

Dynamic Risk-based Process Design and Operational Optimization via Multi-Parametric Programming

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Abstract

We present a dynamic risk-based process design and multi-parametric model predictive control optimization approach for real-time process safety management in process systems. A dynamic risk indicator is used to monitor process safety performance considering fault probability and severity, as an explicit function of safety-critical process variables deviation from nominal operating conditions. Process design-aware risk-based multi-parametric model predictive control strategies are then derived which offer the advantages to: (i) integrate safety-critical variable bounds as path constraints, (ii) control risk based on multivariate process dynamics under disturbances, (iii) provide model-based risk propagation trend forecast. A dynamic optimization problem is then formulated, the solution of which can yield optimal risk control actions, process design values, and/or real-time operating set points. The potential and effectiveness of the proposed approach to systematically account for interactions and trade-offs of multiple decision layers toward improving process safety and efficiency is showcased in a real-world example, the safety-critical control of a continuous stirred tank reactor at T2 Laboratories.

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1. Introduction

Process safety management (PSM) is a top priority in process operations to prevent the occurrence of accidents and the resulting severe human and financial losses with pressing impacts on the society and environment [1, 2, 3]. Process safety evaluation methods, such as hazard and operability study (HAZOP) [4] and quantitative risk analysis (QRA) [5], have been widely applied in current industrial practice which identify process hazards and add on protection layers (e.g., dike, relief valve) based on a static nominal design configuration. However, the recent burgeoning trends toward industry decarbonization, digital innovation, and advanced manufacturing have posed new challenges and opportunities to PSM with more complex, integrated, and automated plants under increasingly dynamic and volatile business, market, and supply chain environments [6]. Online process safety monitoring and smart risk management under disturbances become instrumental during daily process operations to safeguard the promises of real-time decision making for enhanced profitability, energy efficiency, and sustainability [7, 8]. Therefore, it is imperative to bridge the link between safety-critical decision making with systems-based real-time operation, which are normally performed independently without considering the interactions and trade-offs.

To support real-time PSM, quantitative process safety performance indicator is a key enabling factor which should account for the time-variant impacts of process design, operation, and disturbances. Conventional QRA approaches adapt risk as the indicator, estimated as the product of “fault probability” and “consequence severity” for a process facility [9, 10]. Some examples of process faults include reactor runaway, pipe rupture, liquid level sensor failure, etc. which can lead to consequences of fire, explosion, toxic release, etc. These approaches evaluate a steady-state snapshot of the process designs, which cannot provide a dynamic risk picture under operating condition variations. The use of generic failure data also challenges the process-specific estimation preciseness [11, 12]. Dynamic risk analysis strategies [13] are thus developed to address these gaps, the first of which was proposed by Meel and Seider in 2006 [14]. The majority of available dynamic risk analysis approaches [15, 16, 17, 18] employed Bayesian theory to determine the

posterior fault probability distribution based on a prior distribution using generic historical data (e.g., database in [5]), however after any new occurrence of abnormality events or faults during operation. The consequence severity was then updated using a bow-tie, event tree, or fault tree model to capture the “cause-barrier-abnormality-consequence” relationships. The loss of intrinsic physics-based process dynamics and relationships hinders a joint decision making basis to integrate these approaches with process control and real-time optimization.

From the aspect of process control, the prevention of fault occurrence is mostly addressed by controlling safety-critical process variables (typically state variables) within their bounds. Model predictive control (MPC) has been leveraged to this purpose which was first proposed by Leveson and Stephanopoulos in 2013 [19]. However, an overall process safety performance indicator is missing which can assess the cumulative impacts of state transitions and disturbances. Research efforts have also been made to theoretically characterize a safe and stable dynamic state space operating region. Albalawi et al. [20, 21] developed a safeness index-based Lyapunov economic MPC (LEMPC) strategy, in which a safety zone was defined in the state space as a subset of the stability region determined by the Lyapunov level set. In the case of process states deviating from the safety zone, LEMPC control actions would drive states back to safety zone in finite time. Another work by Venkidasalapathy and Kravaris [22] used pertinent systems theory to calculate a set of initial states, starting from which the safety-critical constraints could be satisfied at all times despite potential safety threatening disturbances during dynamic operation. It remains a critical yet open research question on how to quantify the bounding conditions of uncertainties, for both process disturbances and modeling uncertainties, within which the above robust and safe control can be theoretically guaranteed.

To detect fault proactively at an early developing stage, Ahooyi et al. [23] developed a model-predictive safety system approach which could generate predictive alarm signals based on whether the process can satisfy its operability constraints using the most aggressive, feasible, manipulated input profiles. Bhadriraju et al. [24] applied model predictive control for fault prognosis, in which the control moving horizon estimation was utilized to predict potential fault occurrence. The forecast of fault propagation trajectory thus accounted for the process mechanistic, closed-loop control actions, and disturbances utilizing the real-time information of both measured and unmeasured process variables. In case of any potential fault occurrence during the next output

horizon, alarms would be triggered ahead of time. A dynamic risk indicator was incorporated in [24]. A major limitation of these approaches lies in the tight coupling of controller output horizon with fault prognosis horizon (i.e., equal to each other). In this way, the fault prognosis capability is restricted by the online control requirement and computational power. Process design represents another key factor affecting process safety performance, which has mostly been investigated at steady state from the aspect of inherently safer design. Recent works [25, 26, 27, 28] indicated that process safety considerations could result in significant and non-intuitive design changes in the optimal process solution, compared to that driven by economics and sustainability objectives. This highlights the need to fully integrate the decision making of process design, real-time operation, and dynamic risk management across multiple time scales, despite a unified methodology is currently lacking.

To address the above challenge, in this work we develop a dynamic risk-based process design and operational optimization approach based on the PAROC (PARametric Optimization and Control) framework [29]. The remaining of the paper is organized as follows: Section 2 introduces the proposed risk-based optimization approach integrating dynamic risk analysis and multi-parametric control. Section 3 demonstrates this approach step-by-step for the safety-critical control of a real-world T2 continuous stirred tank reactor. Section 4 presents concluding remarks and future work.

2. The Risk-based Optimization Approach

2.1. Overview of the Proposed Approach

This work aims to develop a general methodology framework for risk-based process design and operational optimization. A schematic of the proposed framework is shown in Fig. 1, which features:

- A dynamic risk-based multi-parametric model predictive controller for optimal risk control at short term. The dynamic risk indicator is formulated as an explicit function of safety-critical process variable deviations during online operation.
- A dynamic optimizer at long term for safety and economics optimization. The time scale for the dynamic optimizer is independent of the controller time scale.

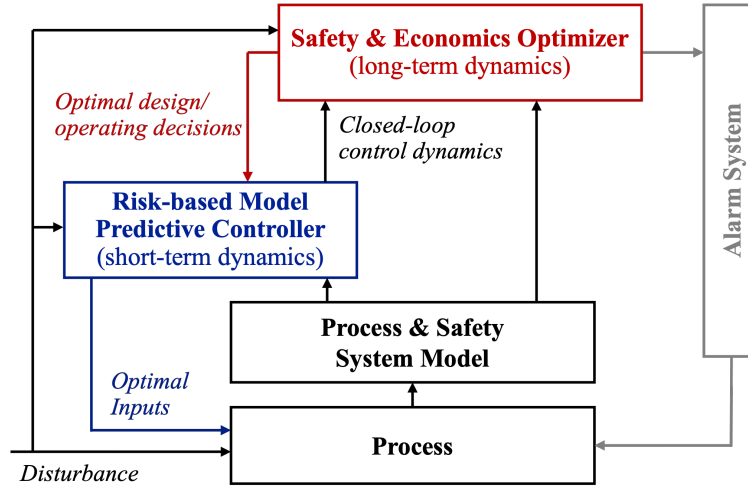
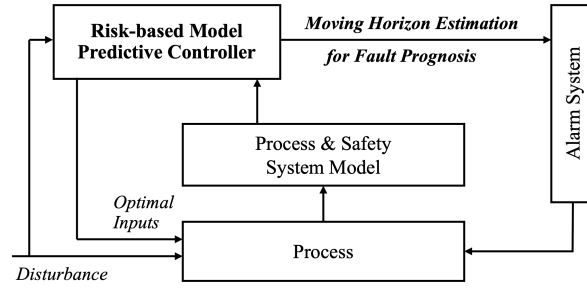


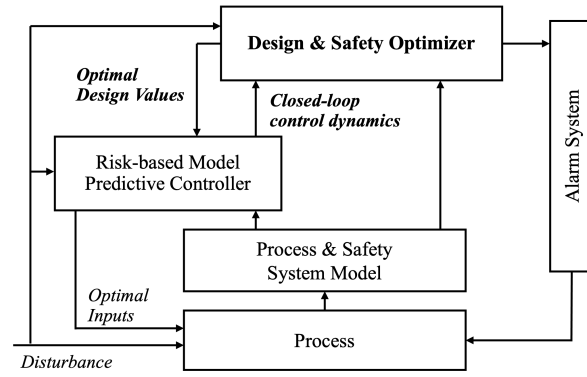
Figure 1: The integration of risk-based process design and control optimization.

106 Note that the decision making of controller and optimizer is fully inte-
 107 grated: the controller takes optimal design and/or operating decisions from
 108 the optimizer, while the optimizer is aware of the closed-loop dynamics. The
 109 potential and versatility of the framework are demonstrated through the fol-
 110 lowing three classes of applications and illustrated in Fig. 2:

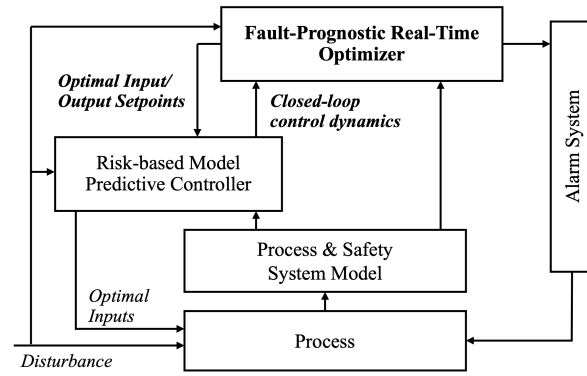
- 111 • Application 1: Model-based risk control and fault prognosis via moving
 112 horizon estimation – In this case, long-term dynamic optimizer is not
 113 used. The controller will forecast potential fault occurrence during the
 114 next output horizon and to raise alarms when necessary.
- 115 • Application 2: Simultaneous design and control optimization with dy-
 116 namic risk considerations – This can be utilized to investigate interac-
 117 tions and trade-offs between design variables, operating variables, and
 118 safety performance metrics at the early design stage.
- 119 • Application 3: Integrated fault-prognostic real-time optimization and
 120 risk-based control – This can support online operation (i.e., design is
 121 fixed at this stage) to guide optimal state transitions while preserving
 122 desired process safety under disturbances. In this case, fault prognosis
 123 is achieved by the dynamic optimizer which can forecast into a longer
 124 time horizon independent of control output horizon as needed in certain
 125 processes with fast dynamics, long shutdown time, etc.



Application 1: Risk control and fault prognosis via MPC moving horizon estimation.



Application 2: Risk-based simultaneous design and control optimization.



Application 3: Fault-prognostic real-time optimization and control.

Figure 2: Applications of the general framework for risk-based design and operation.

126 A (mixed-integer) dynamic optimization problem (Eq. 1) is formulated
 127 to mathematically realize Fig. 1, synergizing dynamic risk analysis [30] and
 128 multi-parametric control [31]. Specifically, Eq. 1a defines the objective func-
 129 tion for the optimizer which can account for process safety, product quality,
 130 and/or economics (e.g., design and/or operating costs). Optimization vari-
 131 ables can include design variables and input/output set point values when
 132 applicable. Control actions at each time step are not optimization variables
 133 as they are calculated from the explicit functions of multi-parametric control
 134 law given in Eqs. 1e-f. Eqs. 1b-c describe the mechanistic dynamic model
 135 for the process and safety system (typically nonlinear). Design variables,
 136 continuous and/or discrete, can be included to consider their impact on con-
 137 trol and risk. Eq. 1d adapts the dynamic risk indicator developed by Bao
 138 et al. [30], which quantifies risk as an explicit function of safety-critical vari-
 139 ables deviation from nominal conditions to instantly reflect real-time process
 140 safety performance changes. This method will be detailed later in Section
 141 2.2. Eqs. 1e-f give the multi-parametric model predictive control (mp-MPC)
 142 laws, calculated as piecewise affine functions of parameters including design
 143 variables and the risk indicator. The ability to obtain the mp-MPC control
 144 laws offline *in priori* serves as the key to connect two separate time scales
 145 (i.e., controller and optimizer) in a single dynamic optimization problem as
 146 well as to maintain tractable online computational load. The derivation of
 147 multi-parametric control laws will be detailed in Section 2.3. Eqs. 1g-i de-
 148 fine the process operating constraints for state, input, and output variables
 149 in terms of lower and upper bounds. More generalized time-varying process
 150 operating constraints, i.e. $g(x(t), u(t), d(t)) \leq 0$, can also be incorporated
 151 when applicable following prior work [32, 33]. Eqs. 1j-k define the bounds
 152 for design and risk variables. A dual layer of process safety management is
 153 actually provided by this approach: (i) the control of dynamic risk as an
 154 overarching process safety performance indicator, (ii) the bounded operation
 155 of safety-critical variables via mp-MPC path constraints.

$$\min_{De, Y, y^R} F = \int_0^\tau P(x(t), y(t), u(t), d(t), De, Y, RI(t)) dt \quad (1a)$$

$$\text{s.t.} \quad dx(t)/dt = f(x(t), y(t), u(t), d(t), De, Y) \quad (1b)$$

$$y = g(x(t), u(t), d(t), De, Y), \quad Y \in \{0, 1\}^q \quad (1c)$$

$$RI = s(x(t), u(t), d(t), De, d(t)) \quad (1d)$$

$$u_k = K_i \theta_k + r_i, \quad \theta_k \in CR_i = \{CR_i^A \theta \leq CR_i^b\} \quad (1e)$$

$$\theta_k = [x_k, y_k, y_k^R, d_k, De, RI_k] \quad (1f)$$

$$\underline{x} \leq x(t) \leq \bar{x} \quad (1g)$$

$$\underline{u} \leq u(t) \leq \bar{u} \quad (1h)$$

$$\underline{y} \leq y(t) \leq \bar{y} \quad (1i)$$

$$\underline{De} \leq De \leq \overline{De} \quad (1j)$$

$$\underline{RI} \leq RI(t) \leq \overline{RI} \quad (1k)$$

156 where $x(t)$ are states, $u(t)$ are input variables, $y(t)$ are output variables, $d(t)$
 157 are disturbances, Y are binary variables, De are process design variables,
 158 $RI(t)$ is dynamic risk indicator, $\theta(t)$ are parameters for mp-MPC, CR are
 159 critical regions. Subscripts k denotes discrete time step, i is index for critical
 160 regions. CR^A and CR^b are coefficient matrices to define critical regions.
 161 Superscript R denotes set point.

162 2.2. Dynamic Risk Modeling

163 In what follows we discuss the dynamic risk model proposed by Bao et al.
 164 [30] and its extension in this work to enable risk-based control. Abnormality
 165 identification is first performed for the specific process system by surveying
 166 historical incident cases and/or performing near miss studies, which aims to
 167 identify any potential faults and the associated safety-critical process vari-
 168 ables $x(t)$. The real-time data of $x(t)$ can be either directly measurable via
 169 online monitoring or implicitly inferential via the mechanistic dynamic pro-
 170 cess model (Eqs. 1b-c). The risk indicator $RI(t)$ is defined in terms of the
 171 real-time deviation of $x(t)$ from nominal operating conditions. Two factors
 172 are considered for risk assessment, i.e. fault probability $P(x(t))$ and conse-
 173 quence severity $S(x(t))$ as shown in Eq. 2.

$$RI(t) = P(x(t)) \times S(x(t)) \quad (2)$$

174 Fault Probability

175 The safety-critical process variables $x(t)$ are assumed to follow statistical
 176 distributions, e.g. normal distribution characterized by the means (μ) and
 177 standard deviations (σ). The values of μ and σ are determined from the
 178 survey of industrial practice, historical cases, and open literature. μ stands
 179 for the $x(t)$ nominal operating points. $\mu \pm 3\sigma$ defines the upper and lower

control limit (UCL, LCL). Statistically, 99.7% of the $x(t)$ values would fall within this three-sigma region (i.e., three-sigma rule). The fault probability $P(x(t))$ is calculated as the probability density function of normal distribution (Eq. 3), particularly stressing the fault occurrence possibility when $x(t)$ deviate away from the three-sigma region.

$$P(x(t)) = \begin{cases} \phi\left[\frac{x(t)-(\mu+3\sigma)}{\sigma}\right] = \int_{-\infty}^{x(t)} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{[t-(\mu+3\sigma)]^2}{2\sigma^2}} dt, & \text{when } x(t) \geq \mu \\ \phi\left[\frac{x(t)-(\mu-3\sigma)}{\sigma}\right] = \int_{-\infty}^{x(t)} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{[t-(\mu-3\sigma)]^2}{2\sigma^2}} dt, & \text{when } x(t) < \mu \end{cases} \quad (3)$$

185 Consequence Severity

Consequence severity $S(x(t))$ quantifies the severity of the potential hazard due to $x(t)$ deviation. An exponential function is used to calculate $S(x(t))$ as shown in Eq. 4. Importantly, the consequence severity will grow increasingly faster as the safety-critical variables deviate further away from the nominal operating point.

$$S(x(t)) = \begin{cases} 100^{\frac{x(t)-(\mu+3\sigma)}{x(t)-\mu}}, & \text{when } x(t) \geq \mu \\ 100^{\frac{(\mu-3\sigma)-x(t)}{\mu-x(t)}}, & \text{when } x(t) < \mu \end{cases} \quad (4)$$

Using Eqs. 3-4, the overall form of risk indicator $RI(t)$ follows a pseudo-exponential function. To provide a more concrete idea, Fig. 3 depicts a generic dynamic risk profile for $RI(t)$ against $x(t)$. The occurrence of a fault is defined by the risk exceeding a pre-specified threshold value determined from historical case analyses.

Major advantages of this dynamic risk model are summarized below:

- 197 • Instantaneity – Fault probability and severity data are updated instantly based on safety-critical process variable changes, which can effectively support real-time process safety monitoring
- 200 • Standardization – At $\mu \pm 3\sigma$, $P(x(t))$ is mathematically set at 0.5 and $S(x(t))$ at 1 which provide a uniform basis to benchmark various processes design and operating conditions,
- 203 • Multivariate – $RI(t)$ can capture the independent or dependent interactions between multiple process variables, e.g. via the use of multivariate joint distribution function developed in [34],

- Prediction – A linear trend risk propagation forecast is utilized in [30]. Model-based forecast will be implemented in this work taking advantage of the model predictive control and optimization capabilities.

To enable the use of linear model predictive control in Section 2.3, piecewise linearization is performed to the $RI(t)$ function as illustrated in Fig. 4. Note that the linearized $RI(t)$ values are over-estimators of the original nonlinear $RI(t)$ values. Binary variables can be introduced to reformulate the piecewise $RI(t)$ functions into a unified mathematical form as shown in Eq. 5. In addition, the piecewise functions can actually characterize distinct operating regions based on the varying risk propagation speeds. The process and risk control priorities can thus be auto-adjusted, e.g. to majorly sustain stable operation in Region 1, to start prioritizing risk control in Region 2, and to adapt increasingly aggressive risk control in Regions 3 and 4.

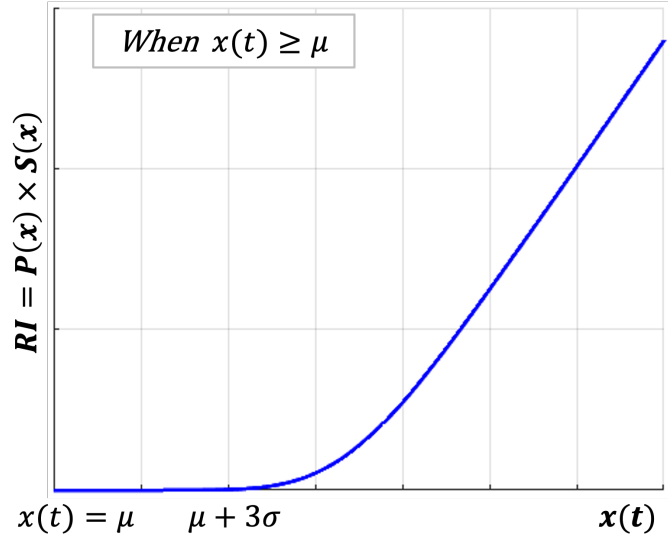


Figure 3: Dynamic risk modeling.

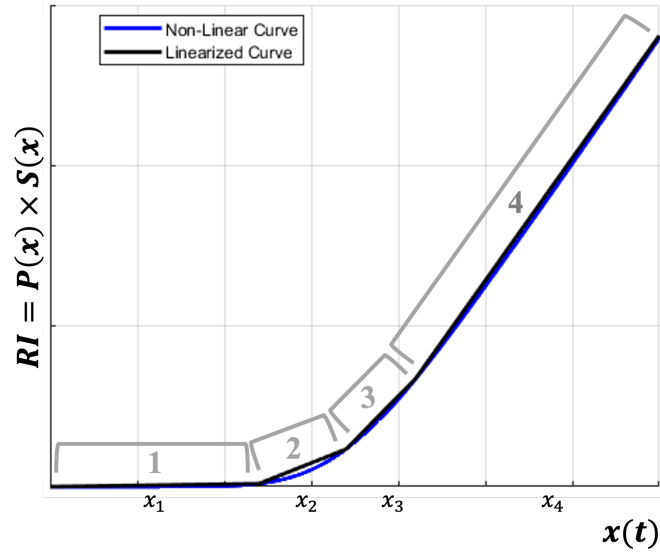


Figure 4: Piecewise linearization of dynamic risk.

$$RI(t) = \begin{cases} m_1x(t) + b_1, & x(t) \in [\underline{x}_1, \bar{x}_1) \\ m_2x(t) + b_2, & x(t) \in [\underline{x}_2, \bar{x}_2) \\ m_3x(t) + b_3, & x(t) \in [\underline{x}_3, \bar{x}_3) \\ m_4x(t) + b_4, & x(t) \in [\underline{x}_4, \bar{x}_4] \end{cases} \quad (5)$$

$$\begin{aligned} & RI(t) = Mx(t) + b \\ & \sum_i m_i y_i(t) = M, \quad \sum_i b_i y_i(t) = b \\ \iff & \sum_i x_i y_i(t) = x, \quad \sum_i y_i(t) = 1 \\ & \underline{x}_i y_i(t) \leq x_i(t) \leq \bar{x}_i y_i(t), \quad y_i(t) \in \{0, 1\} \\ & i \in \{1, 2, 3, 4\} \end{aligned}$$

219 *2.3. Design-dependent Risk-based mp-MPC*

220 The MPC problem with dynamic risk considerations is formulated as Eq.
 221 6. Eq. 6a defines the control objective function for output/input setpoint
 222 tracking and disturbance rejection. Eqs. 6b-c represent the discrete state
 223 space model linearized from the original mechanistic process model which can
 224 be obtained using model reduction, system identification, machine learning
 225 techniques, etc. [35, 36, 37] The piecewise linearized RI functions can thus be
 226 readily integrated with the linear process state space model by treating risk
 227 as an output variable (Eqs. 6c-e). Eqs. 6f-h are the path constraints for state,
 228 input, output, design, and risk variables. It is also worth clarifying that the
 229 linearized process model and dynamic risk model are only employed here for
 230 the design of a linear model predictive controller. The closed-loop controller
 231 validation and long-term dynamic optimization are conducted against the
 232 original mechanistic-based nonlinear models (Eqs. 1b-d). Design variables
 233 De can be explicitly considered in two forms depending on the specific process
 234 system: (i) in the state space model [38, 39], and (ii) in path constraints as
 235 the upper or lower bounds of process variables. The case study in Section 3
 236 will investigate design considerations belonging to the latter case.

$$\min_u J = x_N^T P x_N + \sum_{k=1}^{OH-1} ((y_k - y_k^R)^T Q R_k (y_k - y_k^R)) \quad (6a)$$

$$+ \sum_{k=0}^{CH-1} (u_k - u_k^R)^T R_k (u_k - u_k^R) \quad (6b)$$

s.t. $x_{k+1} = Ax_k + Bu_k + C[d_k; De]$

$$\begin{bmatrix} y_k \\ RI_k - b \end{bmatrix} = \begin{bmatrix} D \\ M \end{bmatrix} x_k + \begin{bmatrix} E \\ 0 \end{bmatrix} u_k \quad (6c)$$

$$\sum_i m_i y_i = M \quad \sum_i b_i y_i = b \quad \sum_i x_i y_i = x \quad (6d)$$

$$\sum_i y_i = 1 \quad \underline{x}_i y_i \leq x_i \leq \bar{x}_i y_i \quad y_i \in \{0, 1\} \quad (6e)$$

$$\underline{x} \leq x_k \leq \bar{x} \quad \underline{u} \leq u_k \leq \bar{u} \quad (6f)$$

$$\underline{y} \leq y_k \leq \bar{y} \quad \underline{d} \leq d_k \leq \bar{d} \quad (6g)$$

$$\underline{De} \leq De \leq \bar{De} \quad \underline{RI} \leq RI_k \leq \bar{RI} \quad (6h)$$

237 where P is terminal weight, QR and R are controller weights, CH and OH
 238 are respectively control and output horizons.

239 MPC problems can be reformulated into multi-parametric quadratic pro-
 240 gramming (mp-QP) problems as Eq. 7. More theoretical fundamentals on
 241 multi-parametric programming and its application in model predictive con-
 242 trol can be found in the recent authored book from Pistikopoulos et al. [31].
 243 Due to the existence of binary variables for piecewise risk functions, the
 244 design-dependent risk-based MPC formulation in Eq. 6 will finally result in
 245 a mixed-integer multi-parametric quadratic programming (mp-MIQP) prob-
 246 lem. Decomposition-based mp-MIQP solution algorithms can be used which
 247 leverage global optimization to identify candidate binary solutions and ac-
 248 celerate mp-QP solution efficiency via parallel computation [40].

$$\begin{aligned}
 \min_u \quad & f(u, \theta) = \frac{1}{2}u^T Qu + u^T H^T \theta + \theta^T Q_\theta \theta + c_u^T u + c_\theta^T \theta + c_c \\
 \text{s.t.} \quad & Nu \leq b + F\theta \\
 & CR^A \theta \leq CR^b \\
 & u \in \mathbb{R}^n, \quad \theta \in \mathbb{R}^m, \quad Q \succ 0
 \end{aligned} \tag{7}$$

249 The multi-parametric solution of Eq. 7 generates an optimal partition
 250 of the parameter space into a list of critical regions CR . Each critical re-
 251 gion is dictated by a unique active set of constraints to attain optimality
 252 in Eq. 7. As shown in Eq. 8, the optimal control actions on each criti-
 253 cal region can be explicitly expressed as an affine function of the parameter
 254 set. To the interest of this work, the parameters include states, outputs,
 255 setpoints, and disturbances as well as design variables (continuous and/or
 256 discrete) and risk indicator. Therefore, the MPC problem which typically
 257 requires online dynamic optimization can be replaced by an online function
 258 evaluation process using the optimal multi-parametric/explicit control laws
 259 generated offline *in priori*. A closed-loop validation step is performed to test
 260 the resulting mp-MPC controller for process and risk control and enhance
 261 the tuning parameters if necessary.

$$\begin{aligned}
 u_k &= K_i \theta_k + r_i \quad \theta_k \in CR^i = \{CR_i^A \theta \leq CR_i^b\} \\
 \theta_k &= [x_k, y_k, y_k^R, d_k, De, RI_k]
 \end{aligned} \tag{8}$$

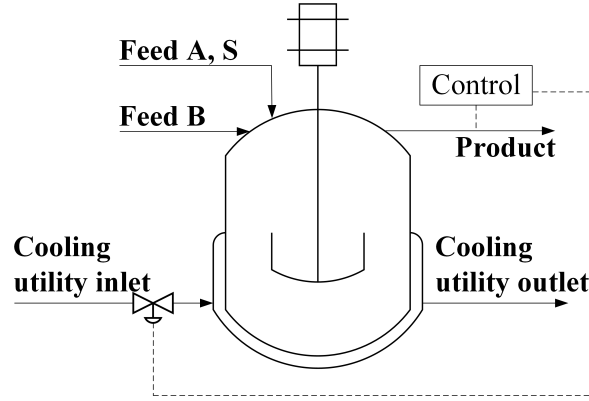


Figure 5: The T2 CSTR process.

increasing reaction rates (particularly of reaction 2). The research objectives of this case study are to:

1. Control the risk at a desired level under disturbances.
2. If thermal runaway cannot be prevented, attenuate the risk propagation speed and consequence severity while raising the alarm ahead of fault occurrence time for operator response (e.g., ≥ 10 minutes).
3. Identify the optimal design configuration and closed-loop control actions under disturbances with constrained dynamic risk.
4. Achieve optimal and safe operation via real-time optimization with process-tailored fault prognosis horizon.

In the second objective, the 10-min fault prognosis horizon is used as an indicative minimum allowable operator response time based on industrial practice [42]. The underlying assumption is that, if alarm can be raised 10 minutes before actual fault occurrence, operators can prudently perform the shutdown. Longer time horizon can also be used (e.g., ≥ 20 minutes) to enable ample time for response and/or to tailor process-specific requirements [43]. Though the proposed methodology framework is generally applicable, it is a trade-off decision on how to optimally determine the fault prognosis horizon (and also the risk threshold to raise alarm). A longer fault prognosis horizon enables more time for operator response while may result in more conservative control actions based on estimated disturbances and process conditions in future time steps.

3.2. T2 CSTR Risk-based Optimization

In what follows, we present the step-by-step application of the proposed approach to this T2 CSTR case study. The step-wise procedure is summarized in Fig. 6 based on the PAROC framework [29].

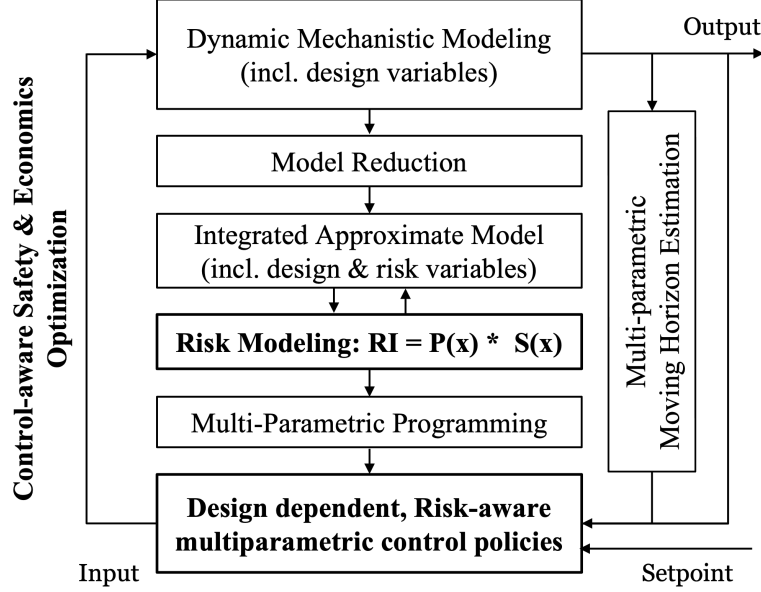


Figure 6: The step-wise procedure for controller and optimizer implementation.

3.2.1. Mechanistic Dynamic Modeling

We first develop a mechanistic dynamic model for the process (and safety) system of interest. An ideal CSTR with constant reactor volume is assumed to be in use. The nonlinear dynamic model is given in Eq. 9. Eqs. 9a-c describe the dynamic mass balances for reactants A, B, and S. Eq. 9d describes the dynamic energy balance in reactor. A list of the major process variables are summarized in Table 9. The kinetics and process parameter values are provided in Appendix A.

$$\frac{dC_A(t)}{dt} = \frac{F_{A,in}}{V} - \frac{q_{out}}{V} C_A(t) - k_1(T(t)) C_A(t) C_B(t) \quad (9a)$$

$$\frac{dC_B(t)}{dt} = \frac{F_{B,in}}{V} - \frac{q_{out}}{V} C_B(t) - k_1(T(t)) C_A(t) C_B(t) \quad (9b)$$

$$\frac{dC_S(t)}{dt} = \frac{F_{S,in}}{V} - \frac{q_{out}}{V}C_S(t) - k_2(t)C_S(t) \quad (9c)$$

$$\frac{dT(t)}{dt} = \frac{q_{out}}{V}(T_{in}(t) - T(t)) + \frac{\sum(-\Delta H_k)r_k(t)}{\rho c_p} - \frac{UA_x(T(t) - T_c)}{\rho c_p V} \quad (9d)$$

Table 1: List of CSTR process variables.

Symbol	Definition	Variable(s)	Physical Description	Unit
$x(t)$	States	C_A, C_B, C_S	Concentrations	mol/l
		T	Reactor temperature	K
$u(t)$	Input	U	Heat transfer coefficient	$\text{kJ}/(\text{K} \cdot \text{h} \cdot \text{m}^2)$
$d(t)$	Disturbance	T_0	Feed inlet temperature	K
De	Design	U_{max}	Maximum heat transfer coefficient	$\text{kJ}/(\text{K} \cdot \text{h} \cdot \text{m}^2)$

329 The mechanistic dynamic model is built in both MATLAB[®] and gPROMS[®]
330 ModelBuilder in preparation for the next step analyses. An open-loop simu-
331 lation is performed to study reactor dynamics particularly regarding thermal
332 runaway risk. A disturbance step change of $\Delta T_{in} = 25$ K is introduced at
333 $t = 0$ which can be resulted by a severe malfunction of the feed preheater.
334 As shown in Fig. 7, reactor temperature increases to the high risk region
335 after ~ 9 hours ($T \geq 480$ K), following which thermal runaway occurs.

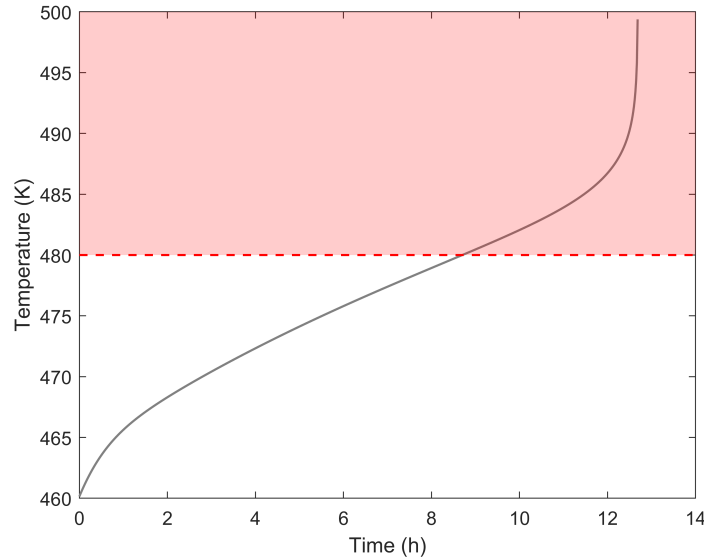


Figure 7: T2 CSTR open-loop simulation.

336 *3.2.2. Model Reduction*

337 The nonlinear CSTR model is then linearized around steady state using
 338 Jacobian matrices calculated by MATLAB[®] jacobian function. A discrete
 339 linear state space model is obtained as given in Eq. 10.

$$\begin{bmatrix} \bar{C}_A \\ \bar{C}_B \\ \bar{C}_S \\ \bar{T} \end{bmatrix}_{k+1} = A \begin{bmatrix} \bar{C}_A \\ \bar{C}_B \\ \bar{C}_S \\ \bar{T} \end{bmatrix}_k + B\bar{U}_k + C\bar{T}_{in,k}, \quad T_s = 1 \text{ min} \quad (10)$$

340 where the deviation variables are defined as $\bar{X} = X - X_{ss}$. The coefficient
 341 matrix values are:

$$A = \begin{bmatrix} 0.9506 & -0.0047 & 0 & -0.0003 \\ -0.0484 & 0.9943 & 0 & -0.0003 \\ 0 & 0 & 0.9990 & -1.5740 \times 10^{-6} \\ 0.6970 & 0.0678 & 0.0002 & 1.0030 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.0007 \end{bmatrix} \quad C = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.0010 \end{bmatrix}$$

342 The linearized CSTR model is validated against the original nonlinear
 343 model in Fig. 8. The two models match well up to $T \approx 480$ K while the
 344 linearized model cannot capture the temperature surge in high risk region.
 345 This also highlights the importance to apply the original nonlinear model for
 346 closed-loop control validation in Section 3.2.4 and dynamic optimization in
 347 Section 3.2.5.

348 In case that significant deviations exist between the Jacobian-based lin-
 349 earization model and the original nonlinear model, linear state space model
 350 can be generated using other techniques such as model reduction, system
 351 identification, and machine learning [35, 36, 37]. Moreover, in the current
 352 work, the approximated model is generated by sampling over the entire ex-
 353 pected operating region. Strategies have also been developed in recent inte-
 354 grated design and control literature which aimed to approximate and validate
 355 the overall dynamic optimization problem against a specific operating point
 356 (e.g., worst-case variability point) using trust-region approach [44], back-off
 357 approach [45, 46], etc. Online model updating and nonlinear model pre-

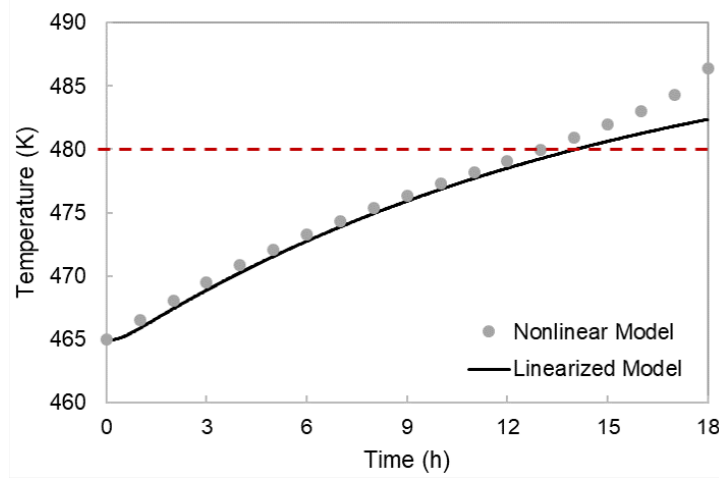


Figure 8: Comparison of linearized vs. nonlinear CSTR model.

dictive control provide alternative ways to overcome model approximation errors, which are briefly discussed in the concluding remarks section.

3.2.3. Dynamic Risk Modeling

We proceed to model the dynamic risk as a function of reactor temperature, namely the safety-critical variable for this T2 CSTR process. Following the methodology presented in Section 2.2, the reactor temperature is assumed to follow normal distribution. The nominal operating temperature is adapted at $\mu = 460$ K as per open literature data. The standard deviation is set at $\sigma = 5$ K which gives the upper control limit as $UCL = \mu + 3\sigma = 475$ K. The risk threshold is defined at $RI \geq 2.82$ according to the high risk region at $T \geq 480$ K. In other words, if the risk value exceeds 2.82 during operation, a fault occurs. The dynamic risk can thus be quantified using Eqs. 3 and 4 (only $x \geq \mu$ is of interest). To linearize the risk model as per Fig. 4, four piecewise affine functions are identified with the expressions listed in Eq. 11.

$$\overline{RI}_k = RI_k - b = M \begin{bmatrix} \overline{C}_A \\ \overline{C}_B \\ \overline{C}_S \\ \overline{T} \end{bmatrix}_k \quad (11)$$

$$M = \begin{bmatrix} 0 \\ 0 \\ 0 \\ m \end{bmatrix}^T, m = \begin{cases} 0.0078, & T \in [460, 472] \\ 0.2147, & T \in [472, 477] \\ 0.5496, & T \in [477, 481] \\ 0.7629, & T \in [481, 495] \end{cases}, b = \begin{cases} -0.1165, & T \in [460, 472] \\ -0.7374, & T \in [472, 477] \\ -0.0662, & T \in [477, 481] \\ 1.2120, & T \in [481, 495] \end{cases}$$

Up to this step, an integrated linear state space model has been obtained for dynamic process and risk modeling, which will be used for the next step controller design:

$$\begin{bmatrix} \bar{C}_A \\ \bar{C}_B \\ \bar{C}_S \\ \bar{T} \end{bmatrix}_{k+1} = A \begin{bmatrix} \bar{C}_A \\ \bar{C}_B \\ \bar{C}_S \\ \bar{T} \end{bmatrix}_k + B\bar{U}_k + C\bar{T}_{in,k} \quad (12)$$

$$\bar{RI}_k = M \begin{bmatrix} \bar{C}_A \\ \bar{C}_B \\ \bar{C}_S \\ \bar{T} \end{bmatrix}_k \quad T_s = 1 \text{ min}$$

3.2.4. Design-dependent Risk-based Multi-Parametric Controller design

A mp-MPC problem is formulated as per Eq. 6 using the above integrated linear state space model. Risk is incorporated as the output variable and bounded via the path constraints. The cooling system design variable U_{max} is considered through the path constraints $\underline{U} \leq U \leq U_{max}$. This step is implemented in MATLAB[®] using the POP Toolbox [47].

We first investigate dynamic risk management solely with mp-MPC and fault prognosis relying on moving horizon estimation. An output horizon of $OH = 10$ is selected which allows a 10-min risk forecast horizon (or fault prognosis). In other words, the controller computes the optimal action at the current time point by optimizing the disturbance rejection performance over the next 10 min. However, if the risk is projected to exceed the threshold value ($RI \geq 2.82$) during the next output horizon, an alarm will be raised around 10 minutes earlier to alert the operator. This will enable the operator to prudently plan for abnormality response or process shutdown. The mp-

390 MPC tuning parameters are listed in Table 2 and path constraints in Table
391 3 (in terms of deviation variables).

392 Since only 4 binary variables are involved in this case study while being
393 mutually exclusive, we solve the resulting mp-MIQP problem by enumerating
394 all the four possible integer solutions. A superset of four mp-QP solution
395 maps is generated and the solution of mp-MIQP is determined by searching
396 through the mp-QP maps (illustrated in Fig. B1). Each mp-QP problem
397 is solved to have 59 critical regions with 8 parameters. The parameter set
398 includes $\overline{C}_A$, $\overline{C}_B$, $\overline{C}_S$, and $\overline{T}$ as 4 state variables, $\overline{RI}$ as output variable, $\overline{RI}^R$
399 setpoint, $\overline{T}_{in}$ as disturbance, and $\overline{U}_{max}$ as design variable.

Table 2: mp-MPC tuning parameters.

OH	CH	QR	R
10	1	10^4	10^{-6}

Table 3: mp-MPC path constraints.

	$\overline{C}_A, \overline{C}_B, \overline{C}_S, \overline{T}$	$\overline{RI}$	$\overline{U}$	$\overline{T}_{in}$
Max	10, 10, 10, 25	25	$\overline{U}_{max}^*$	100
Min	-10, -10, -10, -15	-3	-55^*	-20

* $\overline{U}_{max}$ is the design variable

** CSTR heating duty is also available ($U < 0$)

400 The resulting mp-MPC controller is applied to the nonlinear CSTR and
401 risk model for closed-loop control using the following three scenarios:

402 • Scenario 1: Control at low risk level

403 The CSTR initial states are at $C_A = 0.4$ mol/l, $C_B = 1.5$ mol/l, $C_S = 2.5$
404 mol/l, and $T = 460$ K. A step change of the disturbance is introduced at $t = 0$
405 with $\Delta T_{in} = 25K$. As shown in Fig. 9, the open-loop process (i.e., without
406 controller) enters the high risk region after approximately 9 hours. On the
407 other hand, the risk-based multi-parametric controller can effectively control
408 the CSTR at low risk level ($RI \approx 0.00175$) with a set point at $RI^R = 0$.

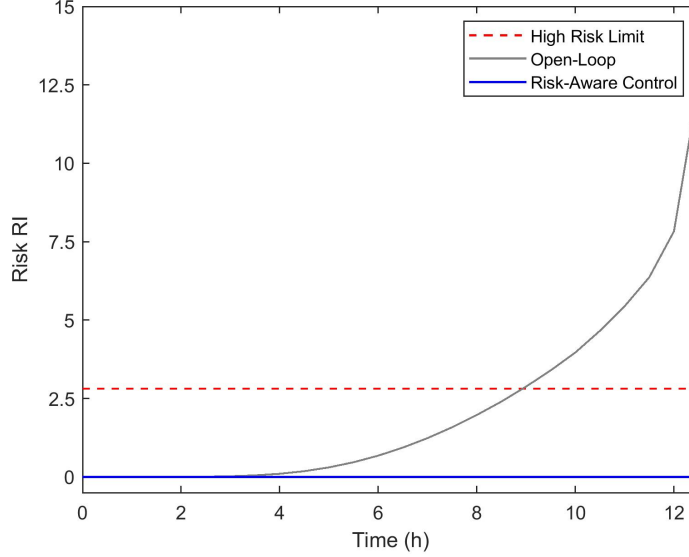


Figure 9: Scenario 1 – Closed-loop control at low risk level.

409 • Scenario 2: Control at medium risk level
 410 Sometimes it may be desired to operate the reactor at higher temperatures
 411 despite the increase of risk, for example to attain higher productivity. Given
 412 this, the controller is tested against a risk set point of $RI^R = 0.74$ which lies
 413 between the upper control limit and the high risk limit. Fig. 10 shows the
 414 closed-loop results. The controller adapts an initial heating step to rapidly
 415 increase the reactor temperature to the desired temperature setpoint but is
 416 able to stabilize the process afterwards in prevention of further risk surge.

417 • Scenario 3: Fault prognosis and alarm raising
 418 In certain cases, the process risk cannot be prevented from entering the
 419 high risk region due to notably large disturbances, insufficient cooling water
 420 availability, or more stringent risk limit. With the mp-MPC output horizon
 421 as 10 min, a ten-minute fault prognosis horizon can be achieved using model-
 422 based risk forecast. An example is shown in Fig. 11. At $t = 11.13$ h,
 423 the mp-MPC forecasts that the risk will enter the high risk region in the
 424 next 10 minutes. An alarm is thus raised to alert the operator. At $t =$
 425 11.35 h, the real-time risk value reaches the high risk region, i.e. a fault
 426 happens. The 13.2 minutes between alarming raising and fault occurrence

427 will be crucial for the operator to take response actions in a proactive manner.
 428 Importantly, the controller can continue mitigating the risk at the same time,
 429 to significantly reduce its propagation speed and fault severity compared to
 430 open-loop operation.

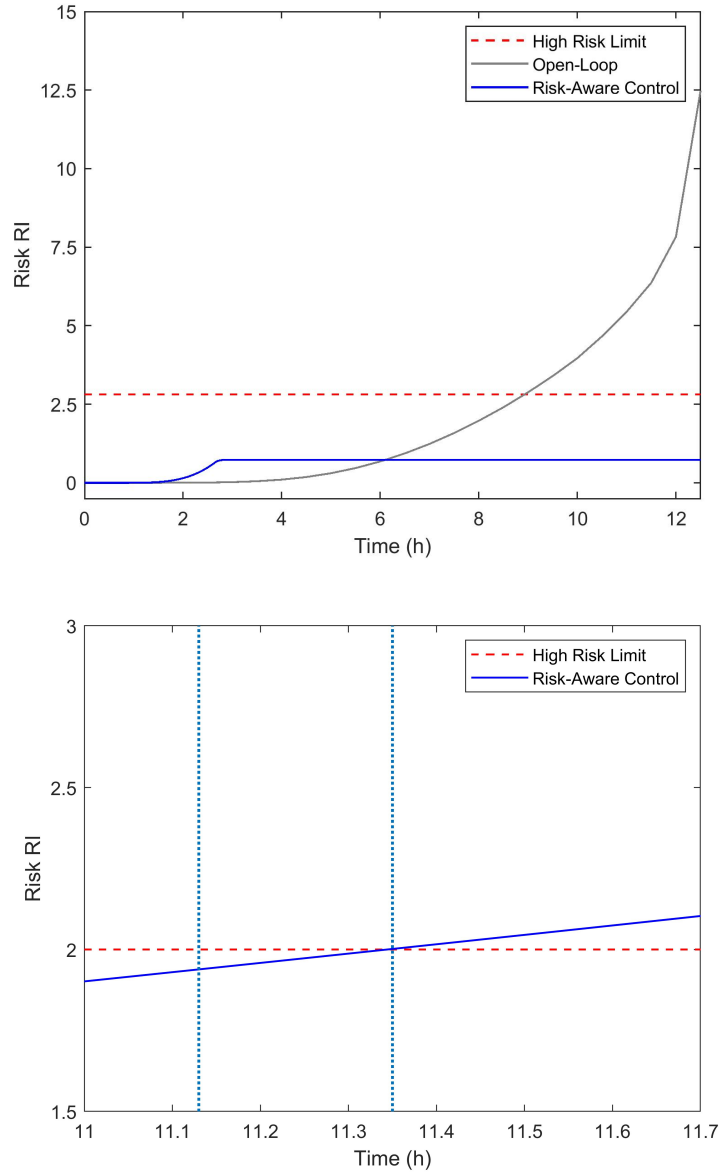


Figure 11: Scenario 3 – Control-aware fault prognosis and alarm raising.
 $t = 11.13$ h: alarm raised, $t = 11.35$ h: fault happens

431 We have demonstrated the proposed approach for dynamic risk manage-
 432 ment and fault prognosis exclusively via predictive control, which restricts
 433 the risk forecast horizon equal to control output horizon. In certain pro-
 434 cess systems such as with very fast dynamics, it is challenging to achieve a
 435 sufficient fault prognosis horizon for the operator from process safety man-
 436 agement aspect. In Section 3.2.5, we will augment the risk-based controller
 437 with a control-aware dynamic optimizer to empower decision making across
 438 distinct time scales. This will also extends the strategy for integrated design,
 439 economics, and safety optimization. To relax the control decision horizon,
 440 in what follows we will use a new mp-MPC controller with $OH = 2$ and
 441 $CH = 1$. Closed-loop validations are performed which have justified the risk
 442 control efficacy under Scenarios 1 and 2. The corresponding explicit control
 443 laws are included in Appendix B.

444 3.2.5. Control-aware Safety and Economics Optimization

445 We investigate two classes of applications for the integration of risk-based
 446 mp-MPC with dynamic optimization as per Fig. 1: (i) simultaneous risk-
 447 based process design and control optimization, (ii) fault-prognostic real-time
 448 optimization and control. For this specific case study, the economic consid-
 449 erations comprise the process design and operating cost (i.e., utility cost).
 450 As the operation of this exothermic CSTR is considered at high temperature
 451 under potential runaway risk, we assume that the product specification for
 452 C (the only liquid product) will always be satisfied when reactor tempera-
 453 ture is above 460 K. In certain other case studies, off-spec products may be
 454 generated due to insufficient control far from the set point (e.g., if higher
 455 temperature results in less productivity or selectivity) while it takes time for
 456 risk to propagate to high limit. Economic losses during this off-spec period
 457 can be readily incorporated to the objective function.

458 This step is implemented in gPROMS[®] ModelBuilder using CVP_SS as
 459 the dynamic optimization solver. The multi-parametric control laws de-
 460 rived from Section 3.2.4 are exported from MATLAB[®] and embedded in
 461 gPROMS[®].

462 • Application 1: Simultaneous Risk-based Design and Control

463 The control-aware dynamic optimization formulation for this T2 CSTR case
 464 study is given in Eq. 13. In the objective function, $\int_0^\tau U dt/\tau$ quantifies the
 465 average operating cost given that U is a pseudo-linear function of cooling
 466 water flowrate. WU_{max} indicates the design cost. U_{max} is considered as

467 a *time-invariant* design variable. W is a weighting factor to balance the
 468 operating and design cost, which currently takes the value of 1. The dynamic
 469 optimization is performed under a worst-case scenario (e.g., with step change
 470 $\Delta T_{in} = 25$ K) over $\tau = 100$ h to ensure the risk dynamics reach a new steady
 471 state (though much longer than enough).

472 U_{max} is treated as the only degree of freedom for this optimization prob-
 473 lem. Note that the multi-parametric control laws generated offline will com-
 474 pute the optimal control actions based on the real-time values of states,
 475 disturbances, risk indicator, and design variable. In other words, the mp-
 476 MPC control laws are designed a priori but the optimal control actions are
 477 calculated in real time and affected by the design variable (see Appendix
 478 B for an example of mp-MPC explicit control laws). The values of process
 479 initial states and risk set point remain same with the above control studies.
 480 By varying the risk tolerance at the end point of τ , a list of optimal cost
 481 objective values are obtained with the associated design variable values. For
 482 example, to achieve the end-point $RI \leq 0.00175$ (i.e., low risk level control
 483 in Scenario 1), the optimal design variable U_{max} is $48.2 \text{ kJ}/(\text{K} \cdot \text{h} \cdot \text{m}^2)$
 484 compared to the nominal value used earlier as $55 \text{ kJ}/(\text{K} \cdot \text{h} \cdot \text{m}^2)$. Fig. 12
 485 quantitatively depicts this trade-off to assist decision making for the optimal
 486 design and operation of safety-critical process systems.

$$\begin{array}{ll}
 \min & F = \int_0^\tau U(t) dt / \tau + W U_{max} & \text{Cost-objective function} \\
 \text{s.t.} & dx(t)/dt = f(x(t), u(t), d(t), De, Y) & \text{Nonlinear CSTR model (Eq. 9)} \\
 & RI = s(x(t), u(t), d(t), De, Y) & \text{Nonlinear risk model (Eqs. 2-4)} \\
 & u_k = K_i \theta_k + r_i & \text{Multi-parametric control laws} \\
 & \theta_k \in CR_i = \{CR_i^A \theta \leq CR_i^b\} & (OH = 2, CH = 1) \\
 & \theta_k = [x_k, y_k, y_k^R, d_k, De, RI_k] & (\text{Section 3.2.4, Eq. B1}) \\
 & \underline{x} \leq x(t) \leq \bar{x} & \text{Below are path constraints} \\
 & \underline{u} \leq u(t) \leq U_{max}, \quad \underline{y} \leq y(t) \leq \bar{y} \\
 & \underline{De} \leq De \leq \overline{De}, \quad \underline{RI} \leq RI(t) \leq \overline{RI}
 \end{array} \tag{13}$$

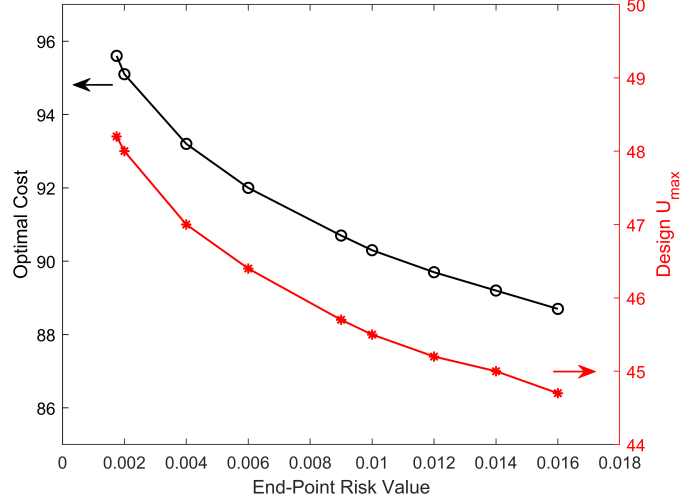


Figure 12: Optimal cost versus end-point risk limit.

487 • Application 2: Fault-Prognostic Real-Time Optimization and Control
 488 Herein, we consider U_{max} as a *time-variant* variable which can be adjusted
 489 in real time. In this context, the dynamic optimizer in Eq. 13 takes the
 490 role of real-time optimization to determine the cost-optimal U_{max} value si-
 491 multaneously with dynamic risk management and fault prognosis. If the
 492 risk is predicted to exceed the threshold, an alarm will be raised in advance.
 493 It is worth highlighting that the dynamic optimizer is aware of the closed-
 494 loop process, risk, and control dynamics. As illustrated in Fig. 13, the
 495 fault-prognostic optimizer forecast horizon τ is selected to be 30 min as an
 496 example. The resulting optimal U_{max} is then passed to the risk-aware con-
 497 troller. However, when applicable, the dynamic optimizer tends to strictly
 498 meet the end-point risk tolerance in exchange for lower design and operating
 499 cost which will then challenge the risk control in the next 30 min. Given
 500 this, the fault-prognostic optimizer is set to be activated in every 20 min to
 501 start the next round economics and safety optimization. Note that the char-
 502 acteristic times can be flexibly selected tailored to the process-specific need.
 503 For consistency, we again test the strategy to operate this T2 CSTR under
 504 a step change of $\Delta T_{in} = 25$ K with $RI \leq 0.00175$ at end of every optimizer
 505 forecast horizon τ . The results are shown in Fig. 14, in which the real-time
 506 U_{max} values can effectively guide the controller for dynamic risk management
 507 with cost-optimality at each step.

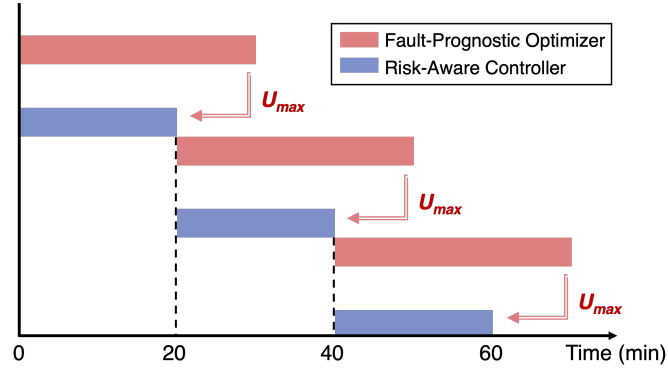


Figure 13: Integration scheme for fault-prognostic optimizer and risk-aware controller.

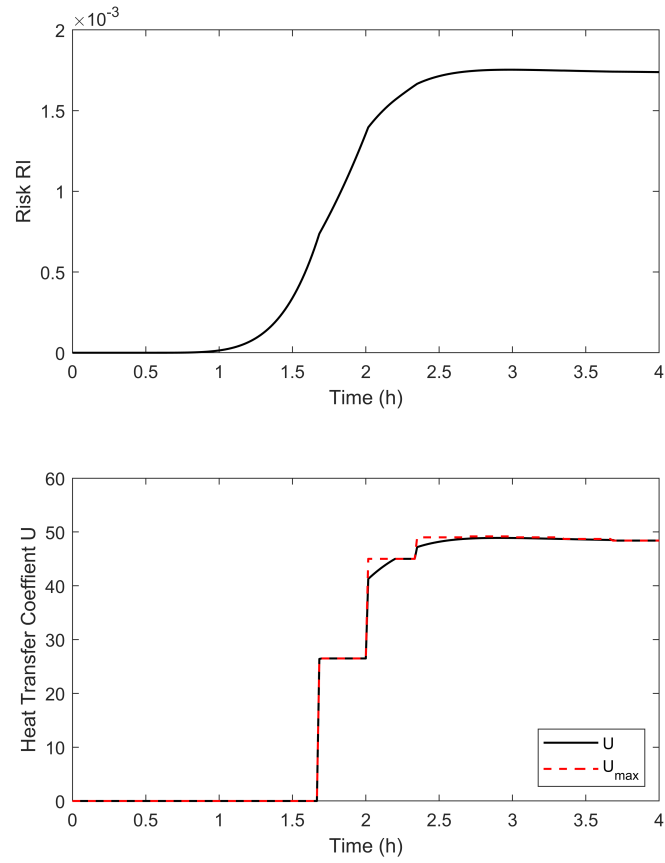


Figure 14: Real-time optimization of cooling water consumption.

508 4. Concluding Remarks

509 We have introduced a general framework for process design and opera-
510 tional optimization with online process safety monitoring and proactive risk
511 management. A model-based risk control strategy is developed via multi-
512 parametric programming, which also enables fault prognosis and alarm man-
513 agement via moving horizon estimation. The risk-based control is further
514 augmented with dynamic optimization to address optimal decision making
515 across multiple decision layers with potentially distinct characteristic time
516 scales (i.e., design, real-time optimization, control, and fault prognosis). The
517 efficacy and applicability of the approach has been demonstrated on the
518 safety-critical operation of an exothermic CSTR at T2 Laboratory, Inc.

519 The current work utilizes linear model predictive control which inevitably
520 introduces model approximation errors against the original nonlinear mecha-
521 nistic model. Robust (multi-parametric) model predictive control provides a
522 classical solution to address bounded errors due to model approximation [48].
523 Online model updating offers another option which can leverage Bayesian ap-
524 proach [49], neural network [50, 51], and other machine learning techniques
525 [52] to achieve reliable model predictive control by continuously learning pro-
526 cess mechanistic. More recently, nonlinear model predictive control (NMPC)
527 has gained increasing momentum with significant algorithmic improvement
528 to speed up computational times and enhanced fundamental understanding
529 on stability and robustness properties [53, 54, 55]. NMPC strategies have
530 been developed for highly nonlinear processes [56], economic MPC [57, 58],
531 and integration with design or higher level operational decisions (e.g., real-
532 time optimization [59, 60], simultaneous design and control [44]). Multi-
533 parametric programming has also been extended to obtain explicit NMPC
534 laws in prior work, e.g. for convex quadratically constrained control prob-
535 lems [61] and for generalized nonlinear process control based on balancing
536 of empirical gramians [36]. Ongoing work is addressing the comparison of
537 multi-parametric linear control versus nonlinear control particularly in the
538 safety-critical chemical processes.

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544 **Appendix A: T2 CSTR Reaction Kinetics and Process Parameters**

545 The reaction rate expressions and CSTR reactor parameter definitions
 546 are given below adapted from [22].

547 Reaction 1: $r_1 = -k_1 C_A C_B$, where $k_1 = k_{10} \exp(-\frac{E_1}{RT})$

548 Reaction 2: $r_2 = -k_2 C_S$, where $k_2 = k_{20} \exp(-\frac{E_2}{RT})$

Table A1: List of CSTR process parameters.

Variable	Description	Value	Unit
$F_{A,in}$	Feed flowrate (A)	1050	mol/h
$F_{S,in}$	Feed flowrate (S)	525	mol/h
$F_{B,in}$	Feed flowrate (B)	1250	mol/h
ρ_{AS}	Feed molar density (AS)	7.33	mol/l
ρ_B	Feed molar density (B)	36	mol/l
ρ	Mixture molar density in CSTR	7.31	mol/l
k_{10}	Rate constant (reaction 1)	4×10^{14}	l/mol
k_{20}	Rate constant (reaction 2)	1×10^{84}	1/h
E_1	Activation energy (reaction 1)	1.28×10^5	J/mol/K
E_2	Activation energy (reaction 2)	8×10^5	J/mol/K
ΔH_1	Heat of reaction 1	-45400	J/(mol B)
ΔH_2	Heat of reaction 2	-3.2×10^5	J/(mol S)
V	Reactor volume	4000	l
C_p	Average specific heat	430.91	J/mol/K
T_c	Coolant temperature	373	K
A_x	Heat transfer area	5.3	m ²

549 Appendix B: Multi-Parametric/Explicit Control Laws

550 This appendix presents more detail for the design-dependent risk-aware
 551 mp-MPC with $OH = 2$, $CH = 1$ (Section 3.2.4). The resulting mp-MIQP
 552 problem is solved by enumerating 4 mp-QP problems, which are partitioned
 553 as per the temperature ranges for RI piecewise linearization. Namely, T
 554 respectively in $[460, 472]$, $[472, 477]$, $[477, 481]$, and $[481, 495]$ (Eq. 11). The
 555 conceptualization is illustrated in Fig. B1. Each mp-QP problem is solved
 556 to have 11 critical regions with 8 parameters.

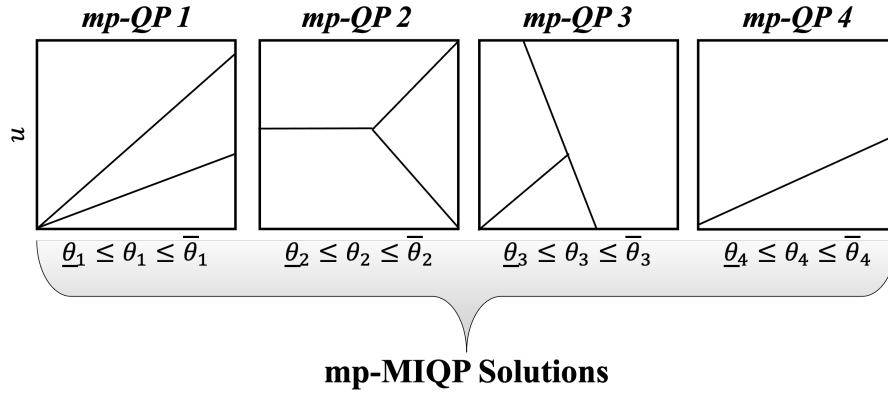


Figure B1: mp-MIQP solutions as a superset of mp-QPs.

557 To provide a more concrete idea on the form of multi-parametric/explicit
 558 control laws, the optimal control solution on Critical Region 1 of the mp-QP
 559 problem for $T \in [460, 472]$ is listed below. Fig. B2 further gives a geometrical
 560 view of critical regions at the CSTR initial states. θ_5 is the deviation variable
 561 for *disturbance* (i.e., $\bar{T}_{in}$) and θ_6 is the deviation variable for *risk* (i.e., $\bar{RI}$).

562 Critical Region 1 (CR01):

- 563 • $\theta = [\bar{C}_A, \bar{C}_B, \bar{C}_S, \bar{T}, \bar{T}_{in}, \bar{RI}^R, \bar{U}_{max}]$
- 564 • $u = K_1 \theta + r_1$
- 565 $K_1 = [130.34, 12.68, 0.033, 0.56, 0.19, 2.41 \times 10^4, -2.41 \times 10^4, 0]$
- 566 $r_1 = 0$
- 567 • $CR_1 = \{CR_1^A \theta \leq CR_1^b\}$
- 568 Constraints for upper and lower bounds of parameters are skipped for
- 569 brevity, but can be found in Table 3.

$$\begin{aligned}
& \begin{bmatrix} 0.0247 & 0.0024 & 6.3e-6 & 0.041 & 3.7e-5 & -0.7063 & 0.7063 & 0 \\ -0.0949 & 0.9955 & -4.4e-8 & -0.0006 & -2.6e-7 & 0.0049 & -0.0049 & 0 \\ 0.024 & 0.0023 & 6.3e-6 & 0.0206 & 3.7e-5 & -0.7068 & 0.7068 & 0 \\ -0.0247 & -0.0024 & -6.3e-6 & -0.041 & -3.7e-5 & 0.7063 & -0.7063 & 0 \\ 0.0949 & -0.9955 & 4.4e-8 & 0.0006 & 2.6e-7 & -0.0049 & 0.0049 & 0 \\ -0.024 & -0.0023 & -6.3e-6 & -0.0206 & -3.7e-5 & 0.7068 & -0.7068 & 0 \\ 0.0038 & 0.0004 & 9.8e-7 & 1.6e-5 & 5.7e-6 & 0.7071 & -0.7071 & 0 \\ -0.0038 & -0.0004 & -9.8e-7 & -1.6e-5 & -5.7e-6 & -0.7071 & 0.7071 & 0 \\ 0.0053 & 0.0005 & 1.4e-6 & 2.3e-5 & 8.0e-6 & 0.9882 & 0.1528 & 0 \\ -0.0053 & -0.0005 & -1.4e-6 & -2.3e-5 & -8.0e-6 & -0.9882 & -0.1528 & 0 \\ -0.0486 & 0.9988 & 0 & -0.0003 & 0 & 0 & 0 & 0 \\ 0.0486 & -0.9988 & 0 & 0.0003 & 0 & 0 & 0 & 0 \end{bmatrix} \\
& CR_1^A = \\
& CR_1^b = \begin{bmatrix} 0.4094 \\ 10.0670 \\ 0.2045 \\ 0.6141 \\ 10.0670 \\ 0.3068 \\ 0 \\ 0.0016 \\ 1.1411 \\ 3.4232 \\ 10.0455 \\ 10.0455 \end{bmatrix}
\end{aligned}$$

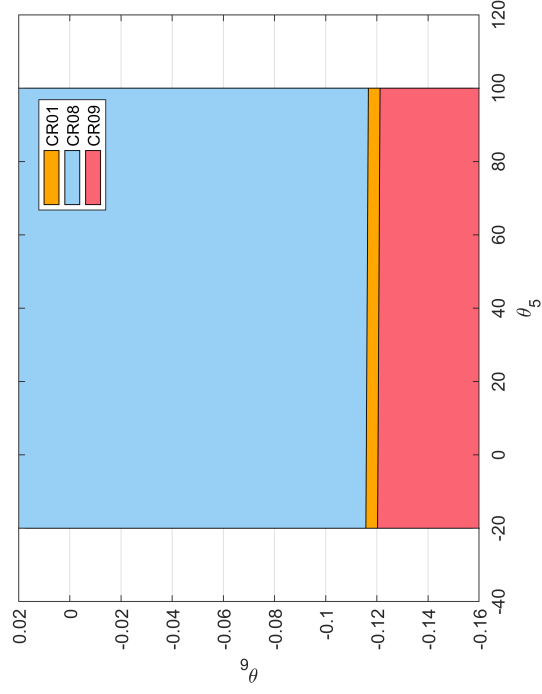


Figure B2: Partitioning of parameter space at CSTR initial states.

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