Distribution of Chores with Information Asymmetry

Extended Abstract

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ABSTRACT

One well-regarded fairness notion in dividing indivisible chores is envy-freeness up to one item (EF1), which requires that pairwise envy can be eliminated by the removal of a single item. While an EF1 and Pareto optimal (PO) allocation of goods can always be found via well-known algorithms, even the existence of such solutions for chores remains open, to date. We take an *epistemic* approach to identify such allocations utilizing information asymmetry by introducing *dubious chores* – items that inflict no cost on receiving agents but are perceived to be costly by others. On a technical level, dubious chores provide a more fine-grained approximation of envy-freeness than EF1. We show that finding allocations with minimal number of dubious chores is computationally hard. Nonetheless, we prove the existence of envy-free and fractional PO allocations for n agents with only $2n-2$ dubious chores and strengthen it to $n-1$ dubious chores in four special classes of valuations.

KEYWORDS

Fair division; Resource allocation; Computational social choice

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1 INTRODUCTION

Fair allocation of resources is a fundamental role of any economy with considerable applications in healthcare, charitable donations, waste management, and task allocation [2, 8, 11, 12, 20, 27–29]. In recent years, significant research has investigated axioms and algorithmic techniques for allocating chores [5, 10, 14, 16, 23]. One canonical notion, *envy-freeness up to one item* (EF1), asserts that pairwise envy between agents can be eliminated by counterfactually removing a single item [11, 25]. An EF1 allocation of goods always exists and can be computed efficiently with economic efficiency notions such as Pareto optimality (PO) [7, 13]. In contrast, for chores, even the existence of such allocations remains open to date, leading to several works on restricted domains [3, 15, 19, 24].

1.1 Our Contributions

We take a different, *epistemic*, approach that utilizes *information asymmetry* to differentiate the allocation of chores that agents are informed about from the allocation they actually receive. The difference is an over-representation about which tasks agents complete, above-and-beyond those they are actually assigned. Our contribution is a novel fairness notion, DEF-k which uses dubious chores, items that are perceived as costly but inflict no actual cost on the receiving agent. Conceptually, dubious chores provide a natural approach when no envy-free solution exists. This is a particularly convenient solution in settings where agents do not have direct means of communication and cannot verify the exact costs incurred by each agent, such as with distributed computing of high-complexity problems or decentralized training of large language models. Technically, DEF-k provides a more fine-grained approximation of envy-freeness than EF1. This enables progress towards addressing open problems in fair allocation of indivisible chores, such as the existence and computation of EF1 and PO. Hence, the significance in DEF-k is in identifying allocations with minimal k. [1]

1.2 Preliminaries

An *instance* $\mathcal{I} = \langle \mathcal{N}, \mathcal{M}, \mathcal{V} \rangle$ is defined by a set of n *agents* $\mathcal{N}$, a set of m *chores* $\mathcal{M}$, and additive *valuations* $\mathcal{V} = (v_i)_{i \in \mathcal{N}}$ that specify the disutility $v_i(S)$ that each agent has for a subset of chores $S \subseteq M$. Additionally, we consider four special cases of restricted valuations: (1) *binary* where every agent values each item at 0 or -1, (2) *identical* where every agent has the same value per chore, (3) *bivalued* where every agent values each item at either x or y, where $x < y < 0$, (4) *two-types of chores* where chores fall into two types and every agent values chores of the same type identically.

An *allocation* $A = (A_i)_{i \in \mathcal{N}}$ is an n-partition of the chores $\mathcal{M}$ among the agents. An allocation satisfies *envy-freeness up to one chore* (EF1) if $\forall i, h \in \mathcal{N}$, $\exists c \in A_i : v_i(A_i \setminus \{c\}) \geq v_i(A_h)$. An allocation is *fractionally Pareto optimal* (fPO) if there is no complete fractional allocation (where chores may be partially allocated) that makes all agents weakly better-off and no agent worse-off.

[1] More details can be found in the full version of the paper [22].

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Definition 1. *A* dubious chore c' *allocated to agent* $i \in \mathcal{N}$ *is a copy of the chore* $c \in \mathcal{M}$ *such that* $v_h(\{c'\}) = v_h(\{c\})\ \forall h \neq i$ *and* $v_i(\{c'\}) = 0$. *An allocation* A *is* envy-free up to k dubious chores *(*DEF-k*) if there exists* k *dubious chores that, upon being additionally allocated, the augmented allocation is envy-free.*

Finally, a *price vector* $p : \mathcal{M} \to \mathbb{R}$ is a payment that each agent receives for doing a chore. Given allocation A, the pair (A, p) is a *Fisher market equilibrium* (FME) if $A_i \subseteq \arg\min_{c \in \mathcal{M}} |v_i(\{c\})|/p(c)$, $\forall i \in \mathcal{N}$. It is known that any such A is fPO [26]. A Fisher market equilibrium (A, p) is *price envy-free up to one item (pEF1)* if $\forall i, j \in \mathcal{N}$, either $A_i = \emptyset$ or $\exists c \in A_i\ :\ p(A_i \backslash \{c\}) \leq p(A_j)$.

2 FAIRNESS WITH DUBIOUS CHORES

We begin by analyzing fairness with dubious chores without any efficiency requirement. We start with the decision problem of finding the optimal k for which a DEF-k allocation exists.

Theorem 1. *Given an instance* $\mathcal{I} = \langle \mathcal{N}, \mathcal{M}, \mathcal{V} \rangle$, *for every fixed constant* $k \in \mathbb{Z}_{\geq 0}$, *deciding if the instance admits a* DEF-k *allocation* A *is NP-complete, even when valuations are identical or binary.*

Theorem 1 is proved using a reduction from `Partition` for identical valuations and `Set Splitting` for binary valuations [17]. This implies that minimizing k for DEF-k existence is NP-hard as well. Moreover, since hardness holds for $k = 0$, this problem has no polynomial-time constant approximation scheme unless P = NP.

Corollary 1. *Given instance* $\mathcal{I} = \langle \mathcal{N}, \mathcal{M}, \mathcal{V} \rangle$, *unless P=NP, there is no polynomial-time algorithm that gives a constant approximation for the problem of finding a* DEF-k *allocation with minimal* k.

Theorem 1 holds when k is a fixed constant. If we do not fix k, even the verification problem of whether a given allocation could be augmented with k dubious chores to become envy-free is computationally intractable.

Theorem 2. *Given an instance* $\mathcal{I} = \langle \mathcal{N}, \mathcal{M}, \mathcal{V} \rangle$ *with binary valuations and an allocation* A, *deciding if allocation* A *is* DEF-k *is NP-complete.*

This is proved via a reduction form `Restricted Exact Cover by 3-Sets (RX3C)` [21]. In lieu of this computational hardness, we establish upper-bounds on k for DEF-k existence. While EF1 allocations can be computed in polynomial time [3, 9], such allocations may require many dubious chores to become envy-free.

Proposition 1. *Given an instance* $\mathcal{I} = \langle \mathcal{N}, \mathcal{M}, \mathcal{V} \rangle$, *every* EF1 *allocation* A *is also* DEF-$n(n-1)$.

This follows since for each pairwise envy relation, the envied agent can dubiously receive the "worst" chore of the envious agent to remove the envy. This bound can be improved as follows.

Theorem 3. *Given an instance* $\mathcal{I} = \langle \mathcal{N}, \mathcal{M}, \mathcal{V} \rangle$, `Round Robin` *returns a* DEF-$(n-1)$ *allocation.*

The improvement arises since each agent chooses the best of the remaining chores for each round. Hence, in each round, there is a single agreed-upon "worst" chore, the latest chore allocated, that could eliminate all envy-relations when dubiously allocated. We note that not every EF1 allocation is DEF-$(n-1)$, nor is any DEF-$(n-1)$ allocation EF1. Furthermore, the existence of *any* DEF-k allocation is tight at $k = n-1$; that is, DEF-$(n-2)$ may not exist.

3 FAIRNESS AND EFFICIENCY

We proceed to demonstrate the existence and computation of DEF-k allocations with fractional Pareto optimality.

Theorem 4. *Given an instance* $\mathcal{I} = \langle \mathcal{N}, \mathcal{M}, \mathcal{V} \rangle$, *there always exists a* DEF-$(2n-2)$ *and* fPO *allocation.*

Our approach is based on Barman and Krishnamurthy [6]'s demonstration that a FME always exists. Let i^* denote the agent with the highest priced bundle $p(A_{i^*})$ and c denote the chore with the highest price $p(c)$; we prove that giving each agent besides i^* two dubious copies of c yields envy-freeness. We note the same algorithm was utilized by Akrami et al. [1] to attain an EF1 and fPO allocation by introducing at most $n-1$ actual copies of chores. Next, we focus on four restricted domains of preferences.

Theorem 5. *For any instance* $\mathcal{I} = \langle \mathcal{N}, \mathcal{M}, \mathcal{V} \rangle$ *with binary, identical, or bivalued valuations, or two-types of chores, there exists a* DEF-$(n-1)$ + fPO *allocation that can be computed in polynomial time.*

A DEF-$(n-1)$ + fPO allocation may be attained for each case, intuitively, as follows. For binary valuations, all the 0-valued chores may be allocated, followed by the -1-valued chores as evenly-distributed as possible. Identical valuations are by-definition fPO; allocated them with `Round Robin` attains DEF-$(n-1)$. For bivalued valuations, we prove that a pEF1 FME (A, p) is DEF-$(n-1)$; Ebadian et al. [15] and Garg et al. [18]'s algorithms find A in polynomial time. For two-types of chores, Aziz et al. [4] find an allocation A with certain properties in polynomial time; we identify a price vector p such that (A, p) is a pEF1 FME.

4 DISCUSSION

We have proposed a novel epistemic framework for fair allocations of chores. While our approach may appear similar to EF1 and duplicating chores, there are significant differences with these models. First, agents in our approach measure whether an allocation is fair based on their information, which is subjective and may differ across agents. This contrasts EF1, which requires agents to counterfactually reason about other agents' bundles. Moreover, DEF-k offers a more fine-grained approximation of EF than EF1. Whereas the latter recognizes any allocation *close to* an EF1 allocation, DEF-k precisely identifies a trade-off between fairness and transparency. That is, a DEF-0 allocation is necessarily EF, while at most k units of transparency must be compromised to make a DEF-k allocation envy-free for $k > 0$.

Second, our approach contrasts recent work by Akrami et al. [1] who introduced real copies of chores (i.e., "*surplus*") into the original fair division instance. This differs from our approach on a conceptual level because (i) duplicates introduce *real* additional cost on the receiving agents, reducing overall welfare, and (ii) more duplicates than dubious chores may be needed to eliminate envy.

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