

Algorithmic Acceleration of the Chebyshev-based Boundary Integral Equation Method for 3D Maxwell Problems using the Interpolated Factored Green Function

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A predominant majority of electromagnetic devices required for modern applications in today's society, such as high-speed wireless communications and integrated photonics, are difficult to model due to lack of analytical solutions, requiring full-wave numerical solution of Maxwell's equations. Popular numerical approaches for solving Maxwell's equations computationally include finite-difference, finite element, and boundary integral equation methods. Finite-difference and finite element methods, which are volumetric approaches, are often limited to being used for solving small to moderately sized problems due to resulting in prohibitively large and poorly conditioned linear systems. Boundary integral equation (BIE) methods, on the other hand, only require meshing and solving for unknowns on the surfaces between objects of different materials and can result in systems of equations with significantly fewer unknowns, especially in scenarios with high volume-to-surface area ratios. However, it is well known that BIE methods result in dense, rather than sparse, linear systems, which require $O(N^2)$ operations to solve iteratively when conditioned well or $O(N^3)$ to solve using dense linear algebra solvers, such as LU decomposition. Fortunately, numerous algorithmic acceleration approaches exist (e.g., FMM, Butterfly, AIM), but they are either specialized for low-frequency problems or require Fast Fourier Transforms (FFTs), which are challenging to parallelize effectively and limit their scaling to very large problems. The Interpolated Factored Green's Function (IFGF) was recently introduced for acoustic scattering [JCP 2021, Bauinger and Bruno] as an alternative algorithmic acceleration approach which also achieves $O(N \log N)$ computational complexity but can be parallelized very effectively due to relying on nested polynomial interpolation rather than on FFTs for propagating information from sources to targets. The IFGF works equally well for low and high-frequency problems and does not require different treatment for the high-frequency regime. The IFGF method works by factoring out a "centered factor" from the Green function kernel, which does not depend on the source coordinates and makes the resulting kernel (referred to as the "analytic factor") easy to interpolate with high accuracy over directional cone structures by using low-order polynomial interpolants. On its own, this interpolation approach can achieve $O(N^{3/2})$ time complexity in the best-case scenario; however, similar to multi-level approaches such as the MLFMM, this can be brought down to $O(N \log N)$ by producing an octree of boxes and leveraging nested interpolation to reuse interpolants produced at lower (smaller boxes) levels for the higher ones. In this work, we extend the IFGF method for the first time to the fully vectorial 3D Maxwell case. We will begin by introducing the general IFGF method, followed by covering details associated with its extension to Maxwell problems. Examples will be presented for dielectric scattering problems using the well-conditioned N-Müller formulation and timing data will be shown that confirms the $O(N \log N)$ computational complexity of the approach.

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