

Steady-State Social Distancing and Vaccination[†]

By CHRISTOPHER AVERY, FREDERICK CHEN, AND DAVID MCADAMS*

This paper analyzes an economic-epidemiological model of infectious disease where it is possible to become infected more than once and individual agents make endogenous choices of social distancing and vaccine adoption. Protective actions adopted by any one person reduce future risks to other people. The positive externalities associated with these behaviors provide motivation for vaccine and social-distancing subsidies, but subsidizing one protective action reduces incentives for other protective actions. A vaccine subsidy increases vaccine adoption and reduces steady-state infection prevalence; a social distancing subsidy can either increase or reduce steady-state infection prevalence. (JEL D62, D91, I12, I18)

The COVID-19 pandemic sparked renewed interest in the economics of infectious disease.¹ Given the prevailing initial view that prior infection provided lasting immunity against future infection, most COVID-related papers have focused on the “Susceptible-Infected-Recovered” (SIR) model. During the course of the pandemic, however, it has become apparent that it is possible to contract COVID more than once, both because of waning immunity and the development of new pathogen variants (Giannitsarou, Kissler, and Toxvaerd 2021). In a recent editorial, Columbia professor Jeffrey Shaman discussed the possible transition of COVID-19 from pandemic to endemic phase.²

Motivated by this background, we consider a “Susceptible-Infected-Recovered-Susceptible” (SIRS) epidemiological model in which recovered agents eventually become susceptible to reinfection. As suggested by Peltzman (1975), a central challenge for disease control is that incentivizing one protective action decreases incentives for other protective actions. We therefore augment the SIRS epidemiological model with a game theoretic model in which each individual has the opportunity to

*Avery: Harvard Kennedy School (email: christopher_avery@hks.harvard.edu); Chen: Wake Forest University (email: chenfh@wfu.edu); McAdams: Duke University, Fuqua School of Business and Economics Department (email: david.mcadams@duke.edu). Dirk Bergemann was coeditor for this article. We are grateful to Glenn Ellison, Joshua Gans, Michael Kremer, Nolan Miller, and Chris Snyder and to seminar participants at INSEAD and Tufts and to anonymous referees for comments and advice. Chris Avery is grateful to INSEAD for hosting him as a visiting scholar during the time that this paper was written. We gratefully acknowledge financial support under National Science Foundation grant DMS-2230074.

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¹Recent surveys include Avery et al. (2020); McAdams (2021); and Bloom, Kuhn, and Prettnner (2022). See also Philipson (2000); Gersovitz and Hammer (2003); and Fenichel et al. (2011).

²Jeffrey Shaman, “What Will Our Covid Future Be Like? Here Are Two Signs to Look Out For,” *New York Times*, March 4, 2022, <https://www.nytimes.com/2022/03/04/opinion/endemic-covid-future.html>.

mitigate risk through social distancing or vaccination. Our theoretical framework yields a unique steady-state Nash equilibrium, which facilitates analysis of policy interventions such as vaccine subsidies or mandates and highlights the simple economics underlying the dynamics of infectious disease. In particular, individuals make cost-benefit trade-offs for risk mitigation, the equilibrium level of vaccine adoption is determined by familiar supply and demand dynamics, and some comparative static results turn on the elasticity of demand for vaccination.

The paper is most closely related to several previous studies: Chen et al. (2011); Chen (2012); and Toxvaerd (2019) conduct equilibrium analysis in a “Susceptible-Infected-Susceptible” (SIS) model with endogenous social distancing; Reluga and Galvani (2011) study equilibrium adoption of vaccination in an SIS model; Chen (2006) and Chen and Cottrell (2009) study incentives for vaccination and abstinence to reduce the risk of contracting HIV; Rowthorn and Toxvaerd (2020) study the optimal timing and trade-offs between treatment and vaccination in an SIS model.

The paper proceeds as follows. Section II describes the model. Section III provides steady-state equilibrium analysis with endogenous social distancing and exogenous vaccination, including the case when no vaccine is available. Section IV expands the equilibrium analysis to allow for both endogenous vaccination and social distancing and also considers the effects of policy applications such as subsidies and vaccine mandates. Section V concludes.

I. The Model

We consider an economic-epidemiological model of an endemic infectious disease, combining an SIRS model of epidemiological dynamics with an economic model in which agents make individually optimal decisions regarding personal social distancing and whether to get vaccinated.

A. Epidemiological Framework

An endemic infectious disease circulates among a fixed population of agents having unit mass. At each point in time $t \in \mathbb{R}$, each agent is either susceptible (S), infected (I), recovered and immune (R), or vaccinated (V) and knows their health status. Let $S(t)$, $I(t)$, $R(t)$, and $V(t)$, respectively, be the mass of susceptible, infected, recovered and immune, and vaccinated agents at time t . We refer to $I(t)$ as the “infection prevalence” at time t . Each susceptible agent i becomes infected once exposed to an infected agent, which occurs at rate $\beta I(t)[1 - x_i(t)]$, where $\beta > 0$ is the transmission rate and $x_i(t) \in [0, 1]$ is agent i ’s chosen level of social distancing. Infected agents recover at rate $\gamma > 0$ and then enjoy immunity from infection for known length of time $t_R \geq 0$, after which they return to the susceptible state. Similarly, a newly vaccinated agent is immune for a known length of time $t_V > 0$, at which point they may choose to renew their vaccination.

For analytical simplicity, we focus on settings where the mass of vaccinated agents is constant over time; that is, $V(t) = V$ for all t . In Section II, V is treated as an exogenous parameter. In Section III, V emerges endogenously as the mass of agents who choose to become and remain vaccinated in steady-state Nash equilibrium.

At each point in time, there is a flow of agents into the infected state as susceptible people are exposed and become infected, a flow into the temporarily immune state as infected people recover, and a flow into the susceptible state as recovered people lose their temporary immunity. Epidemiological dynamics for the system as a whole are governed by two differential equations:

$$(1) \quad I'(t) = \beta I(t)[1 - x(t)]S(t) - \gamma I(t),$$

$$(2) \quad R'(t) = \gamma I(t) - \gamma I(t - t_R),$$

plus the adding-up condition that $S(t) + I(t) + R(t) = 1 - V$, where $x(t)$ is average social distancing of susceptible agents at time t . (In equation (2), $\gamma I(t - t_R)$ is the flow of agents returning to susceptibility at time t after having been immune for length of time t_R .)

If $\beta(1 - V) \leq \gamma$, then each infected person exposes less than one unvaccinated person on average even without social distancing and $I(t)$ necessarily falls toward zero over time. We focus on the case when $V < 1 - \gamma/\beta$, creating the potential for persistent disease transmission.

B. Economic Model

Each agent seeks to minimize the expected present value of lifetime costs of sickness, social distancing, and vaccination. All agents discount future payoffs at discount rate $r > 0$. Infected agents incur flow cost $d > 0$ due to the disease. Susceptible agents who choose social distancing x incur flow cost $c(x)$ from foregone activity. We identify several properties of $c(x)$ that arise from principles of time-use optimization:

- $c(0) = c'(0) = 0$. Absent any fear of infection, people engage in ordinary activity ($x = 0$) and are indifferent at the margin whether to increase or decrease activity.
- $c'(x) > 0$ and $c''(x) > 0$. Since people can prioritize activities according to benefit per unit time, optimal social distancing forgoes the least valuable activities first.³

Let $C_h(t)$ be the expected lifetime cost for unvaccinated agents upon entering health status $h \in \{S, I, R\}$.⁴ Upon recovery from infection, an agent enjoys immunity and incurs no costs for length of time $t_R \geq 0$ before returning to the susceptible state. Thus,

$$(3) \quad C_R(t) = e^{-rt_R} C_S(t + t_R).$$

³Toxvaerd (2019) analyzes a related model and produces results similar to Proposition 1 and Corollary 2 with linear rather than strictly convex costs of social distancing.

⁴Agents' expected lifetime costs at time t depend on the subsequent epidemic trajectory. Our analysis is simplified by the fact that we focus on steady states where infection prevalence and susceptible-agent social distancing are constant.

While infected, agents incur flow cost d from the disease and recover at rate γ , at which point their subsequent expected lifetime cost changes from $C_I(t)$ to $C_R(t)$. Thus,

$$(4) \quad C'_I(t) = -d + \gamma[C_I(t) - C_R(t)] + rC_I(t).$$

Voluntary Social Distancing.—A susceptible agent i who chooses social distancing $x_i(t)$ incurs flow cost $c(x_i(t))$ and transitions to the infected state at rate $\beta I(t)[1 - x_i(t)]$. Given $C_S(t)$ and $C_I(t)$, such an agent chooses $x_i(t)$ to minimize $c(x_i(t)) + \beta I(t)[1 - x_i(t)][C_I(t) - C_S(t)]$, trading off the current cost of social distancing versus the benefit of avoiding infection. A susceptible agent's dynamic programming problem is given by (3–4) and

$$(5) \quad C'_S(t) = -\min_{x \in [0,1]} \{c(x) + \beta I(t)(1 - x)[C_I(t) - C_S(t)]\} + rC_S(t).$$

(A detailed derivation of susceptible agents' optimization problem is given in online Appendix C.)

Voluntary Vaccination.—In Section III with endogenous vaccination, each agent i is modeled as having a random cost of vaccination c_{iV} , drawn iid across agents from a distribution with support $[0, \bar{c}_V]$,⁵ continuous pdf $f(\cdot)$, and cdf $F(\cdot)$. Vaccination provides full protection from infection for period of time t_V , after which agent i becomes susceptible and may be vaccinated again at additional cost c_{iV} .

II. Steady-State Equilibrium with Exogenous Vaccination

In this section, we characterize the set of steady-state equilibria, taking the mass V of vaccinated agents as exogenous and small enough to allow for persistent disease transmission; that is, $0 \leq V < 1 - \gamma/\beta$. We have three main findings. First, a steady-state equilibrium exists and is unique. Second, any policy that reduces the cost of social distancing or that increases the fraction of the population that is vaccinated will result in strictly fewer infections in the new steady-state equilibrium. Third, an increase in social distancing from the steady-state equilibrium level unambiguously increases social welfare in the special case without temporary immunity ($t_R = 0$) but need not do so if $t_R > 0$.

In a steady-state equilibrium, the percentages of people in each state $\{S, I, R, V\}$ and the individually optimal social distancing level for susceptible agents are constant over time.

⁵Our analysis with endogenous vaccination is easily extended to a richer setting in which some agents have negative vaccination cost.

DEFINITION 1: A steady-state equilibrium with exogenous vaccination level $0 \leq V < 1 - \gamma/\beta$ is characterized by infection prevalence I^* , temporary immunity level R^* , and susceptible-agent social distancing x^* such that

- (i) (I^*, R^*, x^*, V) satisfies the steady-state conditions $I'(t) = 0$ and $R'(t) = 0$, which require that $R^* = \gamma t_R I^*$ and

$$(6) \quad \beta(1 - x^*)(1 - I^* - \gamma t_R I^* - V) = \gamma.$$

- (ii) x^* is individually optimal for susceptible agents in this steady state.

Let $x^*(I)$ be the individually optimal social distancing level for susceptible agents in the steady state with infection prevalence I . Let $C_S^*(I)$ and $C_I^*(I)$, respectively, denote the steady-state values of $C_S(t)$ and $C_I(t)$ for an agent who chooses the individually optimal social distancing level $x^*(I)$ whenever susceptible in the steady state with infection prevalence I . By (3–5),

$$(7) \quad C_I^*(I) = \frac{d + \gamma e^{-rt_R} C_S^*(I)}{\gamma + r},$$

$$(8) \quad C_S^*(I) = \frac{c(x^*(I)) + \beta I[1 - x^*(I)]C_I^*(I)}{\beta I[1 - x^*(I)] + r}.$$

Combining (7) and (8) gives

$$(9) \quad C_S^*(I) = \frac{(\gamma + r)c(x^*(I)) + \beta I[1 - x^*(I)]d}{\beta I[1 - x^*(I)](\gamma + r - \gamma e^{-rt_R}) + r(\gamma + r)}.$$

The first-order condition for individually optimal social distancing $x^*(I)$ in a steady state with infection prevalence I is

$$(10) \quad c'(x^*(I)) = \beta I[C_I^*(I) - C_S^*(I)],$$

where $C_I^*(I) - C_S^*(I)$ can be interpreted as the “harm of becoming infected” for a susceptible agent. Replacing $x^*(I)$ with $x = 0$ in (8) gives the bound $C_S^*(I) \leq \beta I C_I^*(I)/(\beta I + r)$, so $C_S^*(I) < C_I^*(I)$, which implies that $x^*(I) > 0$ for each I by properties of $c(\cdot)$.

With no social distancing ($x^* = 0$), steady-state infection prevalence would be $(1 - V - \gamma/\beta)/(1 + \gamma t_R) > 0$ by (6) and because $c'(0) = 0$, susceptible agents would not choose $x = 0$, a contradiction. Similarly, complete social distancing $x^* = 1$ would eliminate the disease and induce susceptible agents to choose full activity, another contradiction. Thus, there is partial social distancing ($0 < x^* < 1$) and the first-order condition (10) holds with equality in any steady-state equilibrium.

An increase in infection prevalence directly increases the marginal benefit of distancing for susceptible agents. Yet, a susceptible agent who avoids being infected at any given instant (“now”) faces a higher risk of being infected later, so an increase

in infection prevalence indirectly reduces the marginal benefit of distancing. If this indirect effect were stronger than the direct effect, an increase in infection prevalence would reduce the incentive of susceptible agents and create the potential for multiple equilibria. Proposition 1 rules out this possibility.⁶

PROPOSITION 1: *For each $V < 1 - \gamma/\beta$, there is a unique steady-state equilibrium with exogenous vaccination and associated infection rate $I^*(V) > 0$.*

PROOF:

For convenience, define

$$(11) \quad C_S(I, x) \equiv \frac{(\gamma + r)c(x) + \beta I(1 - x)d}{\beta I(1 - x)\theta + r(\gamma + r)},$$

where $\theta = \gamma + r - \gamma e^{-rT_R}$. $C_S(I, x)$ is the expected lifetime cost incurred by a susceptible agent in the steady state with infection prevalence I who chooses social distancing level x whenever susceptible. First, we show that there is a unique x that minimizes $C_S(I, x)$ for any given I . Using the quotient rule,

$$(12) \quad \frac{\partial C_S(I, x)}{\partial x} = \left\{ [(\gamma + r)c'(x) - \beta Id] [\beta I(1 - x)\theta + r(\gamma + r)] \right. \\ \left. + \beta I\theta [(\gamma + r)c(x) + \beta I(1 - x)d] \right\} / [\beta I(1 - x)\theta + r(\gamma + r)]^2.$$

Using (11) to substitute for the last term in the numerator,

$$(13) \quad \frac{\partial C_S(I, x)}{\partial x} = \frac{(\gamma + r)c'(x) - \beta Id + \beta I\theta C_S(I, x)}{\beta I(1 - x)\theta + r(\gamma + r)}.$$

If $\partial C_S(I, x)/\partial x = 0$, the numerator of (13) is zero and so, by the quotient rule, $\partial^2 C_S(I, x)/\partial x^2$ takes the same sign as $(\gamma + r)c''(x)$, which is positive since c is strictly convex. Thus, $\partial C_S(I, x)/\partial x = 0$ implies that $\partial^2 C_S(I, x)/\partial x^2 > 0$ and hence $C_S(I, x)$ has a unique minimum in x for each I . Moreover, the properties of $c(x)$ and $C_S(I, x)$ imply that this solution $x^*(I)$ is continuous and positive for all $I > 0$.

Next, we show that $x^*(I)$ is strictly increasing in I so long as $x^*(I) < 1$. The first-order condition $\partial C_S(I, x)/\partial x = 0$ can be rewritten as

$$(14) \quad c'(x) = \frac{\beta I[d - \theta C_S(I, x)]}{\gamma + r}.$$

⁶Multiple steady-state equilibria can exist in richer models with economic complementarities of activity (McAdams, Song, and Zou 2023) or with transmission crowding effects (Chen 2012).

From (11),

$$(15) \quad \theta C_S(I, x) = \frac{(\gamma + r)c(x) + \beta I(1 - x)d}{\beta I(1 - x) + \frac{r}{\theta}(\gamma + r)}.$$

Thus,

$$(16) \quad d - \theta C_S(I, x) = \frac{(\gamma + r)\left[\frac{rd}{\theta} - c(x)\right]}{\beta I(1 - x) + \frac{r}{\theta}(\gamma + r)}.$$

Multiplying both sides by βI ,

$$(17) \quad \beta I[d - \theta C_S(I, x)] = \frac{(\gamma + r)\left[\frac{rd}{\theta} - c(x)\right]}{(1 - x) + \frac{r}{\beta\theta} \frac{\gamma + r}{I}}.$$

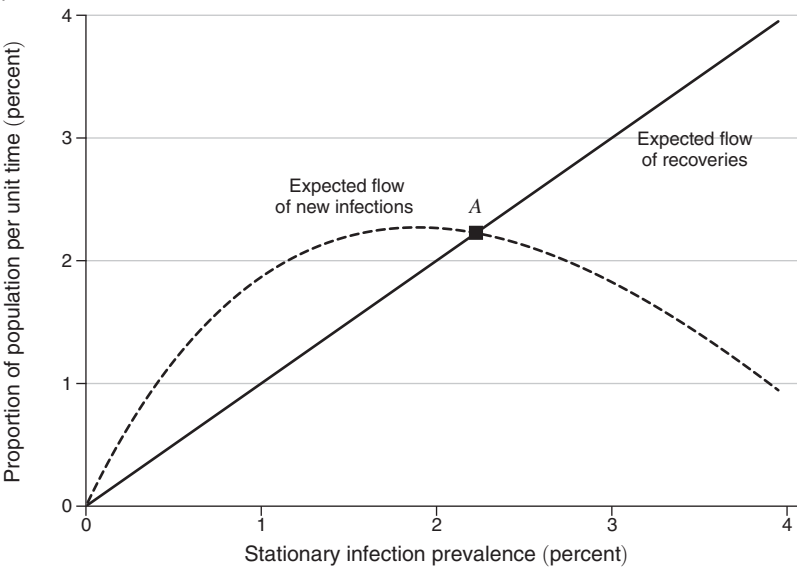
By (12), $\partial C_S(I, x)/\partial x < 0$ at $x = 0$; so, $x^*(I) > 0$ and (14) implies that $d - \theta C_S(I, x^*(I)) > 0$ for any $I > 0$. Thus, for each I and $x = x^*(I)$, the left-hand side of (17) is positive and hence the right-hand side of (17) must also be positive. The only term on the right-hand side of (17) that varies with I is $(\gamma + r)/I$. Holding x fixed at $x^*(I)$, a slight increase in I yields an increase in the marginal value of social distancing; that is, the marginal benefit of social distancing is now greater than its marginal cost at $x = x^*(I)$. Since $C_S(I, x)$ has a unique minimum in x for each I and $c(\cdot)$ is strictly convex, we conclude that $x^*(I)$ is strictly increasing in I so long as $x^*(I) < 1$. (Depending on disease severity d , there may be an infection level $\bar{I}$ such that $x^*(I) = 1$ for all $I \geq \bar{I}$. But such high infection levels cannot be sustained in any steady-state equilibrium since then susceptible agents would isolate themselves completely and there would be no new infections, a contradiction.)

To complete the proof of Proposition 1, we show that there is a unique I that supports a steady-state equilibrium. Let $g(I)$ denote the net expected flow into the infected state per infected agent in the steady state with infection prevalence I . By (1), $g(I) = \beta[1 - x^*(I)]S - \gamma$, where $S = 1 - I - R - V$ and $R = \gamma t_R I$. In any steady-state equilibrium, $g(I) = 0$ by (6). Given that $x^*(0) = 0$ and our maintained assumption that $V < 1 - \gamma/\beta$, we have $g(0) > 0$. On the other hand, $g((1 - V)/(1 + \gamma t_R)) = -\gamma < 0$. Since $x^*(I)$ is continuous and strictly increasing in I , $g(I)$ is continuous and strictly decreasing in I . Thus, there is a unique $I^* \in (0, (1 - V)/(1 + \gamma t_R))$ that solves $g(I^*) = 0$. ■

Example 1: Suppose that $V = 0, \beta = 3, d = 1, \gamma = 1, t_R = 20, c(x) = 0.05x^2$, and $r = -\ln(0.95)$.⁷

⁷These parameter values have been chosen to be roughly in line with SARS-CoV-2 in 2020. Liu et al. (2020) estimate a basic reproduction number $R_0 = \beta/\gamma$ of around 3. Cevik et al. (2021) estimate length of contagiousness as about ten days, normalized to $1/\gamma = 1$. Our choice of $t_R = 20$ means that adaptive immunity lasts 20 times longer than contagiousness—that is, a bit more than six months—consistent with the low end of the estimate range in Milne et al. (2021). Of course, given the complexity of SARS-CoV-2's biology and the emergence of numerous variants (Liu and Rocklöv 2022), one should not interpret any of the numerical exercises here as being about SARS-CoV-2 specifically.

Panel A. New infections versus new recoveries given any stationary infection prevalence



Panel B. Infection-prevalence trajectory leading to the steady-state equilibrium

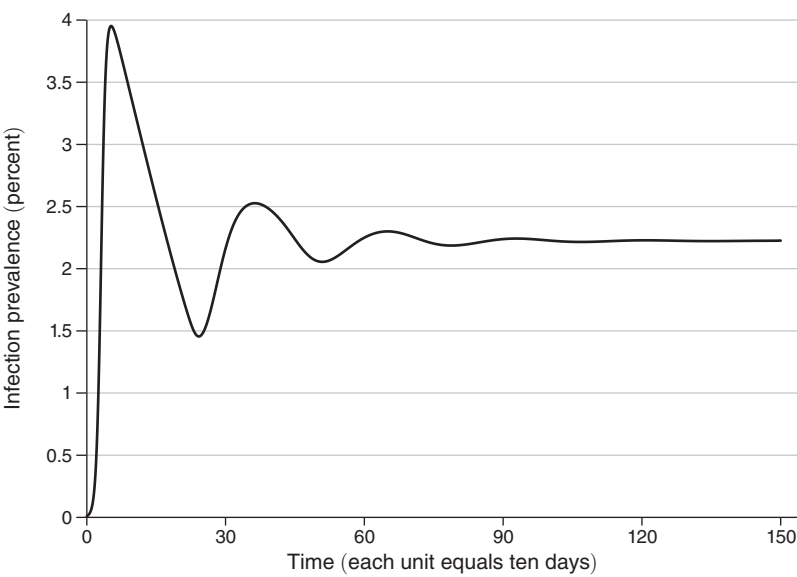


FIGURE 1. STEADY-STATE EQUILIBRIUM

In a steady-state equilibrium, the flow of new infections (dashed line in Figure 1, panel A) must equal the flow of recoveries (solid line in Figure 1, panel A). Figure 1, panel A compares these flows in Example 1 for any given steady-state infection prevalence I , accounting for how susceptible agents’ optimal social distancing

intensity $x^*(I)$ varies with I .⁸ The unique steady-state equilibrium corresponds to Point A, with stationary infection prevalence approximately equal to 2.226 percent.⁹

Figure 1, panel B illustrates an equilibrium epidemic trajectory in Example 1, starting from low infection prevalence 0.01 percent and eventually converging to the steady-state level. After an initial increase of infection prevalence to the steady-state level, there are fewer immune agents and more susceptible agents than in the steady state, so the infection trajectory “overshoots” the steady-state level and then gradually converges, as shown in Figure 1, panel B.

Figure 1, panel A suggests two corollaries that follow almost immediately from the proof of Proposition 1.

COROLLARY 1: *Any exogenous increase in (perfect) vaccination strictly reduces the infection prevalence in the unique steady-state equilibrium.*

PROOF:

Vaccination reduces the size of the susceptible population but has no effect on $x^*(I)$, the individually optimal level of social distancing given stationary infection prevalence I . An exogenous increase in vaccine adoption therefore shifts the new-infection curve down, while having no effect on the new-recovery curve. Thus, $g(I)$ strictly declines for each I as a result of an exogenous increase in vaccination, which in turn causes a reduction in the value of I , satisfying the steady-state equilibrium condition $g(I) = 0$. ■

COROLLARY 2: *A reduction in the cost of social distancing from $c(x)$ to $c_1(x)$ where $c_1(0) = c'_1(0) = 0$ and $c'_1(x) < c'(x)$ for each $x > 0$ reduces the steady-state equilibrium infection prevalence.*

PROOF:

By (16), the first-order condition $c'(x) = \{\beta I[d - \theta C_S(I, x)]\}/(\gamma + r)$ for individually optimal social distancing can be written as

$$(18) \quad c'(x) = \frac{\beta I \left[\frac{rd}{\theta} - c(x) \right]}{\beta I(1 - x) + \frac{r}{\theta}(\gamma + r)}.$$

Note that the marginal value of social distancing declines with $c(x)$ for each (x, I) . Since the marginal cost of social distancing is lower and the marginal value of social distancing is greater with $c_1(x)$ than with $c(x)$, $x_1^*(I) > x^*(I)$ for each I . This shifts the new-infection curve down and to the left, yielding a reduction in equilibrium steady-state infection prevalence as in Corollary 1. ■

⁸ When social distancing costs are quadratic as in Example 1, equations (7)–(10) imply that in any steady-state equilibrium, (i) $C_I^*(I)$ and x^* are each linear functions of $C_S^*(I)$ and (ii) $C_S^*(I)$ is the solution to a quadratic equation. See online Appendix A for details.

⁹ Online Appendix B provides all numerical details for Example 1, including how we approximated the equilibrium trajectory in Figure 1, panel B (using a backward-shooting algorithm as in Farboodi, Jarosch, and Shimer (2021)) and the steady-state equilibria in Figure 1, panel A; Figure 2; and Figure 3, panels A and B.

Infection Prevalence and Social Welfare.—Let $B(I)$ denote the “burden of the disease,” the aggregate lifetime costs of the entire population, in the steady state with infection prevalence I .¹⁰ Expected lifetime costs are $C_I(I)$ for mass I of infected agents; $C_S(I)$ for mass $S = 1 - V - (1 + \gamma t_R)I$ of susceptible agents; $e^{-rs}C_S(I)$ for temporarily immune agents who have $s \sim U[0, t_R]$ time remaining until they return to being susceptible again;¹¹ and zero for mass V of vaccinated agents. Overall,

$$(19) \quad B(I) \equiv IC_I(I) + [1 - V - (1 + \gamma t_R)I]C_S(I) + \gamma I \int_0^{t_R} e^{-rs} ds C_S(I).$$

Given that there are positive externalities for social distancing (because avoiding infection indirectly protects others), one might expect underprovision of social distancing in equilibrium so that the steady-state infection prevalence is higher than the social optimum. We show that this is indeed true (i.e., $B'(I^*) > 0$) if recovered agents are immediately susceptible to reinfection (Proposition 2) but not in general with temporary immunity (Example 2).

PROPOSITION 2: *Suppose that $t_R = 0$ so that recovered agents are not temporarily immune. Then $B'(I^*) > 0$.*

PROOF:

Define $I(x)$ to be the prevalence I that satisfies the steady-state condition $\beta(1 - x)[1 - (1 + \gamma t_R)I] = \gamma$. Note that $I^* = I(x^*)$, where x^* is susceptible agents’ social distancing in the unique steady-state equilibrium. We can express the burden of the disease in (19) equivalently as a function of social distancing x :

$$\begin{aligned} B(x) \equiv & \underbrace{I(x) \left[\frac{d + \gamma e^{-rt_R} C_S(I(x), x)}{\gamma + r} \right]}_{\text{infected agents}} \\ & + \underbrace{[1 - V - (1 + \gamma t_R)I(x)] C_S(I(x), x)}_{\text{susceptible agents}} + \underbrace{\gamma I(x) \left(\int_0^{t_R} e^{-rs} ds \right) C_S(I(x), x)}_{\text{currently immune agents}}. \end{aligned}$$

Since $I'(x) < 0$, we need to show that $B'(x^*) < 0$ in the case when $t_R = 0$. The derivative $B'(x)$ can be usefully decomposed into four terms:

$$(20) \quad B'(x) = (A) \frac{\partial C_S(I(x), x)}{\partial x}$$

$$(21) \quad + I'(x)(A) \frac{\partial C_S(I(x), x)}{\partial I}$$

$$(22) \quad + I'(x) \left[\frac{d}{\gamma + r} + \frac{\gamma}{\gamma + r} e^{-rt_R} C_S(I(x), x) - C_S(I(x), x) \right]$$

¹⁰In this steady state, susceptible agents choose social distancing $x_{ss}(I) \equiv 1 - (\gamma/\beta)/[1 - V - (1 + \gamma t_R)I]$ by (6) and hence incur flow cost $c(x_{ss}(I))$ while susceptible.

¹¹Because there is a constant flow of agents into the recovered state, the amount of time since recovery for a randomly selected agent still in that state is uniformly distributed on $[0, t_R]$.

$$(23) \quad + I'(x) \left[\gamma C_S(I(x), x) \left(\int_0^{t_R} e^{-rs} ds - t_R \right) \right],$$

where

$$A \equiv I(x) \frac{\gamma}{\gamma + r} e^{-rt_R} + [1 - V - (1 + \gamma t_R) I(x)] + \gamma I(x) \left(\int_0^{t_R} e^{-rs} ds \right) > 0.$$

The term in (20) is zero at $x = x^*$ because $\partial C_S(I^*, x^*)/\partial x = 0$ due to susceptible agents' individual optimization in the steady-state equilibrium. The term in (21) is negative at $x = x^*$ because $I'(x) < 0$ for all x and $\partial C_S(I(x), x)/\partial I > 0$ at $x = x^*$ (see online Appendix E for a proof of the latter inequality). The term in (22) is negative at $x = x^*$ because $d/(\gamma + r) + \gamma e^{-rt_R} C_S(I(x), x)/(\gamma + r)$ is the expected lifetime cost of an infected agent, which is greater than $C_S(I(x), x)$. Finally, the term in (23) is zero if $t_R = 0$ (but positive if $t_R > 0$ because $\int_0^{t_R} e^{-rs} ds < t_R$). We conclude as desired that $B'(x^*) < 0$. ■

When $t_R > 0$ so that recovered agents enjoy temporary immunity, susceptible agents discount the future benefit that they will get due to immunity (after eventually recovering from infection) when deciding how much to socially distance. Consequently, depending on the duration of temporary immunity and other model parameters, susceptible agents may choose a level of social distancing *greater* than the level that minimizes the steady-state burden of the disease.

Example 2: Suppose that $V = 0, \beta = 3, d = 1, \gamma = 1, t_R = 100, c(x) = 0.05x^2$, and $r = -\ln(0.95)$.

Example 2 is a variation of our COVID-inspired Example 1, with duration of temporary immunity increased to $t_R = 100$ —that is, about 1,000 days. The unique steady-state equilibrium in Example 2 has social distancing $x^* = 0.131$ and infection prevalence $I^* = 0.00610$. The level of social distancing that minimizes the steady-state burden of the disease is $0.104 < x^*$, resulting in a higher steady-state infection prevalence $0.00622 > I^*$.¹² Details for these computations are provided in online Appendix D.

III. Equilibrium with Endogenous Vaccination

This section extends the model to incorporate interactions between social distancing and vaccination decisions. We begin by extending the definition of steady-state equilibrium to require individually optimal vaccination decisions. In a stationary setting, incentives for a susceptible agent do not change with time; so, each agent either never chooses to be vaccinated or gets vaccinated and revaccinated at the first moment that they become susceptible.

¹² Steady-state equilibrium overdistancing occurs here so long as t_R exceeds a threshold of about 60—that is, 600 days.

DEFINITION 2: A steady-state equilibrium with endogenous vaccination is characterized by infection level I^* , temporary immunity level R^* , susceptible-agent social distancing x^* , and vaccination level V^* such that

- (i) (I^*, R^*, x^*, V^*) satisfies the steady-state conditions $R^* = \gamma t_R I^*$ and
- $$(24) \quad \beta(1 - x^*)(1 - I^* - \gamma t_R I^* - V^*) = \gamma.$$
- (ii) x^* is individually optimal for susceptible agents in this steady state; that is,
- $$(25) \quad x^* \in \arg \min_{x \in [0,1]} C_S(I^*, x).$$
- (iii) Fraction V^* of newly susceptible agents find it individually optimal to become vaccinated; that is,
- $$(26) \quad F(C_S^*(I^*)(1 - e^{-rt_V})) = V^*,$$

where $C_S^*(I^*) \equiv C_S(I^*, x^*)$ and $F(\cdot)$ is the cdf for vaccination cost.

Vaccination allows agents to avoid infection for t_V units of time without social distancing. In a steady state with infection prevalence I , a susceptible agent with vaccination cost c_{iV} benefits from adopting the vaccine if $c_{iV} + e^{-rt_V} C_S^*(I) < C_S^*(I)$, or $c_{iV} < C_S^*(I)(1 - e^{-rt_V})$.¹³ The fraction of newborn agents who find it optimal to vaccinate (“vaccine demand”) is $D_V(I) = F(C_S^*(I)(1 - e^{-rt_V}))$, which is continuous and strictly increasing in I .

PROPOSITION 3: There is a unique steady-state equilibrium with endogenous vaccination.

PROOF:

For any fixed $V < 1 - \gamma/\beta$, let $I(V) > 0$ be the infection level in the unique steady-state equilibrium with exogenous vaccination level V , and let $\bar{I} \equiv I(0)$. For all $I \in (0, \bar{I}]$, let $SS_V(I)$ be the vaccination level that induces equilibrium steady-state infection prevalence I ; that is, $I(SS_V(I)) = I$. Because $I(V)$ is continuous and strictly decreasing (Corollary 1), $SS_V(I)$ is also continuous and strictly decreasing.

A steady-state equilibrium with endogenous vaccination exists with infection prevalence I and vaccination level V if and only if $SS_V(I) = D_V(I) = V$. For all $I \approx 0$, we have $SS_V(I) \approx 1 - \gamma/\beta > D_V(I) \approx 0$. On the other hand, $SS_V(\bar{I}) = 0 < D_V(\bar{I})$. Since $SS_V(I) - D_V(I)$ is continuous and strictly decreasing, there is a unique I^* such that $SS_V(I^*) = D_V(I^*) \equiv V^*$, as desired. ■

¹³With vaccination cost c_{iV} , the expected lifetime cost of adopting the vaccine and renewing it whenever immunity runs out is $c_{iV} + e^{-rt_V} c_{iV} + e^{-2rt_V} c_{iV} + \dots = c_{iV}/(1 - e^{-rt_V})$.

A. Subsidizing Vaccination versus Subsidizing Social Distancing

The externalities associated with vaccination and social distancing provide motivation for policy interventions.¹⁴ There may also be societal benefits beyond our model from reductions in steady-state infection prevalence (e.g., workplace productivity gains and less-burdened health systems) and/or from increasing the steady-state vaccination level (e.g., blunting the severity of any new-variant outbreak). However, interventions that promote social distancing or vaccination can have quite different effects on steady-state infection prevalence, after accounting for individuals' simultaneous social distancing and vaccination choices. Most strikingly, we show that a social distancing subsidy can sometimes have the ironic effect of *increasing* steady-state infection prevalence. In particular, consider a lump-sum subsidy S_V for susceptible agents who get vaccinated ("vaccine subsidy") and flow subsidy $S(x)$ that reduces social distancing cost to $c_1(x) = c(x) - S(x)$ as in Corollary 2 ("social distancing subsidy").

PROPOSITION 4: *A subsidy for vaccination increases vaccine adoption and reduces infection prevalence in the steady-state equilibrium. A subsidy for social distancing reduces vaccine adoption and could either increase or reduce infection prevalence in the steady-state equilibrium.*

PROOF:

Let I_V denote the infection prevalence in the steady-state equilibrium with endogenous vaccination and no subsidy. A vaccine subsidy increases the demand for vaccination but has no effect on the level of vaccination required for a steady-state equilibrium at infection prevalence I . Define the demand for vaccination with the subsidy as $D_{V,S}(I)$. Since $D_{V,S}(I) > D_V(I)$ for each I and $D_V(I_V) = SS_V(I_V)$, we know that $D_{V,S}(I_V) > SS_V(I_V)$. Therefore, $D_{V,S}(I)$ and $SS_V(I)$ intersect at some $I < I_V$, proving the desired result.

By contrast, a subsidy for social distancing has two effects. First, following the logic of Corollary 2, the subsidy reduces the marginal cost of social distancing and therefore reduces the number of new infections for any stationary infection prevalence I without vaccination. Because of this shift in the new-infection curve, the subsidy reduces $SS_V(I)$, the vaccination rate required to produce a steady-state equilibrium with infection prevalence I . Second, the subsidy reduces the expected future cost $C_S^*(I)$ for a susceptible person and thus reduces $D_V(I)$, the demand for vaccination given any stationary infection prevalence I . These changes each cause the steady-state vaccination level to fall but have opposite effects on the steady-state infection prevalence. Figure 3 provides examples of both possibilities, that steady-state infection prevalence may rise or fall. ■

¹⁴ As observed by Brito, Sheshinski, and Intriligator (1991), a universal vaccination requirement can reduce the equilibrium utility of susceptible agents who are affected by that rule. Geoffard and Philipson (1997) suggest that infectious diseases tend to remain endemic because reductions in prevalence erode individual incentives for precautionary behavior.

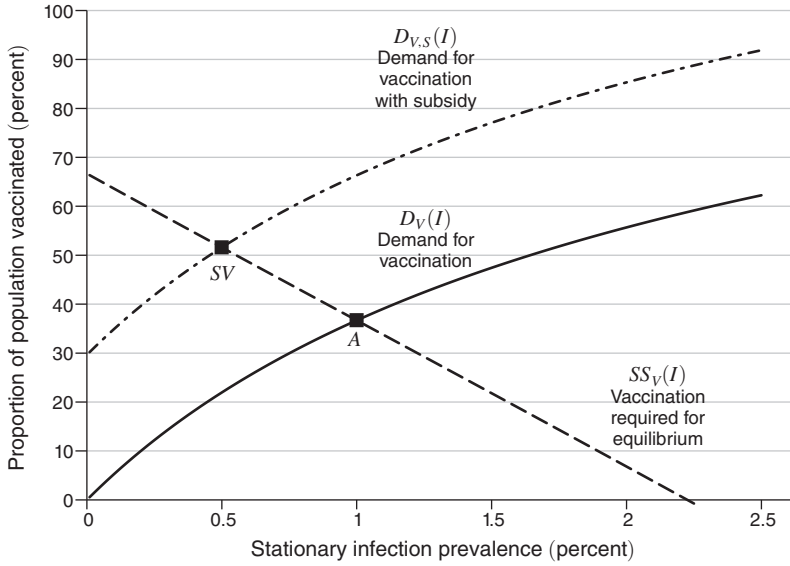


FIGURE 2. THE UNAMBIGUOUS EFFECT OF A VACCINE SUBSIDY

Figure 2 depicts the effect of a vaccine subsidy on steady-state vaccine adoption and infection prevalence in Example 1, with agents' cost of vaccination c_{iV} uniformly distributed on $(0, 0.6468)$ and a subsidy of 0.1917 for susceptible agents for each vaccination. Under the original conditions with no subsidy, the level of vaccination required for a steady-state equilibrium is decreasing while demand for vaccination is increasing in infection prevalence. The intersection of these two curves at point A represents the baseline equilibrium with a stationary infection prevalence of 2.226 percent and approximately 37 percent of the population adopting the vaccine. For reasons explained in the proof of Proposition 4, the subsidy shifts the vaccine demand curve $D_V(I)$ up and to the left, which increases vaccination and reduces infection prevalence at the new steady-state equilibrium point SV.

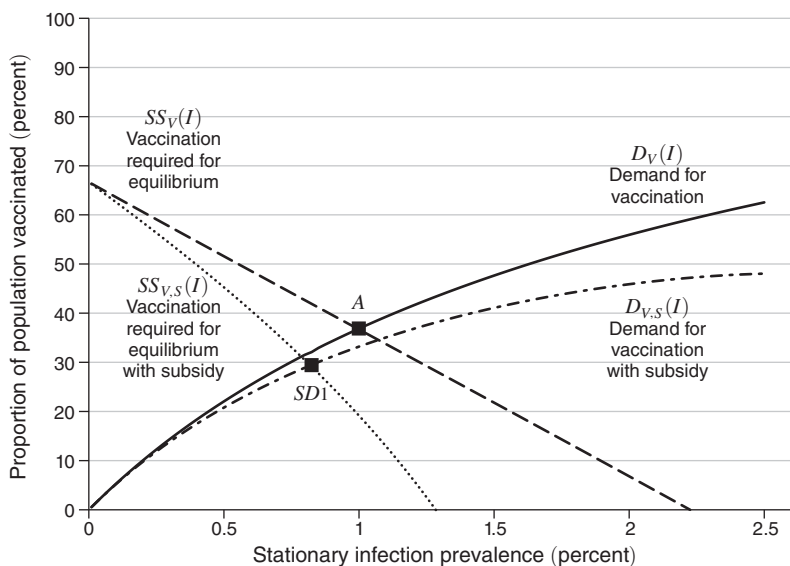
Figure 3 illustrates the possible equilibrium impacts of a subsidy that reduces the cost of social distancing, in the context of Example 1 with distancing costs $0.025x^2$ after the subsidy, but with different vaccination-cost distributions.¹⁵

The social distancing subsidy shifts the vaccine demand curve $D_V(I)$ down and to the right and shifts the level of vaccination required for equilibrium $SS_V(I)$ down and to the left. The steady-state equilibrium shifts from point A to point SD on both panels. As described in Proposition 4, a social distancing subsidy unambiguously reduces steady-state vaccination but may decrease (Figure 3, panel A) or increase (Figure 3, panel B) steady-state infection prevalence. When demand for vaccination is relatively inelastic, as in Figure 3, panel A, the effect of the shift in $SS_V(I)$ predominates and so a social distancing subsidy reduces steady-state infection prevalence.

By contrast, when demand for vaccination is relatively elastic, as in Figure 3, panel B, the primary effect of a social distancing subsidy is to reduce demand for

¹⁵In Figure 2 and Figure 3, panel A, $c_{iV} \sim U(0, 0.6468)$. For Figure 3, panel B, we use a much tighter distribution $c_{iV} \sim U(0.22411, 0.26)$.

Panel A. An example with falling infection prevalence



Panel B. An example with rising infection prevalence

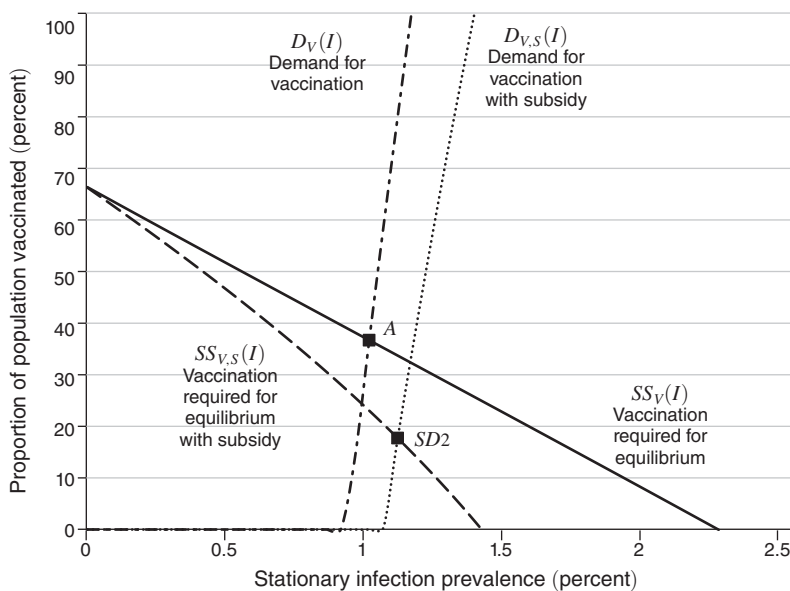


FIGURE 3. THE AMBIGUOUS EFFECT OF A SOCIAL DISTANCING SUBSIDY

vaccination. In this case, the subsidy leads to increased infection prevalence at the new steady-state equilibrium point $SD2$.

The difference between the effects of vaccine and social distancing subsidies arises because reducing the cost of vaccination only indirectly affects incentives for social distancing (by causing steady-state infection prevalence I to fall), while changes in the marginal cost of social distancing directly affect incentives for vaccination (by reducing $C_S^*(I)$ and hence the marginal value of vaccination). Thus, as suggested by

Corollary 1, a vaccine subsidy has an unambiguous effect on steady-state infection prevalence, whereas a social distancing subsidy has an ambiguous effect in the fashion described by Peltzman (1975).

B. Vaccine Mandates

In November 2021, Austria announced a nationwide lockdown focused only on the unvaccinated, who were driving an upsurge of infection at the time (Horowitz and Eddy 2021). Many governments, colleges, and employers imposed similar “vaccine mandates,” restricting unvaccinated people’s ability to participate in socioeconomic activity. Formally, we model a vaccine mandate as imposing a minimum level of social distancing on all unvaccinated agents. Many vaccine mandates imposed during the COVID pandemic were temporary measures, such as New York City’s vaccination requirement for indoor restaurant dining from August 2021 until March 2022. But such mandates could also be maintained in perpetuity. What impact would that have on the steady-state equilibrium?

PROPOSITION 5: *A vaccine mandate reduces infection prevalence and could either increase or reduce vaccination in the steady-state equilibrium.*

PROOF:

By increasing susceptible-agent and infected-agent social distancing, a vaccine mandate reduces the number of new infections corresponding to any stationary infection prevalence I and vaccination level V . This results in a decline in $SS_V(I)$, the required level of vaccination for a steady-state equilibrium with infection prevalence I . Second, by imposing additional costs on all unvaccinated agents, the mandate increases $C_S^*(I)$, the expected lifetime costs of (unvaccinated) susceptible agents for any given I .¹⁶ This increases $D_V(I)$, the demand for vaccination. Since D_V is increasing and SS_V is decreasing in I , steady-state infection prevalence must fall, but the impact on the steady-state vaccination level is ambiguous. (Numerical examples similar to Figure 3 to demonstrate that the equilibrium steady-state vaccination level may rise or fall are easily constructed but omitted to save space.) ■

In January 2022, France barred unvaccinated people not just from enclosed spaces such as long-distance trains but also open-air cafés and other public places where the risk of transmission is relatively low. The goal of this policy, in the words of President Emmanuel Macron, was to “annoy the unvaccinated.”¹⁷ Such restrictions are effectively equivalent to a financial penalty on unvaccinated people and, as such, have the same unambiguous effect as a vaccine subsidy—both increasing steady-state vaccination and reducing steady-state infection prevalence

¹⁶For any *fixed* stationary infection level I , a susceptible agent’s lifetime expected cost is minimized by choosing their individually optimal social distancing level $x^*(I)$. So, a vaccine mandate that forces (unvaccinated) susceptible people to choose $x > x^*(I)$ necessarily increases lifetime expected costs. In addition, a vaccine mandate imposes social distancing and thus additional costs on infected and temporarily immune people, further increasing susceptible agents’ lifetime expected costs since they will now also be losing out on future economic benefits while in those other disease states.

¹⁷“Les non-vaccinés, j’ai très envie de les emmerder. Et donc, on va continuer de le faire, jusqu’au bout.”

(Proposition 4). On the other hand, a policy that more judiciously restricts activity, only barring unvaccinated people from the highest-risk and lowest-value activities, might ironically reduce the steady-state level of vaccination (Proposition 5).

IV. Discussion and Conclusion

We have presented an economic SIRS model in which agents can engage in social distancing or choose to get vaccinated in order to reduce their chance of getting infected. The model allows us to consider the effects of disease-control policies on social distancing, infection prevalence, and the demand for vaccination during the endemic phase of an infectious disease. It also provides insights about how social distancing and vaccination interact as distinct behavioral responses to determine the steady-state level of infection. In addition, we have shown how the pillars of economic analysis—cost-benefit trade-offs, supply and demand, comparative statics—apply to the dynamics of infectious diseases.

Our basic model can be extended in several directions. One possibility is to examine the effects on agents' behavior and infection prevalence when vaccination reduces but does not eliminate the risk of infection. Chen (2006) and Chen and Cottrell (2009) show that multiple steady-state equilibria can arise when vaccines are imperfect since the benefit of vaccination may not be monotonic in disease prevalence. Similarly in our model, demand for vaccination could be increasing over low prevalences but decreasing over high prevalences if the efficacy of the vaccine is low. This implies that the D_V curve could intersect the SS_V curve at more than one point, resulting in multiple steady-state equilibria.

Although we have modeled the vaccine as prophylactic, that assumption could be adjusted so that vaccination reduces the severity of disease (as represented by cost parameter d) but does not prevent infection. One possibility in this case is that when the rate of vaccination is high in the population—which would be expected when infection prevalence is high—the overall level of social distancing would be low, resulting in more infections. Thus, high prevalence begets high prevalence, creating a positive feedback loop with significant policy implications that could be explored in future work.

It is also possible to use our framework to study other interventions, such as a rule requiring a recent negative test to eat at a restaurant or fly on a commercial airline. By blocking at least some infected people from these activities, such a requirement effectively reduces the transmission rate associated with them. Moreover, the cost of getting a test to comply with the requirement reduces the social distancing cost of forgoing these activities. The overall effect of a testing requirement therefore combines the effects of a social distancing subsidy and a transmission-reducing intervention.

A final extension we will mention, motivated by “vaccine passports” (Hall and Studdert 2021),¹⁸ is to consider the effects of making agents' vaccination status known to others. In our setting, there is random mixing in the population, meaning

¹⁸Diagnostic tests that provide information on a consumer's current health status appear to raise similar issues but with additional nuances because people who test negative can subsequently become infected and because test results serve as private information about health status. See Phelan and Toda (2022) and Deb et al. (2022) for insightful analyses of the equilibrium impact of imperfect testing.

that the types of agents that one encounters is independent of one's vaccination status. Making people's vaccination status known to others could lead to assortative mixing whereby vaccinated and unvaccinated people are more likely to mix amongst themselves.¹⁹ Making information about people's vaccination status public could also change their incentive to get vaccinated by creating social consequences for vaccination. These issues and their implications merit consideration and thorough examination in the future.

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¹⁹ Kremer (1996) shows that differential behavioral responses by different types of agents induce changes in the pool of potential partners that can lead to multiple equilibria for a sexually transmitted disease.

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