Vulnerability of State Variable Filter and Switched Capacitor Filters to Self-oscillatory Mode of Operation

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Abstract— **Vulnerability of some widely used filter structures to an undesired nondestructive stationary nonlinear mode of oscillation is discussed. This undesired mode of operation can be very difficult to detect during design, verification, and testing, yet when triggered the filter operation ceases until power is cycled to resume normal operation. The major non-linearities that cause this undesired mode of operation is also discussed.**

Keywords—switched capacitor filter, two integrator loop filter, state variable filter, filter reliability.

I. Introduction

Some continuous-time filter architectures are known for their excellent sensitivity properties and as a result are employed in a wide range of applications. Despite their popularity, two of the most attractive structures, the Akerberg- Mossberg filter [1] and the KHN biquad filter [2], [3], are known to be plagued by a stationary undesired oscillatory mode of operation. Several other low sensitivity structures [4-7] have been reported to have a similar problem. The newest of these works is over four decades old and there has been little mention of these problems of these concerns for many years. At the time these problems were reported, they were attributable to the nonlinear operation of the operational amplifiers of the 1970's. In these earlier works the undesired mode invariably was discovered with little effort in the laboratory and a heuristic approach involving the strategic placement of one or two diodes was usually effective at eliminating this undesired mode of operation. A quote from [1] summarizes the understanding of this problem in the earlier works "In their unstable mode the filter self-oscillates with maximum amplitude at a frequency close to the pole frequency, probably due to nonlinear effects in the amplifiers".

Since there has been little mention of these concerns in textbooks or research papers for many years, it might be comforting to assume the problem has gone away and is no longer of concern with modern operational amplifiers incorporated in the basic filter structures that are used today. In this paper, it will be shown that the vulnerability is still present but may be much more difficult to recognize than it was in the past. Before addressing this issue, a brief discussion about the undesired oscillatory mode of operation is necessary. In circuit structures where this undesired oscillatory mode of operation is present, the circuit may operate normally until the oscillatory mode is triggered. But once triggered, it is often necessary to cycle the supply voltage to return to the normal mode of operation. Thus, this is not associated with the desired poles of the circuit migrating into the right half-plane but rather with the existence of two or more stationary modes of operation. One being the desired mode and the other being undesired stationary modes of operation. But in contrast to the earlier works, the existence of the undesired stationary mode of operation can be extremely difficult to detect by either simulation or laboratory measurements yet it can be triggered by strategic inputs to the circuit. In critical systems such as transportation, health care, and military infrastructure, there is a growing emphasis on improving reliability. If such a mode of operation were to exist in a circuit, a system could fail if the mode were accidentally triggered by the user or intentionally triggered by an adversary. Aside from the potentially catastrophic effects, it might be difficult to even recognize the problem since cycling power would return the circuit to the normal mode of operation.

In this work it will be shown that three popular second-order filter structures that are representative of what are used today but that are vulnerable to the presence of an undesired stationary mode of oscillation will be discussed. One is basically the Tow-Thomas Biquad [4,5], still a widely used filter structure, and the others are switched-capacitor filters similar to that used today [9] in a commercial filter product. Though Thomas [5] indicated a vulnerability in the Tow-Thomas biquad, he indicated that the problem existed when designing filters to operate at high frequencies and few if any concerns have been expressed about this problem in two integrator loop continuous-time filters in recent works. From the early works on switched-capacitor filters [6-8] till today, there does not appear to be any mention of this potential problem in switched-capacitor structures. But in contrast to the earlier works where the undesired modes of operation were readily detected during testing, the undesired modes of operation can be extremely difficult to detect in the structures discussed in this paper. This unwanted self-oscillatory mode may be the cause of some unexplained system crashes or failures which invariably lead to system reliability issues.

To address this problem, it is necessary to review some basic concepts. Both the vulnerable continuous- time and the switched capacitor filters are discussed in Section II. Simulation results showing the existence of the undesired stationary oscillatory modes of operation are discussed in Section III and experimental results showing the self-oscillatory mode are presented in Section IV.

II. TWO INTEGRATOR LOOP BIQUAD FILTER

A. Continuous-time Tow-Thomas Filter

A basic two-integrator loop filter structure is shown in Fig. 1 using RC components. This filter is realized by inverting the output of a Miller integrator using an inverting amplifier and combining it with a summing lossy integrator. The realized filter is popularly known as the state variable filter with transfer function, f_0, and pole Q given by the expressions:

$$\frac{V_{out}(s)}{V_{in}} = \frac{\frac{R_4}{C_1C_2R_2R_5R_3}}{s^2+\frac{s}{R_QC_1}+\frac{R_4}{C_1C_2R_1R_2R_3}} \tag{1}$$

$$f_o = \frac{1}{2\pi}\sqrt{\frac{R_4}{C_1C_2R_1R_2R_3}} \tag{2}$$

$$Q = R_Q\sqrt{\frac{R_4C_1}{C_2R_1R_2R_3}} \tag{3}$$

This state variable filter, also known as the Tow-Thomas biquad filter [4]-[5], is vulnerable to the existence of a stationary undesired oscillatory mode of operation.

B. Switched capacitor filters

1) Two integrator loop switched capacitor filter

This state variable filter can be converted to a switched capacitor equivalent structure. One such conversion is shown in Fig. 2. Just as in the continuous-time case, the switched capacitor filter is achieved by combining the noninverting integrator and the summing lossy integrator with a summer. The circuits of Fig. 1 and Fig. 2 have corresponding transfer functions when the inverting amplifier gain (R_3=R_4) is unity.

Using equation (4), the well-known approximate relationship between a resistor and a switched capacitor, when the sampling clock frequency f_{CLK} is much greater than the signal frequency f_{SIG}, is given by the expression:

$$R_{EQ} \cong \frac{1}{f_{CLK}C_{EQ}} \tag{4}$$

This switched-capacitor filter structure is also vulnerable to the existence of a stationary undesired oscillatory mode of operation caused by the amplifier non-linearities.

The nonlinear self-oscillatory mode of operation in both the continuous-time and the corresponding switched-capacitor filters is mainly caused by the relationship between the saturation limits and the slew rate of the operational amplifier used in the implementation as well as the choice of the passive element values of the filter.

2) Fleischer-Laker switched capacitor filter

The Fleischer-Laker switched capacitor filter is another example of a switched capacitor filter vulnerable to this type of alternative mode of operation. Two configurations suggested in [10], F-circuit and the E-circuit, shown in Fig.3 and Fig.4, have shown the existence of an alternative mode besides the expected filter mode which has the transfer function of the E-circuit and F-circuit given by (5) and (6) respectively.

$$\frac{V_{02}(z)}{V_{in}(z)} = \frac{(AH-DJ)z^{-2}+(-AG+DI+DJ)z^{-1}-DI}{(BD-AE)z^{-2}+(AC+AE-2BD)z^{-1}+BD} \tag{5}$$

$$\frac{V_{02}(z)}{V_{in}(z)} = \frac{(AH-DJ)z^{-2}+(-AG+DI+DJ)z^{-1}-DI}{(BD)z^{-2}+(AC-2BD-DF)z^{-1}+BD+DF} \tag{6}$$

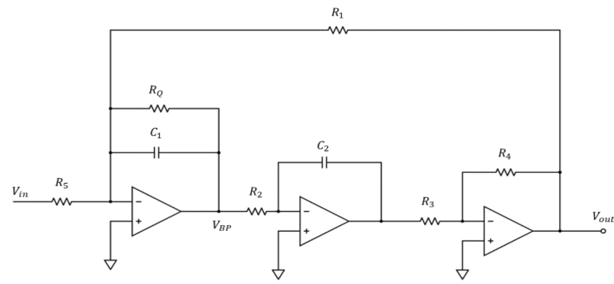


Fig. 1: State variable filter

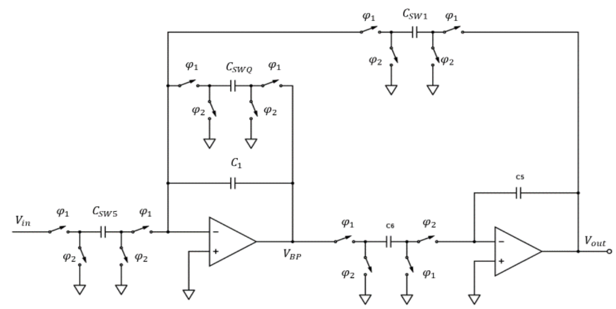


Fig. 2: Switched capacitor two integrator loop filter

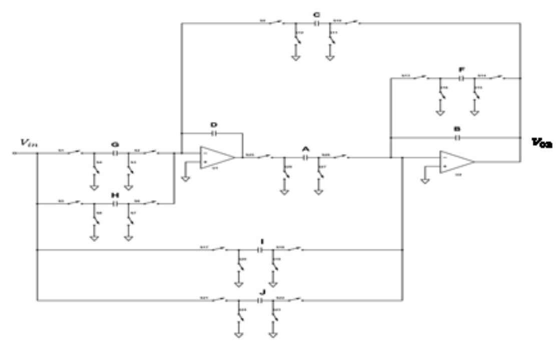


Fig. 3: Fleischer-Laker switched capacitor filter, F-circuit

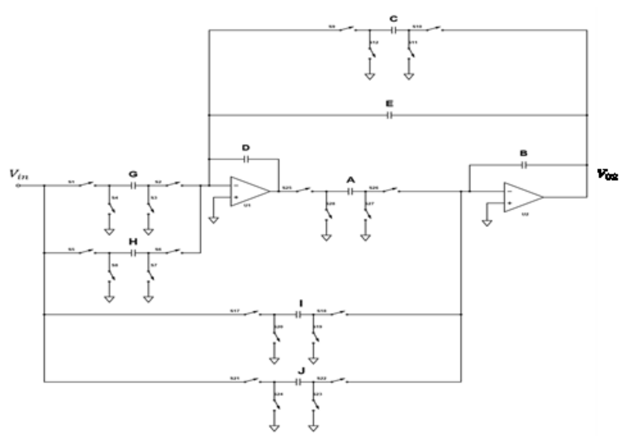


Fig. 4: Fleischer-Laker switched capacitor filter, E-circuit

III. Simulation Results

To demonstrate the existence of the oscillatory mode in all filters, a simple lowpass filter was designed. The filters were triggered with either an input sequence of signals or by setting some initial voltages across the capacitors, to trigger the filter into its alternative mode of operation.

A. *Test circuit*

A second-order lowpass filter was designed with f_0=7.23kHz and Q=15 using both the continuous-time and the switched-capacitor structures of Figs. 1-4. For these filters, the opamp had a positive saturation voltage (V_{DD}) of 15V and a negative saturation voltage (V_{SS}) of -15V, a slewrate of 0.5V/μs (both rising and falling), and a gain-bandwidth product of 1MHz.

Component values for continuous-time state variable filter:

- $R_1 = R_2$ =22kΩ
- $R_3 = R_4$ =10kΩ
- R_5 =55kΩ
- R_Q =330kΩ
- $C_1 = C_2$ =1nF

Component values for the switched-capacitor two integrator loop filter:

- f_{CLK}=1MHz
- $C_{sw1} = C_{sw2}$ =45.45pF
- C_{sw5} =18.18pF
- C_{swQ} = 3pF
- $C_1 = C_2$ =1nF

For the Fleischer-Laker switched capacitor filter, the following parameters were used:

TABLE I

Fleischer-Laker Filter Capacitor Values		
Capacitor	E-circuit	F-circuit
A	243.9pF	220.7pF
B	4.878nF	4.8543nF
C	2.409pF	2.512pF
D	60.9pF	55.16pF
E	3.678pF	Not used
F	Not used	14.709pF
G	1pF	1pF
H	Not used	Not used
I	1pF	1pF
J	1pF	1pF

The simulated magnitude response of the filters implemented above are shown in Fig. 5 and Fig. 6. The significance of this plot is to show that the filters have a desired magnitude response. A simulation of the time-domain response for various sinusoidal inputs showed results that are consistent with the magnitude response shown in Fig. 5 and Fig. 6.

B. *Undesired Nonlinear Oscillation*

For all filters, the input was set to ground (0V) and the initial values of C_1 and C_2 (or B and D in Fleischer-Laker filter) were both set to 15V. The 15V initial conditions were chosen because that voltage will trigger the filter into the undesired oscillatory mode of operation.

Shown in Fig. 7-9 are simulation results of the filters in the undesired oscillatory modes of operation. When the magnitudes of V_{SS} and V_{DD} are reduced beyond a certain point, the undesired oscillatory mode of operation was not observed. From these simulations, it can not be concluded that the undesired mode of operation does not exist for lower supply voltages, it can only be concluded that an oscillatory mode was not triggered. Although it may be the case that the oscillatory mode of operation vanishes for sufficiently small supply voltages. Similarly, when the slew rate of the amplifier is increased beyond a certain value, the oscillatory mode was not triggered but it can not be concluded that the undesired mode of operation does not exist for higher slew rates. Again, it may be the case that the oscillatory mode of operation vanishes for larger slew rates.

Fig. 10 shows the absence of the self-oscillatory mode of operation of the filters with either low saturation voltage, high slew rate, or both. It is important to note that the filter also performs its function as a filter i.e., it has 2 modes of operation.

All these filters can operate normally as a filter without any change to the circuit. Under the right conditions i.e., some input sequence or right initial values across the capacitors, this alternative mode can be triggered. These oscillatory modes are not a result of the poles moving into the right-hand plane. In fact, the poles were verified in simulation to be in the left-hand plane. The same F-circuit that is oscillating after being triggered into the alternative mode is shown in Fig.11 where it is operating as a filter without any modification to the circuit.

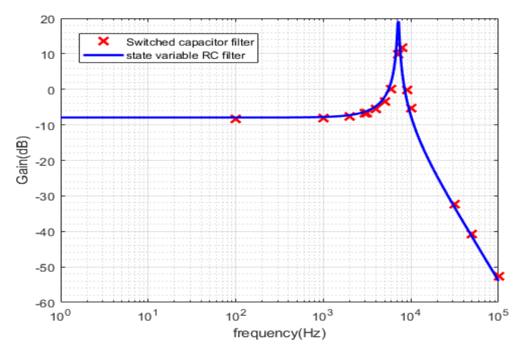


Fig. 5: Amplitude response of filters

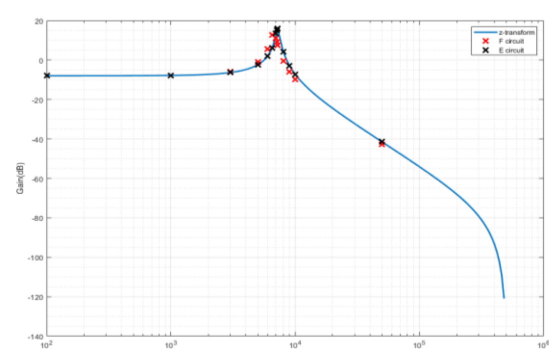


Fig. 6: Amplitude response of Fleischer-Laker filters

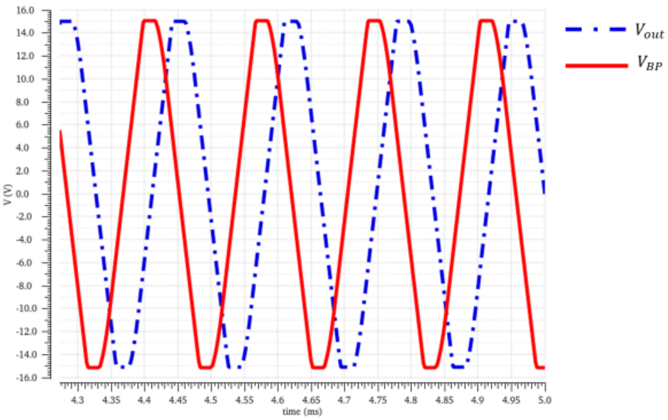


Fig. 7: State variable filter output voltages showing self-oscillation.

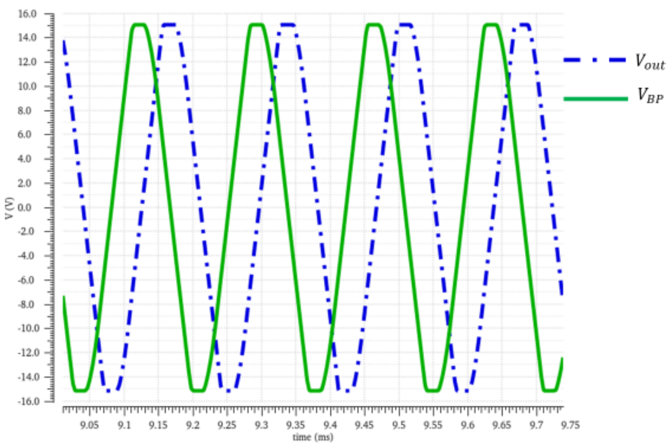


Fig. 8: Two-integrator loop Switched capacitor filter showing self-oscillation.

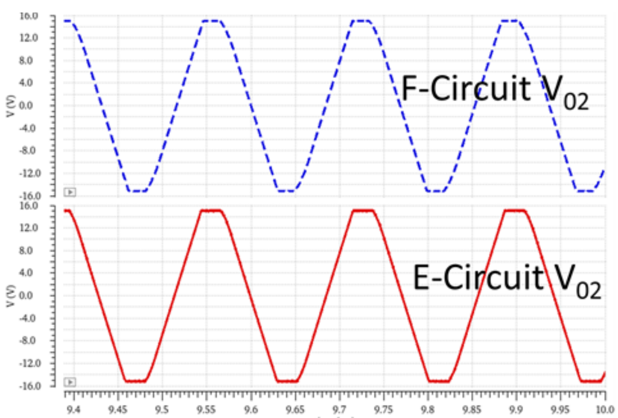


Fig. 9: Fleischer-Laker Switched capacitor filter showing self-oscillation with input grounded after being triggered

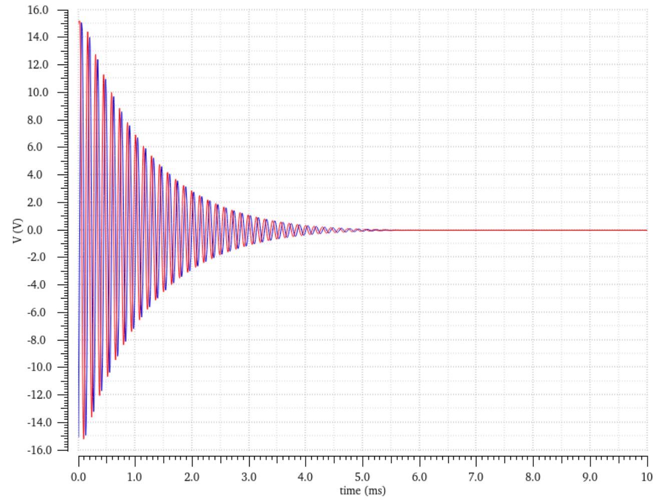
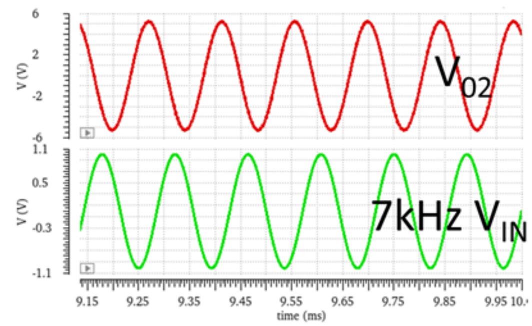
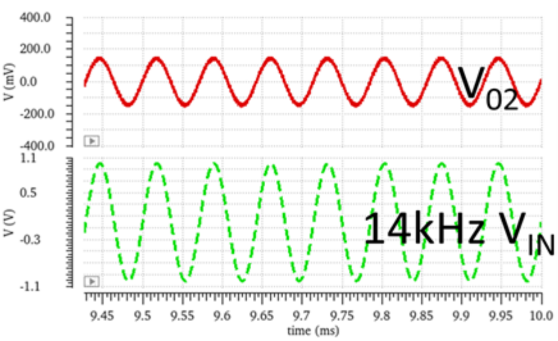


Fig. 10: Higher slew rate/lower saturation voltage



(a)



(b)

Fig.11: Fleischer-Laker F-Circuit operating as a filter in normal operation.

The vulnerability host map of Fig.12 presents some interesting insights about the continuous time Tow-Thomas filter. For various slew-rates and saturation voltages (VDD), the existence of the vulnerability of the filter is observed to be a function of the linear and non-linear parameters (slew-rates and saturation voltages) of the filter. Slew-rate is known to have two effects, first is a phase shift, and the second is a gain reduction [11], while the saturation voltage can act as a form of gain reduction. The relationship between these factors could lead to the existence of oscillatory vulnerability in the filter.

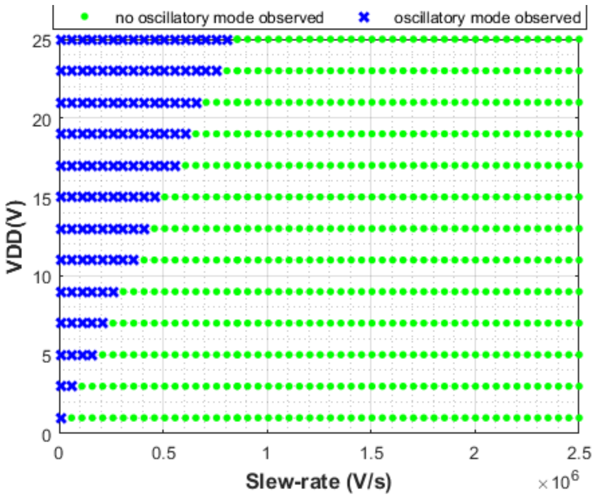


Fig.12: Vulnerability Host Map

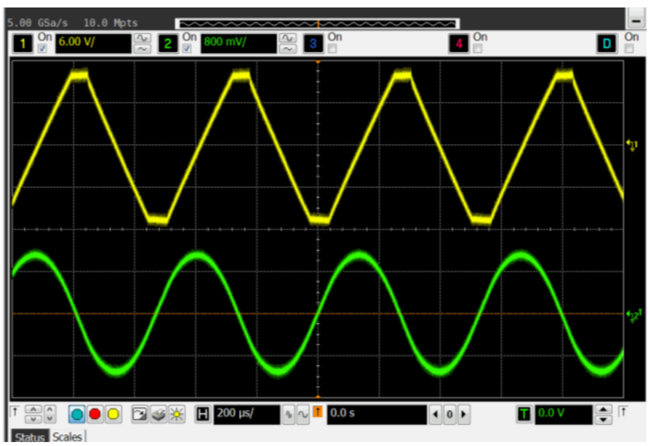


Fig. 13: Experimental results showing self-oscillatory mode of the continuous time state variable filter

IV. Experimental Results

To demonstrate the self-oscillatory mode of operation, the LM741 operational amplifier was used to build the filter shown in Fig. 1. The self-oscillatory mode was triggered by setting the initial voltage of the capacitors to the saturation voltage of the operational amplifier.

For this RC state variable filter, the following parameters were used:

- $R_1 = R_2 = 10k\Omega$
- $R_3 = 22k\Omega$
- $R_4 = 100k\Omega$
- $R_5 = 55k\Omega$
- $R_Q = 68k\Omega$
- $C_1 = C_2 = 9.4nF$

The measured results shown in Fig. 13 demonstrates the self-oscillatory mode in a state variable filter.

V. Conclusion

Three popular filter structures have been identified that possess two stationary modes of operation. One is the normal or desired mode whereby the circuit performs the desired filtering function and the other is an undesired stationary nonlinear oscillatory mode of operation. The presence or absence of the undesired oscillatory mode of operation is strongly dependent upon the saturation limits of the amplifier used in the filter and the slew rate limitation of that amplifier as well as a relationship between these two parameters and the passive elements in the filter.

Acknowledgment

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