause damage to this component [7]. Following [3], we enforce a constraint on the discharge temperature to prevent damaging the compressor

$$T_{dsh} \geq \underline{T}_{dsh}, \tag{1}$$

where T_{dsh} is the discharge temperature super-heat. The constraint (1) requires that the discharge super-heat temperature T_{dsh} is above a minimum super-heat level $\underline{T}_{dsh}$ so that the refrigerant entering the compressor is super-heated. This paper considers the design of an add-on controller that enforces the constraint (1) for the closed-loop AC plant. In future work, we will consider enforcing multiple safety constraints.

B. Reference Governors

This section introduces reference governors which govern the reference to a closed-loop system to enforce constraints [5]. We are interested in RGs since we do not want to replace the efficiency-optimizing existing controller for the AC plant, but rather augment it to ensure constraint enforcement. We consider an RG implemented by the optimization problem

$$r_k^\star = \arg\min_r \quad \|r - \tilde{r}_k\| \tag{2a}$$
$$\text{s.t.} \quad (x_k, r) \in \mathcal{O}, \tag{2b}$$

where $\mathcal{O}$ is a CAPI set for the closed-loop system, $x_k \in \mathbb{R}^n$ is the discrete-time state of the closed-loop system, $\tilde{r}_k \in \mathbb{R}$ is the desired reference and $r_k \in \mathbb{R}$ is the implemented reference. The RG (2) minimizes the deviation $\|r - \tilde{r}\|$ between the desired $\tilde{r}$ and implemented r references subject to the implemented reference r ensuring constraint satisfaction.

The CAPI set $\mathcal{O}$ characterizes state and reference pairs $(x_k, r) \in \mathcal{O}$ that satisfy the constraints. Since the set $\mathcal{O}$ is constraint admissible $C\mathcal{O} \subseteq \mathcal{Y}$ for constant references, any state and reference pair $(x, r) \in \mathcal{O}$ in this set satisfies constraints $y = Cx \in \mathcal{Y}$. Since the set $\mathcal{O}$ is PI, it describes a set of initial conditions $(x(0), r) \in \mathcal{O}$ such that the state and reference trajectory $(x_k, r_k) \in \mathcal{O}$ remain inside this set when the reference is constant $r_{k+1} = r_k$. Thus, if we start inside the CAPI set then we can guarantee constraint satisfaction by keeping the reference constant $r_{k+1} = r_k$. Furthermore, this set describes safe changes to the reference that ensure constraint satisfaction.

The challenges this paper addresses arise from synthesizing a CAPI set for the complex and unmodeled dynamics of the closed-loop AC plant. Typically, RGs are designed using analytic models of the closed-loop system. These models are unavailable for the AC plant since the controller and indoor air dynamics are unmodeled. Furthermore, synthesizing a CAPI set for the nonlinear dynamics of the AC unit is challenging. Our approach will use robust linear system identification to design the RG.

C. AC Constraint Enforcement Problem Statement

The AC-RG design problem is summarized below.

Problem 1: Design an RG (2) to enforce the discharge temperature super-heat constraint (1) for the AC plant.

In this preliminary work, we are interested in empirically demonstrating constraint enforcement. We will design the RG (2) using linear system identification based on data acquired from high-fidelity simulations of the closed-loop RG plant using `Thermosys` [1]. This linear identified model will be used to synthesize a local robust CAPI set $\mathcal{O}$ for the RG (2). We will design an observer to estimate the states from the system output which will be used in the RG.

III. RG Design and Implementation

A. Data-Driven Control Oriented Modeling

In this subsection, we will derive a model through system identification using data obtained from high-fidelity simulations using `Thermosys` [1]. To enforce the constraint (1), we require a model of the dynamics from the reference temperature T_r to the discharge super-heat T_{dsh}. Since the AC plant has nonlinear dynamics, this model will only be valid near the nominal operating point. Thus, the inputs and outputs of the linearized AC plant are shifted

$$r_k = T_{r,k} - T_r^o, \tag{3a}$$
$$y_k = T_{dsh,k} - T_{dsh}^o, \tag{3b}$$

where T_r^o and T_{dsh}^o are the nominal reference temperature and discharge temperature, respectively, around which the linearized dynamics will be valid.

We used `Matlab`'s system identification toolbox [10] to identify a linear discrete-time state-space model of the AC plant dynamics

$$x_{k+1} = Ax_k + Br_k + w_k, \tag{4a}$$
$$y_{k+1} = Cx_k. \tag{4b}$$

where w_k is a disturbance that accounts for modeling errors. The output $y = T_{dsh} - T_{dsh}^o$ is the shifted discharge super-heat which must lie inside the constraint set $\mathcal{Y} = \{y \mid y = Cx \geq \bar{y}\}$ where $\bar{y} = \underline{T}_{dsh} - T_{dsh}^o$.

The modeling errors w_k are bounded by an ellipsoidal set

$$\mathcal{W} = \left\{w \in \mathbb{R}^n : w^\top W w \leq \rho^2\right\} \tag{5}$$

where the parameters $W \succ 0 \in \mathbb{R}^{n\times n}$ and $\rho > 0$ are chosen so that the set (5) contains the observed modeling errors $w_i = x_i^+ - Ax_i - Br_i$ for data $\{x_i^+, x_i, r_i\}_{i=1}^N$ collected from the high-fidelity simulations where x^+ is successor state. The parameters W and ρ can be optimized to the tightest bound $\mathcal{W}$ using an linear matrix inequality (LMI) [2] This model was used to design a CAPI set for the RG to enforce the constraint (1) for the AC plant.

B. Constraint Admissible PI Set Synthesis

The RG (2) requires a CAPI set $\mathcal{O} \subseteq \mathbb{R}^{n+1}$ for the closed-loop AC plant to enforce the constraint (1). We consider a sublevel-set

$$\mathcal{O} = \left\{ \begin{bmatrix} x \\ r \end{bmatrix} : \begin{matrix} V(x,r) \leq (Gr - \bar{y})^2 \\ 0 \leq \ Gr - \bar{y} \end{matrix} \right\}, \tag{6}$$

of the quadratic function

$$V(x,r) = \begin{bmatrix} x \\ r \end{bmatrix}^\top \begin{bmatrix} Q & S \\ S^\top & R \end{bmatrix} \begin{bmatrix} x \\ r \end{bmatrix} \tag{7}$$

where $G = C(I - A)^{-1}B$ is the steady-state gain of (4). The second inequality requires that the steady-state output $y_\infty = Gr \geq \bar{y}$ satisfies the constraints $y_\infty \in \mathcal{Y}$. The design parameters $Q \in \mathbb{R}^{n\times n}$, $S \in \mathbb{R}^{n\times 1}$, and $R \in \mathbb{R}^{1\times 1}$ must be selected to satisfy two properties 1) $\mathcal{O} \subseteq \mathbb{R}^{n+1}$ must be PI and 2) $\mathcal{O} \subseteq \mathbb{R}^{n+1}$ must be constraint admissible.

1) Positive Invariance: The following proposition provides conditions on Q, R, and S so that (6) is PI under the idealistic condition where (4) accurately models the dynamics.

Proposition 1: Let the closed-loop AC plant be stable. If $Q \succ 0$ satisfies the Lyapunov equation

$$A^\top QA - Q \preceq 0 \tag{8}$$

and $S = -Q(I-A)^{-1}B$ and $R = S^\top Q^{-1}S$, then the set (6) is PI for the model (4) under constant references $r_k = r_\infty$ with $w_k = 0$.

Proof: We will show that $(x_{k+1}, r_{k+1}) \in \mathcal{O}$ whenever $(x_k, r_k) \in \mathcal{O}$ for constant references $r_{k+1} = r_k = r_\infty$. Consider the quadratic function (7) defining (6)

$$\begin{aligned} &V(x_{k+1}, r_{k+1}) = \\ &\quad = (Ax_k + Br_\infty + Q^{-1}Sr_k)^\top Q(Ax_k + Br_\infty + Q^{-1}Sr_k) \end{aligned}$$

where $R = S^\top Q^{-1}S$. By definition of $S = -Q(I-A)^{-1}B$, the state $-Q^{-1}Sr_\infty = x_\infty$ is the equilibrium state for the identified model (4) corresponding to the constant reference $r_k = r_\infty$ i.e. x_∞ satisfies $x_\infty = Ax_\infty + Br_\infty$. Thus,

$$V(x_{k+1}, r_{k+1}) = \tilde{x}_k^\top A^\top QA\tilde{x}_k$$

where $\tilde{x}_k = x_k - x_\infty$ and $\tilde{x}_{k+1} = Ax_k + Br_k - x_\infty = Ax_k + Br_k - Ax_\infty - Br_\infty = A\tilde{x}_k$. By the Lyapunov equation (8), we have $\tilde{x}_k^\top A^\top QA\tilde{x}_k \leq \tilde{x}_k^\top Q\tilde{x}_k$. Expanding this term, yields

$$\tilde{x}_k^\top Q\tilde{x}_k = \begin{bmatrix} x_k \\ r_k \end{bmatrix}^\top \begin{bmatrix} Q & S \\ S^\top & R \end{bmatrix} \begin{bmatrix} x_k \\ r_k \end{bmatrix} = V(x_k, r_k).$$

Thus, the quadratic function (7) satisfies

$$V(x_{k+1}, r_{k+1}) \leq V(x_k, r_k) \leq (Gr_{k+1} - \bar{y})^2,$$

where $r_k = r_{k+1}$. Thus, $(x_{k+1}, r_{k+1}) \in \mathcal{O}$ where $r_{k+1} = r_k$. Therefore, $\mathcal{O}$ is PI for the dynamics (4) with constant references $r_k = r_\infty$. ∎

The assumption that the AC plant is stable is reasonable since we are considering the closed-loop system. The PI set (6) shrinks $(y_\infty - \bar{y})^2 \approx 0$ when the reference r_∞ produces a steady-state $y_\infty = Gr_\infty$ near the constraint boundary $y_\infty \approx \bar{y}$.

Proposition 1 means that the set (6) is PI if the identified model (4) accurately models the dynamics of the AC plant. However, this obviously does not hold since the AC plant is nonlinear. The following proposition extends Proposition 1 to provide robustness to the nonlinearity of the AC plant.

Proposition 2: Let the unmodeled closed-loop dynamics $x^+ = f(x,r)$ of the AC plant be stable and satisfy

$$f(x,r) \in \{Ax + Br + w : w \in \mathcal{W}\} \tag{9}$$

where $\mathcal{W}$ is the bound (5) on the modeling errors of the identified model (4). If Q satisfies the LMI

$$\begin{bmatrix} A^\top QA - (1-\alpha)Q & QA \\ A^\top Q & Q - \alpha W \end{bmatrix} \preceq 0 \tag{10}$$

and $S = -Q(I-A)^{-1}B$ and $R = S^\top Q^{-1}S$, then the set

$$\mathcal{O} = \left\{ \begin{bmatrix} x \\ r \end{bmatrix} : \begin{matrix} V(x,r) \leq (Gr - \bar{y})^2 \\ \rho \leq \ Gr - \bar{y} \end{matrix} \right\} \tag{11}$$

is PI for the unknown nonlinear dynamics under constant references $r_k = r_\infty$.

Proof: Consider $(x,r) \in \mathcal{O}$. Then, according to (9) there exists $w \in \mathcal{W}$ such that $f(x,r) = Ax + Br + w$ and $w^\top Ww \leq \rho^2$ according to (5). Thus,

$$\begin{aligned} V(f(x,r),r) &= V(Ax + Bu + w, r) \\ &= \begin{bmatrix} Ax + Br + w \\ r \end{bmatrix}^\top \begin{bmatrix} Q & S \\ S^\top & R \end{bmatrix} \begin{bmatrix} Ax + Br + w \\ r \end{bmatrix} \end{aligned}$$

Defining $\tilde{x} = x + Q^{-1}Sr$ as in Proposition 1, we obtain

$$V(f(x,r),r) = \begin{bmatrix} \tilde{x} \\ w \end{bmatrix}^\top \begin{bmatrix} A^\top QA & QA \\ A^\top Q & Q \end{bmatrix} \begin{bmatrix} \tilde{x} \\ w \end{bmatrix}$$

From the LMI (10), we have

$$\begin{aligned} V(f(x,r),r) &\leq \begin{bmatrix} \tilde{x} \\ w \end{bmatrix}^\top \begin{bmatrix} (1-\alpha)Q & 0 \\ 0 & \alpha W \end{bmatrix} \begin{bmatrix} \tilde{x} \\ w \end{bmatrix} \\ &= (1-\alpha)\tilde{x}^\top Q\tilde{x} + \alpha w^\top Ww \end{aligned}$$

Since $\tilde{x}^\top Q\tilde{x} \leq (\bar{y} - y_\infty)^2$ and $w^\top Ww \leq \rho^2 \leq (\bar{y} - y_\infty)^2$ we have

$$\begin{aligned} V(f(x,r),r) &\leq (1-\alpha)(\bar{y} - y_\infty)^2 + \alpha(\bar{y} - y_\infty)^2 \\ &= (\bar{y} - y_\infty)^2 \end{aligned}$$

That is $(f(x,r),r) \in \mathcal{O}$. Therefore, $\mathcal{O}$ is PI. ∎

Proposition 2 means that if the modeling error bounds (5) are large enough to capture (9) the nonlinear plant dynamics then (11) is a PI for the nonlinear plant. Here, a bounded uncertainty model $\mathcal{W} \subset \mathbb{R}^n$ is valid since the PI set $\mathcal{O} \subset \mathbb{R}^{n+1}$ defines a bounded (compact) region of the state and reference space. Thus, for Lipschitz continuous dynamics $f(x,r)$, the modeling errors must be bounded. The condition $0 < \rho \leq y_\infty - \bar{y}$ ensures that the robust PI set (11) is large enough that the stable closed-loop AC plant dynamics can compensate for the disturbances before the state leaves the set.

2) Constraint Admissibility: The following proposition provides conditions on Q, R, and S so that (6) is constraint admissible.

Proposition 3: If Q satisfy the LMI

$$C^\top C \preceq Q, \tag{12}$$

and $S = -Q(I-A)^{-1}B$ and $R = S^\top Q^{-1}S$, then the set (6) is constraint admissible $[C, 0]\mathcal{O} \subseteq \mathcal{Y}$ for the model (4) under constant references $r_k = r_\infty$.

Proof: We will show that if $(x,r) \in \mathcal{O}$ then $y = Cx \in \mathcal{Y}$. From the conditions on Q, R, S, we have

$$\begin{bmatrix} x \\ r \end{bmatrix}^\top \begin{bmatrix} Q & S \\ S^\top & R \end{bmatrix} \begin{bmatrix} x \\ r \end{bmatrix} = \tilde{x}^\top Q\tilde{x} \leq (\bar{y} - y_\infty)^2$$

where $\tilde{x} = x - x_\infty$, and $y_\infty = Gr$. Since $Q \succeq C^\top C$, we have $\tilde{x}^\top C^\top C\tilde{x} \leq \tilde{x}^\top Q\tilde{x} \leq (\bar{y} - y_\infty)^2$ which implies $|C\tilde{x}| = |y - y_\infty| \leq |y_\infty - \bar{y}| = y_\infty - \bar{y}$, since $y_\infty > \bar{y}$. Thus, $\bar{y} \leq y \leq 2y_\infty - \bar{y}$, where $y = Cx \in \mathcal{Y}$ is supported by $\bar{y} \leq y$. ∎

A similar result holds for the robust PI set (11), if $\mathcal{W}$ captures all the nonlinearity and the output is a linear transformation of the state $y = Cx$. The only difference is that the reference range will decrease so that a margin ρ is kept between the steady state $y_\infty = Gr > \bar{y} - \rho$ and the boundary $\bar{y}$.

C. Observer design

The RG (2) requires the state x_k of the closed-loop AC system which is not measured. Indeed, the state of the identified model (4) is not physically meaningful. Therefore, we use the identified model (4) to estimate the non-physical state

$$\hat{x}_{k+1} = A\hat{x}_k + Br_k + L(y_k - \hat{y}_k) \tag{13a}$$
$$\hat{y}_k = C\hat{x}_k \tag{13b}$$

where $(A - LC)$ is Schur. With the observer dynamics (13) included, the RG is now a dynamic controller as shown in Fig. 1.

D. Reference Governor Implementation

For the CAPI set (6), the RG (2) is a quadratically constrained quadratic program. Although this optimization problem can be solved using off-the-shelf software, it may not be computationally tractable using a low-cost embedded processor. Thus, we provide a closed-form solution to the RG problem (2) described in the following proposition.

Proposition 4: The RG optimization problem (2) has the closed-form solution

$$r^\star = \begin{cases} \bar{r} & \text{if } \tilde{r} > \bar{r} \\ \tilde{r} & \text{if } \underline{r} \leq \tilde{r} \leq \bar{r} \\ \underline{r} & \text{if } \tilde{r} < \underline{r} \end{cases} \tag{14}$$

where $\underline{r}$ and $\bar{r}$ are roots of the quadratic equation

$$(R - G^2)r^2 + 2(x^\top S + G\bar{y})r + x^\top Qx - \bar{y}^2. \tag{15}$$

Proof: From the definition (6) of the CAPI set $\mathcal{O}$, the constraint (2b) can be written as

$$(R - G^2)r^2 + 2(x^\top S + G\bar{y})r + x^\top Qx - \bar{y}^2 \leq 0. \tag{16}$$

Since $R = S^\top Q^{-1} S$ and $Q \succeq C^\top C$, we have $R = B^\top(I - A^\top)^{-1}Q(I-A)^{-1}B \geq B^\top(I-A^\top)^{-1}C^\top C(I-A)^{-1}B = G^2$. Thus, $R - G^2 \geq 0$. Therefore, the quadratic equation is convex and the inequality holds between its roots. Thus, the RG (2) can be written as

$$\min_r \quad \|r - \tilde{r}\|$$
$$\text{s.t.} \quad \underline{r} \leq r \leq \bar{r}$$

which has the closed-form solution (14) since it is a one-dimensional optimization problem. ■

Proposition 4 says that the RG (2) can be implemented by saturating (14) the desired reference $\tilde{r} \in [\underline{r}(x_k), \bar{r}(x_k)]$ with state-dependent bounds. Since (15) is a quadratic equation, these bounds $\underline{r}$, $\bar{r}$ can be found using the quadratic formula. Algorithm 1 summarizes a step-by-step process for implementing the reference governor.

IV. Case Study

In this section, we present a case study that demonstrates our RG's ability to enforce the discharge super-heat temperature constraint (1). In this case study, we considered a AC plant equipped with our RG implemented as in Algorithm 1, simulated in `Thermosys`-4 toolbox for `Matlab`/`Simulink` [1] as shown in Fig. 3.

Algorithm 1 : Implementation of the RG

```
Parameters: Operating point T_r^o, T_dsh^o, output bound T̲_dsh
Normalize the bound: ȳ = T̲_dsh − T_dsh^o
for k = 1, ..., ∞ do
    Input: T̃_{r,k} and T_{dsh,k}
    Normalize:
        r̃_k = T̃_{r,k} − T_r^o
        y_k = T_{dsh,k} − T_dsh^o
    Estimate the state x_k using (13)
    Compute the roots of (15) for thresholds r̲_k, r̄_k
    Saturate the commanded reference r̃_k
        r_k = { r̄_k  if r̃_k > r̄_k
              { r̃_k  if r̲_k ≤ r̃_k ≤ r̄_k
              { r̲_k  if r̃_k < r̲_k
    Denormalize: T_{r,k} = r_k + T_r^o
    Output: T_{r,k}
end for.
```

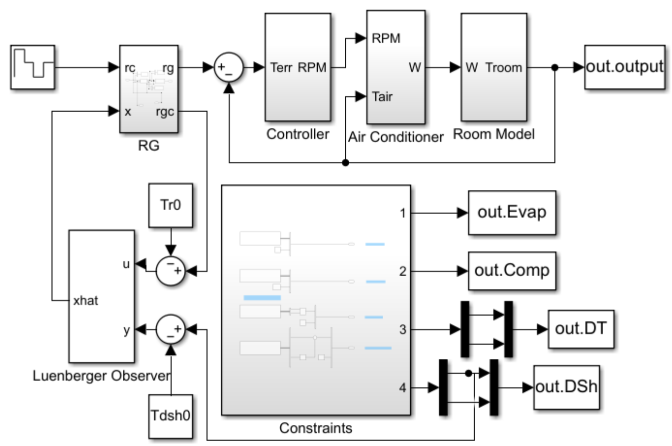

Fig. 3. Simulation in `Thermosys` toolbox, including an AC plant equipped with our RG from Algorithm 1.

A. Control-Oriented Model Validation

In this section, we describe the identification of the linear model (4). We generated two different data sets as train data and test data for system identification. To obtain our data set, we start by simulating the AC plant using `Thermosys` for a duration time of 5 hours. A series of desired room temperatures (references) T_r was provided to the closed-loop system, changing every 10 minutes. We considered an outside temperature of 37°C. We saved 6 signals: the room temperature, evaporator temperature, compressor speed, discharge temperature, and discharge super-heat temperature. From the simulation signals, we observed that there was a constraint violation with the discharge super-heat temperature. The signals from the high-fidelity model do not have a uniform sampling time. To fix this, we interpolated using a uniform time-step of 60 seconds. With the train data, we used the `Matlab`'s system identification toolbox to generate a stable linear model (4) with zero feed-through $D = 0$. Fig. 4 shows the comparison between high-fidelity simulation and

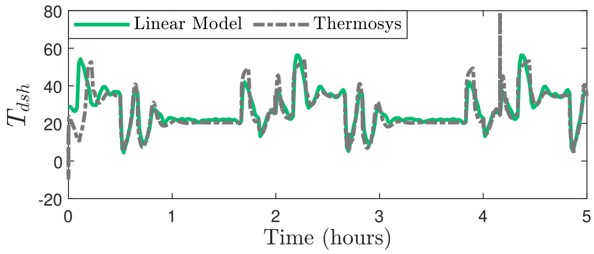

Fig. 4. Comparison of discharge super-heat temperature $\boldsymbol{T_{dsh}}$ for the high-fidelity nonlinear model and the identified linear model (4) for the test data-set.

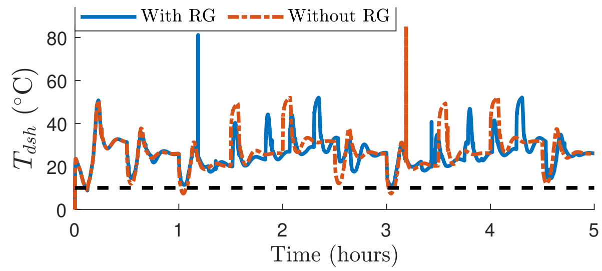

Fig. 5. Comparison of the discharge super-heat temperature $\boldsymbol{T_{dsh}}$ with and without the RG (2). Without the RG, the discharge super-heat temperature violates the constraint, whereas the constraint was enforced when the system is equipped with the RG.

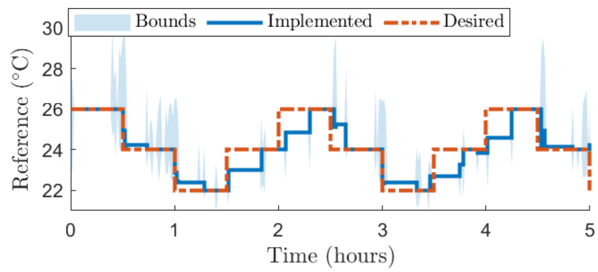

Fig. 6. The desired $\tilde{\boldsymbol{T}}_{\boldsymbol{r}}$ and implemented $\boldsymbol{T_r}$ references as well as the safety bounds $[\boldsymbol{T_r^0} + \underline{\boldsymbol{r}}, \boldsymbol{T_r^0} + \bar{\boldsymbol{r}}]$ on the reference obtained by the RG. The RG slows down the change in the reference if the desired reference $\tilde{\boldsymbol{T}}_{\boldsymbol{r}}$ is not in the safety bounds. These bounds loosen as the closed-loop system approaches steady-state.

the identified model (4). As shown in Fig. 4, the identified model captures the trends of the high-fidelity model, but has modeling errors due to nonlinearity.

B. Reference Governor Validation

In this section, we demonstrate that the presented RG (2) can enforce the discharge super-heat constraint (1) through high-fidelity simulations using Thermosys. The RG was implemented using Algorithm 1. The CAPI sets $\mathcal{O}$ was synthesized with the form (6) where $Q = CC^\top + 0.5I \succ CC^\top$, and $S = -Q(I-A)^{-1}B$ and $R = S^\top Q^{-1} S$ according to Proposition 1 and 3. At each time-step k, the state x_k was estimated using the observer (13).

Fig. 5 shows the discharge super-heat temperature T_{dsh} with and without the RG for a pseudo-random reference temperature profile $\tilde{T}_r$. Without the RG, the discharge super-heat violated the constraint (1) at four separate time instances. In contrast, the discharge super-heat never violated the constraint while the RG was active. Note that in the high-fidelity simulations the RG was not activated until $t = 600$ seconds for two reasons; (i) to allow the start-up transients for the Thermosys model to subside and (ii) to allow the observe to gather sufficient measurements to accurately estimate the state.

Fig. 6 shows how the RG adjusted the reference to the closed-loop AC plant. Fig. 6 plots the desired $\tilde{T}_r$ which is the desired room temperature and the implemented reference which was modified by the RG. In addition, Fig. 6 shows the bounds (14) on the reference $\tilde{T}_{r,k}$ at each time-instance k. These bounds were computed from the CAPI set (6) to guarantee constraint satisfaction. When the desired reference $\tilde{T}_{r,k}$ is rapidly changing, the safety bounds on the reference become tighter. As a result, the RG slows the implemented reference $T_r = r + T_r^0$ to prevent over-stimulating the dynamics of the AC plant which can cause a constraint violation. These bounds become less restrictive as the AC plant converges to the new equilibrium.

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