

Context and Research Objective:

Student growth mindset—the belief that intelligence can be developed with time and effort—shares strong positive associations with mathematics performance (Yeager et al., 2019). Growth-mindset holders attribute their challenges and failures to a lack of effort and show a mastery-oriented pattern by making more efforts or participating in remedial activities (Hong et al., 1999). Thus, it is reasonable to assume that a growth mindset fosters behavioral response (e.g., behavioral engagement) in the setback, further improving math performance. Unfortunately, such a mediation mechanism was rarely investigated. Furthermore, few studies (e.g., Qin et al., 2021) investigated the contextual differences in the mediation mechanism, although recent studies increasingly evaluated the heterogeneity of the growth mindset impact (Yeager & Dweck, 2020).

This study employs causal moderated mediation analysis to address the following two research questions:

1. Does student growth mindset improve math performance through math engagement?
2. Does this causal mediation mechanism vary by teaching quality or student biological sex?

Setting/Participants/Measures:

Participants were 1350 (8th grade: 566, 10th grade: 499, 12th grade: 285; 51% female; 68% White, 27% Black; 47% low income) adolescents from 15 public schools located in a metropolitan area of the northeastern U.S. All students were asked to participate in a multi-year study on student experiences and positive youth development. Table 1 lists constructs and their associated survey items. Table 2 displays the correlation matrix and descriptive statistics.

Outcome-Math Performance is a mathematics semester grade (mean: 82.64, SD: 12.19).

Mediator-Student Classroom Behavioral Engagement is a well-validated classroom behavioral engagement scale in math (Wang et al., 2016) (mean: 3.94, SD: 0.81, $\alpha=0.89$).

Treatment-Growth Mindset is a well-validated Implicit theory of intelligence scale (Dweck et al., 1995; Blackwell et al., 2007) (mean: 3.84, SD: 1.03, $\alpha=0.94$). We set high and low growth mindset levels at one standard deviation above and below the mean ($t_1 = 4.86$ vs. $t_2 = 2.81$), respectively, and focus on assessing the impact of a high vs. low growth mindset.

Moderator-Teaching Quality and Biological Sex

Students' perceived instructional quality of math teachers is a well-validated Instructional Quality scale (Wang et al., 2020) (mean: 3.91, SD: 0.78, $\alpha=0.92$). The student's perspective of teaching

quality predicts math engagement and performance more than the teacher's perspective (Wang et al., 2020). We set high and low teaching quality at one standard deviation above and below the mean ($w_1 = 4.69$ vs. $w_2 = 3.14$). Males and females were assigned 1 and 0 ($w_1 = Male(1)$ vs. $w_2 = Female(0)$).

Research Design and Data Analysis:

We applied a causal moderated mediation analysis method (Qin & Wang, in press) developed under the potential outcome framework.

Definition

To answer our first research question, we focused on the population average natural indirect effect (NIE), defined as the average of individual-specific NIE over all individuals. To answer our second research question, we assessed how the conditional average of NIE changed with differences in teaching quality and biological sex. It is defined as the average of an individual-specific NIE over the individuals within given levels of teaching quality and biological sex. Table 3 displays statistical notation for potential outcomes and defines individual causal mediation effects.

Identification

The identification of causal mediation effects relies on the assumption that there are (a) no unmeasured pretreatment confounding of the treatment-mediator, treatment-outcome, or mediator-outcome relationships and (b) no unmeasured posttreatment confounding of the mediator-outcome relationships within levels of moderators.

Estimation

Based on the above assumptions, we identified causal mediation effects based on the mediator and outcome models in Figure 1. The Monte Carlo method was used to estimate and test causal effects.

Sensitivity Analysis

We conducted a simulation-based sensitivity analysis to assess whether the results were robust to potential unmeasured pretreatment confounding variables. The idea is to simulate an unmeasured pretreatment confounder from its conditional distribution at a given strength and compare the results before and after adjusting for it in the analysis.

We used the R-package *mice* to impute missing values via multiple imputation. For each imputed data set, we used the R-package *mediate* to estimate causal effects, assess sensitivity, and visualize results. The reported results are pooled over ten imputed data sets.

Finding/Results:***Does student growth mindset improve math performance through math engagement on average?***

On average, NIE was significant and positive (See Table 4). When all other possible pathways underlying the impact of a growth mindset on mathematics engagement were held constant at the level under a high growth mindset, the growth mindset-induced increase in engagement significantly increased one's math achievement ($\beta = 0.50$, $SE = 0.20$). Contrary to the indirect effect, the direct effect was insignificant. The proportion mediated was 0.47, suggesting that the effect of a growth mindset on math achievement was primarily transmitted through math engagement.

Does this causal mediation mechanism vary by teaching quality and biological sex?

The mediating role of engagement was more salient with higher teaching quality, and this effect only reached statistical significance when teaching quality was above 3.58 (See Figure 2). The result indicates that higher teaching quality was a prerequisite for a significant mediating role of math engagement. Math engagement played a more critical role in transmitting the impact of a growth mindset on math achievement among girls than among boys though the result is insignificant (See Table 4).

Sensitivity Analysis

The above causal conclusions were made based on the identification assumptions (a) and (b). Assuming no posttreatment confounder of the mediator-outcome relationship, we conducted a simulation-based sensitivity analysis to assess the influence of unmeasured pretreatment confounding (Qin & Wang, in press). The results show that the sign and significance of the original results would not be reversed even if there were a strong unmeasured pretreatment confounder, indicating the robustness of the violation of an assumption (a).

Conclusion:

We found evidence of causal mediation and causal moderated mediation within the links between student growth mindset, engagement, and math achievement that emphasize the importance of contextual support and student characteristics. Policymakers should continue creating quality classroom environments where teachers can facilitate students' motivated behaviors. Future research needs to investigate the robustness of an assumption (b) violation.

Figure 1: Mediator and outcome models and causal effect estimands

The mediator model:

$$Engage_{ij} = \beta_0^m + \beta_t^m Growth_{ij} + \mathbf{X}_{ij} \boldsymbol{\beta}_x^m + r_{ij}^m + u_{0j}^m,$$

$$r_{ij}^m \sim N(0, \sigma_m^2), u_{0j}^m \sim N(0, \tau_{00}^m).$$

where

$$\beta_0^m = \beta_{00}^m + \beta_{01}^m Teaching_{ij} + \beta_{02}^m Sex_{ij} + \beta_{03}^m Meta_{ij}$$

$$\beta_t^m = \beta_{t0}^m + \beta_{t1}^m Teaching_{ij} + \beta_{t2}^m Sex_{ij} + \beta_{t3}^m Meta_{ij}$$

The outcome model:

$$MathScore_{ij} = \beta_0^y + \beta_t^y Growth_{ij} + \beta_m^y Engage_{ij} + \beta_{tm}^y Growth_{ij} * Engage_{ij} + \mathbf{X}_{ij} \boldsymbol{\beta}_x^y + r_{ij}^y$$

$$+ u_{0j}^y,$$

$$r_{ij}^y \sim N(0, \sigma_y^2), u_{0j}^y \sim N(0, \tau_{00}^y).$$

where

$$\beta_0^y = \beta_{00}^y + \beta_{01}^y Teaching_{ij} + \beta_{02}^y Sex_{ij} + \beta_{03}^y Meta_{ij}$$

$$\beta_t^y = \beta_{t0}^y + \beta_{t1}^y Teaching_{ij} + \beta_{t2}^y Sex_{ij} + \beta_{t3}^y Meta_{ij}$$

$$\beta_m^y = \beta_{m0}^y + \beta_{m1}^y Teaching_{ij} + \beta_{m2}^y Sex_{ij} + \beta_{m3}^y Meta_{ij}$$

$$\beta_{tm}^y = \beta_{tm0}^y + \beta_{tm1}^y Teaching_{ij} + \beta_{tm2}^y Sex_{ij} + \beta_{tm3}^y Meta_{ij}$$

Based on the identification assumptions, NIE and NDE can be identified as follows:

$$\delta_{NIE} = (\beta_m^y + \beta_{tm}^y t_1) \beta_t^m (t_1 - t_2),$$

$$\delta_{NDE} = (\beta_t^y + \beta_{tm}^y (\beta_0^m + \beta_t^m t_2 + E[\mathbf{X}_{ij}] \boldsymbol{\beta}_x^m)) (t_1 - t_2),$$

Note 1: $\mathbf{X}_{ij}$ is a vector of pre-treatment confounders for the mediator and outcome model. As Wang et al. (2021) found that metacognition is a significant moderator, metacognition is one of the moderators for a correct specification. However, our fundamental interest lies in the moderation effect of teaching quality and sex. A random intercept is included in both models to account for the clustered data structure that students were nested within 145 classes (intraclass correlation = 0.25).

Note 2: To ease model specifications, we expressed coefficients of mediator and outcome models as a function of teaching quality and metacognition in the hierarchical form (Qin & Wang, in press). The models are equivalent to the following models in the combined form:

$$Engage_{ij} = \beta_{00}^m + \beta_{01}^m Teaching_{ij} + \beta_{02}^m Sex_{ij} + \beta_{03}^m Meta_{ij} + (\beta_{t0}^m + \beta_{t1}^m Teaching_{ij} + \beta_{t2}^m Sex_{ij}$$

$$+ \beta_{t3}^m Meta_{ij}) Growth_{ij} + \mathbf{X}_{ij} \boldsymbol{\beta}_x^m + r_{ij}^m + u_{0j}^m,$$

$$MathScore_{ij} = \beta_{00}^y + \beta_{01}^y Teaching_{ij} + \beta_{02}^y Sex_{ij} + \beta_{03}^y Meta_{ij} + (\beta_{t0}^y + \beta_{t1}^y Teaching_{ij}$$

$$+ \beta_{t2}^y Sex_{ij} + \beta_{t3}^y Meta_{ij}) Growth_{ij} + (\beta_{m0}^y + \beta_{m1}^y Teaching_{ij} + \beta_{m2}^y Sex_{ij}$$

$$+ \beta_{m3}^y Meta_{ij}) Engage_{ij} + (\beta_{tm0}^y + \beta_{tm1}^y Teaching_{ij} + \beta_{tm2}^y Sex_{ij}$$

$$+ \beta_{tm3}^y Meta_{ij}) Growth_{ij} * Engage_{ij} + \mathbf{X}_{ij} \boldsymbol{\beta}_x^y + r_{ij}^y + u_{0j}^y$$

Table 1: List of variables

Variables	Label
Treatment variable (Spring 2018)	
Growth mindset ($\alpha=0.94$)	To be honest, you can't really change how intelligent you are in math. (R)
Implicit theory of intelligence scale (Dweck et al., 1995; Blackwell et al., 2007)	You have a certain amount of math intelligence, and you can't really do much to change it. (R)
	Your math intelligence is something about you that you can't change very much. (R)
	You can learn new things but you can't really change your basic math intelligence. (R)
Mediator variable (Fall 2018)	
Math engagement ($\alpha=0.89$)	I stay focused in math class.
Behavioral engagement scale in math (Wang et al., 2016; Fredricks et al., 2016)	I put effort into learning math.
	I keep trying even if something is hard in math class.
	I complete my math homework on time.
	I don't participate in math class. (R)
	I do other things when I am supposed to be paying attention in math class. (R)
	If I don't understand a task in math class, I give up right way. (R)
Outcome variable (End of Fall 2018)	
Math performance	Math Course Grade (0-100)
Moderator variables	
Teaching quality ($\alpha=0.92$) (Fall 2017)	My math teacher encourages me to solve problems on my own.
Instructional quality scale (Wang et al., 2020)	My math teacher asks me to think about what I have learned at the end of activities.
	My math teacher encourages me to consider different solutions and points of view.
	My math teacher explains it in a new way if I say that I <u>don't</u> understand something.
	My math teacher connects what I am learning to what I already know.
	My math teacher provides challenging work in math class.
	My math teacher suggests ways that I can learn more.
	My math teacher keeps working with me until I understand what we are doing.
	My math teacher gives clear instructions for how to do well in math class.
	My math teacher respects me.
	My math teacher says nice things to me.
	My math teacher helps me when I need help.
	I feel comfortable in math class.
	My math teacher understands how I feel about things in class.

Table 1: List of variables (**continued**)

Male (Spring 2018)	1: Male, 0: Female
Meta cognition (Fall 2017)	I go through the work that I do for math and make sure that it's right. I try to connect what I am learning in math to things I have learned before. I try to understand my mistakes when I get something wrong in math.
Pre-treatment confounders	
Prior math performance (Fall 2017)	Math Course Grade 1 (0-100)
Free/reduced lunch status (AY 2017)	1: free/reduced, 0: paid
Special needs status (AY 2017)	1: Yes, 0: No
Grade 8 indicator (Fall 2018)	1: Yes, 0: No
Grade 12 indicator (Fall 2018)	1: Yes, 0: No
Black (Spring 2018)	1: Yes, 0: No
White (Spring 2018)	1: Yes, 0: No
Math interest (Fall 2017)	I look forward to math class. I enjoy learning new things about math. I feel good when I am in math class. I think that math class is boring. (R)
Prior Growth mindset (Spring 2017)	To be honest, you can't really change how intelligent you are in math. (R) You have a certain amount of math intelligence, and you can't really do much to change it. (R) Your math intelligence is something about you that you can't change very much. (R) You can learn new things but you can't really change your basic math intelligence. (R)
Prior Math engagement (Spring 2017)	I stay focused in math class. I put effort into learning math. I keep trying even if something is hard in math class. I complete my math homework on time I don't participate in math class. (R) I do other things when I am supposed to be paying attention in math class. (R) If I don't understand a task in math class, I give up right way. (R)

Note: R indicates the reversed item. AY stands for the academic year. Value ranges from 1 to 5 if not specified.

Table 2: Bivariate Correlation Matrix and Descriptive Statistics before imputation

Variable	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1. Growth mindset	1															
2. Engagement	0.27	1														
3. Math performance	0.19	0.42	1													
4. Teaching quality	0.27	0.29	0.16	1												
5. Male	-0.10	-0.08	-0.19	-0.05	1											
6. Meta cognition	0.30	0.44	0.24	0.48	-0.09	1										
7. Prior math performance	0.21	0.31	0.53	0.29	-0.18	0.32	1									
8. Free lunch	-0.05	-0.18	-0.29	-0.14	-0.03	-0.16	-0.23	1								
9. Special needs	-0.09	-0.09	-0.15	-0.03	0.08	-0.04	-0.08	0.08	1							
10. Grade 8	0.15	0.09	0.00	0.07	0.00	0.05	0.07	0.09	-0.02	1						
11. Grade 12	-0.12	0.02	0.06	-0.01	-0.04	-0.04	-0.01	0.01	-0.03	-0.44	1					
12. Black	-0.01	-0.14	-0.28	-0.11	-0.01	-0.07	-0.15	0.47	0.03	0.15	-0.04	1				
13. White	0.00	0.15	0.27	0.08	0.03	0.06	0.14	-0.50	-0.01	-0.15	0.04	-0.89	1			
14. Math interest	0.25	0.36	0.17	0.50	0.05	0.55	0.28	-0.10	0.03	0.17	-0.10	0.01	-0.01	1		
15. Prior growth mindset	0.48	0.18	0.15	0.20	-0.04	0.21	0.14	-0.09	-0.06	0.11	-0.13	-0.07	0.07	0.19	1	
16. Prior engagement	0.28	0.51	0.27	0.33	-0.05	0.49	0.29	-0.25	-0.05	0.14	-0.16	-0.21	0.20	0.42	0.31	1
Mean	3.84	3.94	83.59	3.91	0.49	3.80	82.64	0.48	0.10	0.42	0.21	0.27	0.68	3.28	3.90	4.04
Standard Deviation	1.03	0.81	12.21	0.78	0.50	0.91	12.19	0.50	0.30	0.49	0.41	0.45	0.47	1.14	1.01	0.77
N	1309	1304	1264	1139	1350	1298	1341	1350	1350	1350	1350	1350	1350	1302	1325	1332
% of missingness	3.0%	3.4%	6.4%	15.7%	0%	3.9%	0.7%	0%	0%	0%	0%	0%	0%	3.6%	1.9%	1.3%

Note: Bolded values indicate significant at $p < .05$.

Table 3. Definitions of Potential Outcomes and Causal Effects at the individual level

	Notation	Definition (for individual i)
	$Y_i(t_1, M_i(t_1))$	Potential math performance if $T_i = t_1$ (high growth mindset)
	$Y_i(t_2, M_i(t_2))$	Potential math performance if $T_i = t_2$ (low growth mindset)
	$Y_i(t_1, M_i(t_2))$	Potential math performance if $T_i = t_1$ (high growth mindset) yet the engagement takes the value that would result if $T_i = t_2$ (low growth mindset)
	$Y_i(t_2, M_i(t_1))$	Potential math performance if $T_i = t_2$ (low performance) yet the engagement takes the value that would result if $T_i = t_1$ (high growth mindset)
Total Effect	$\delta_{TEi} = Y_i(t_1, M_i(t_1)) - Y_i(t_2, M_i(t_2))$	The total high growth mindset effect on math performance
Natural Indirect Effect (NIE)	$\delta_{NIEi} = Y_i(t_1, M_i(t_1)) - Y_i(t_1, M_i(t_2))$	The high growth mindset effect on the math performance under $T_i = t_1$ (high growth mindset) transmitted solely through the high growth mindset-induced change in the engagement
Natural Direct Effect (NDE)	$\delta_{NDEi} = Y_i(t_1, M_i(t_2)) - Y_i(t_2, M_i(t_2))$	The high growth mindset effect on the math performance if the engagement is held at the level that would be realized under $T_i = t_2$ (low growth mindset)
Pure Indirect Effect (PIE)	$\delta_{PIEi} = Y_i(t_2, M_i(t_1)) - Y_i(t_2, M_i(t_2))$	The high growth mindset effect on the math performance under $T_i = t_2$ (low growth mindset) transmitted solely through the high growth mindset-induced change in the engagement
Total Direct Effect (TDE)	$\delta_{TDEi} = Y_i(t_1, M_i(t_1)) - Y_i(t_2, M_i(t_1))$	The high growth mindset effect on the math performance if the engagement is held at the level that would be realized under $T_i = t_1$ (high growth mindset)
Natural Treatment-by-Mediator Interaction Effect (INT)	$\delta_{INTi} = \delta_{TIEi} - \delta_{PIEi} = \delta_{TDEi} - \delta_{PDEi}$	The difference in how the high growth mindset-induced change in the engagement affects the math performance between the treatment conditions t_1 and t_2 . (high and low growth mindset)

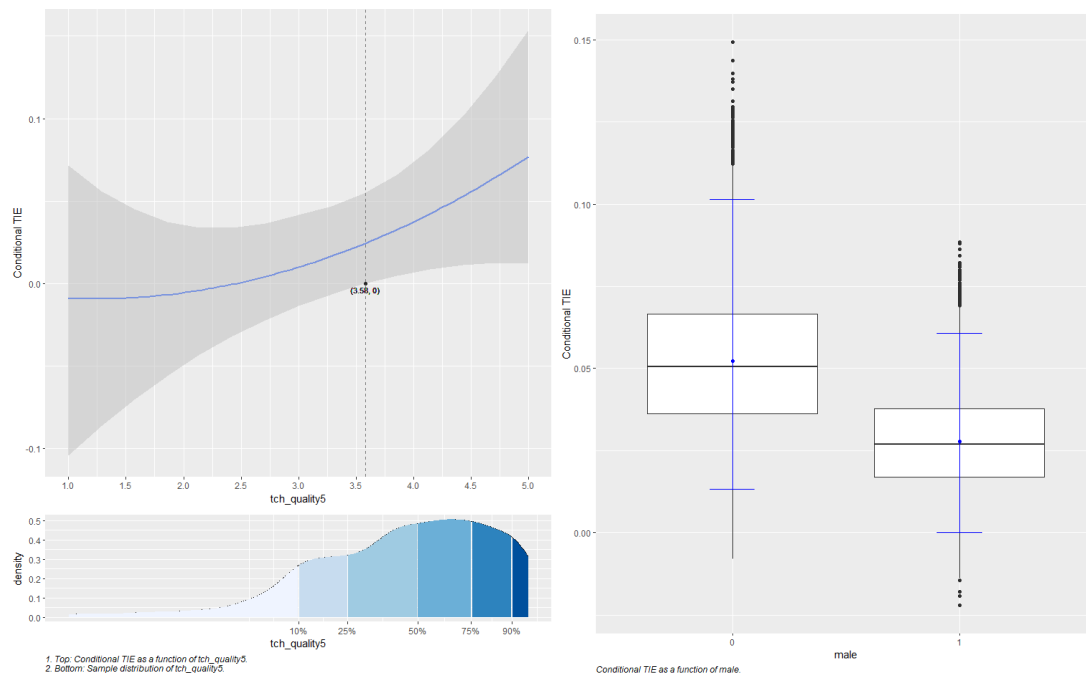
Note: The table is a modified version of the one of Qin and Wang (in press)

Table 4: The pooled estimated population average causal effects ($t_1 = 4.86$ vs $t_2 = 2.81$) moderated by teaching quality ($w_1 = 4.69$ vs. $w_2 = 3.14$) and biological sex ($w_1 = \text{Male}(1)$ vs. $w_2 = \text{Female}(0)$)

	Estimate (SE)	Effect size
NIE	0.50*(0.20)	0.0414
NDE	0.55(0.72)	0.0453
Conditional NIE by teaching quality	0.62†(0.36)	0.0513
Conditional NIE by biological sex	-0.30(0.28)	-0.0239

Note: * $p < 0.05$, † $p < 0.10$. Effect size is calculated by standardizing both independent and dependent variables. High and low teaching quality are respectively set at one standard deviation above and below the mean ($w_1 = 4.69$ vs $w_2 = 3.14$). Other two moderators are not conditioned as specific values.

Figure 2: Conditional NIE by teaching quality and biological sex



Note: The conditional NIE is in effect size. The only dependent variable is standardized. The plot is produced by combining ten imputed data frames.