Learning 3D Matrix Algebra using Virtual and Physical Manipulatives: Statistical Analysis of Quantitative Data Evaluating the Efficacy of the AR-Classroom

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Abstract. The Augmented reality (AR) allows digital information to be overlayed onto a physical plane that users can manipulate. Using the unique capabilities of AR, the AR-Classroom learning tool aims to teach three-dimensional (3D) geometric rotations and their mathematics using virtual and physical manipulatives. In an efficacy experiment, undergraduates completed pre-test measures of matrix algebra and spatial thinking skills, were assigned to interact with virtual ($N = 20$) or physical ($N = 20$) manipulatives in the AR-Classroom, or to complete active control activities ($N = 20$), and then completed post-test measures of matrix algebra and spatial thinking skills. Using a series of ANCOVAs, pre-test accuracy on matrix algebra and spatial thinking tests significantly predicted matrix algebra post-test accuracy. There were no significant group differences indicating that all three groups showed similar improvement in matrix algebra skills. Further ANCOVAs revealed that the virtual and physical conditions showed marginally significant improvements on matrix algebra questions that were taught by the AR-Classroom, specifically rotations and rotations combined with translations. These initial findings indicate that the AR-Classroom may aid students in improving their mathematical skills. Suggestions for future improvements to the AR-Classroom and efficacy experiments on the AR-Classroom are discussed.

Keywords: User Experience (UX), Augmented Reality (AR), Educational Technology, Embodied Learning, Math Learning.

1 Introduction

1.1 Learning using Augmented Reality

Mathematical concepts can be challenging for many students to visualize, particularly when those concepts are abstract, dynamic, and/or involve multi-dimensional spaces [1, 2]. These challenges extend to their instructors as they struggle to find teaching aids that address or alleviate these challenges. If not appropriately addressed, students who struggle with mathematical concepts in younger grades may continue to struggle well into higher education [3, 4, 5]. Difficulties in learning mathematical concepts can impede students from pursing science, technology, engineering, or mathematics (STEM) degrees and careers.

Multiple representations are frequently used to enhance conceptual learning. This is why it is common for STEM instructional material to use multiple representations to illustrate complex or abstract concepts. Another approach is to use embodied learning where students physically interact with either virtual or physical manipulatives to enhance their learning. While numerous interventions have sought to integrate embodied learning with abstract conceptual learning [6], theories on conceptual and embodied learning often conflict about the effectiveness of using virtual or physical manipulatives alone [7]. Therefore, learning interventions designed for teaching abstract mathematical concepts might be more effective when they combine multiple manipulatives and stimulation. Augmented Reality (AR) technology can do just that. AR can provide both situated and embodied approaches to learning [8, 9, 10] as students' learning is contextually situated while the student physically interacts with that specific context. Combining both approaches may enhance knowledge acquisition of challenging concepts.

AR-enabled educational technologies could be an innovative solution to the challenges instructors and students face in engaging with complex mathematical topics. AR can be used to intentionally integrate real and virtual stimuli within immersive and interactive learning experiences [11, 12, 13]. Using AR for the instruction of mathematical concepts has been found to provide an interactive learning process that enhances understanding and visualization [14]. AR also has the potential to simplify complex and abstract mathematical theory thereby allowing learners to interact with the content through virtual and physical stimuli. With this in mind, the AR-Classroom application was developed to leverage AR capabilities combined with physical and virtual manipulatives. This paper investigates the efficacy of the AR-Classroom as a learning intervention targeting geometric transformations and their underlying mathematical theory.

1.2 Foundational Research on the BRICKxAR/T

The development of and research on the AR-Classroom has been informed by usability tests and learning efficacy research conducted on the BRICKxAR/T application. BRICKxAR/T is a predecessor to the AR-Classroom. It was an educational technology application that ran on an iPad so that users could move around and interact with LEGO models [15]. This embodied approach was designed to support learning the mathematical concepts behind geometric transformations. One version of the BRICKxAR/T used AR technology to visualize axes of rotation along with the entries within transformation matrices. Another version did not use AR (non-AR) and served as a comparison for evaluative purposes.

The BRICKxAR/T's user experience was evaluated by a series of usability tests: a benchmark usability test provided initial user interactions with the BRICKxAR/T, recommendations for improvements were made based on the benchmark usability test, then some recommendations were implemented, and finally a follow-up usability test was run to identify changes in user interactions [16]. This series of tests revealed that making changes to initial instructions and a demo video resulted in users rating the app as more intuitive and easier to use, users more often sought out and effectively used in-app instructions, and better understood the relationship between the LEGO model, AR wireframe overlay on the LEGO model, and the rotation matrix. In addition, a learning efficacy experiment conducted on BRICKxAR/T found that scores on a math test significantly improved from pre-test to post-test [17]. Participants who interacted with the AR-enabled workshop tended to show greater improvements compared to the non-AR workshop. Participants reported that the BRICKxAR/T was interesting and useful, and participants engaged with the AR workshop for longer than the non-AR workshop. On the basis of these promising research findings, the AR-Classroom was developed to expand the use of AR technology and to compare the use of physical versus virtual manipulatives teach matrix algebra concepts.

1.3 AR-Classroom Capabilities

The AR-Classroom application was developed in a way that makes challenging and not (typically) visible concepts more interactive, visible, and intuitive [18]. These features are designed to engage students in embodied learning which may lessen the demand on their spatial thinking skills and support more intuitive understanding of mathematical concepts. More specifically, the AR-Classroom utilizes AR technology to teach two-dimensional (2D) and three-dimensional (3D) geometric rotations and their mathematics using virtual and physical manipulatives.

AR-Classroom consists of a model registration tutorial and two workshops: virtual and physical. In the model registration tutorial, users are introduced to registering the LEGO space shuttle with the AR-Classroom. In the workshops, users perform rotations by utilizing the application's X, Y, and Z axes sliders to rotate a virtual model (i.e., virtual workshop, Fig. 1) or by using a physical space shuttle LEGO model (i.e., physical workshop, Fig. 2). Each workshop displays a green wireframe model superimposed onto a LEGO model to represent the rotation transformations, color-coded axis lines, degree or radian representations, Z-axis direction (up versus down) manipulation, different model views, and 2D or 3D matrices.

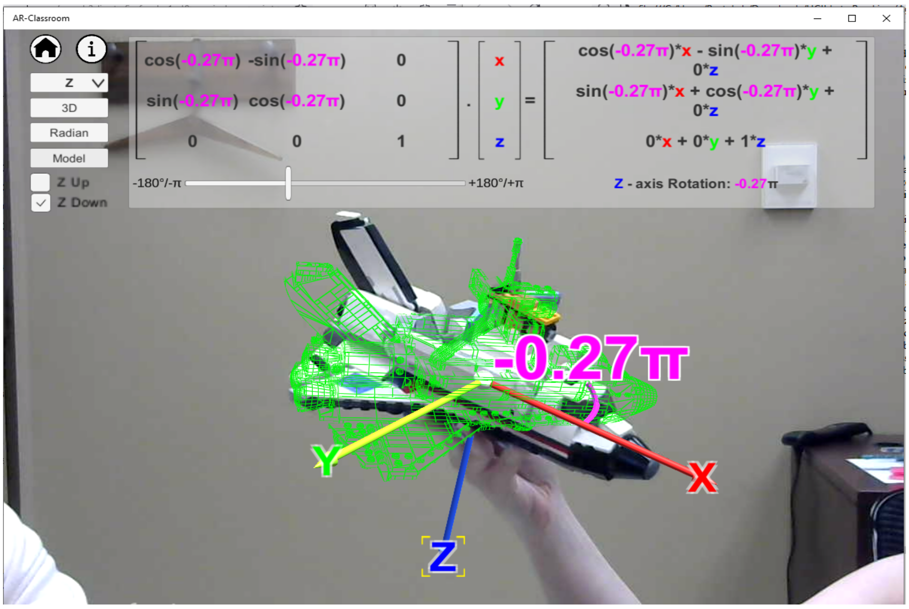


Fig. 1. The AR-Classroom's Virtual workshop with a Z-axis rotation and angle in radians.

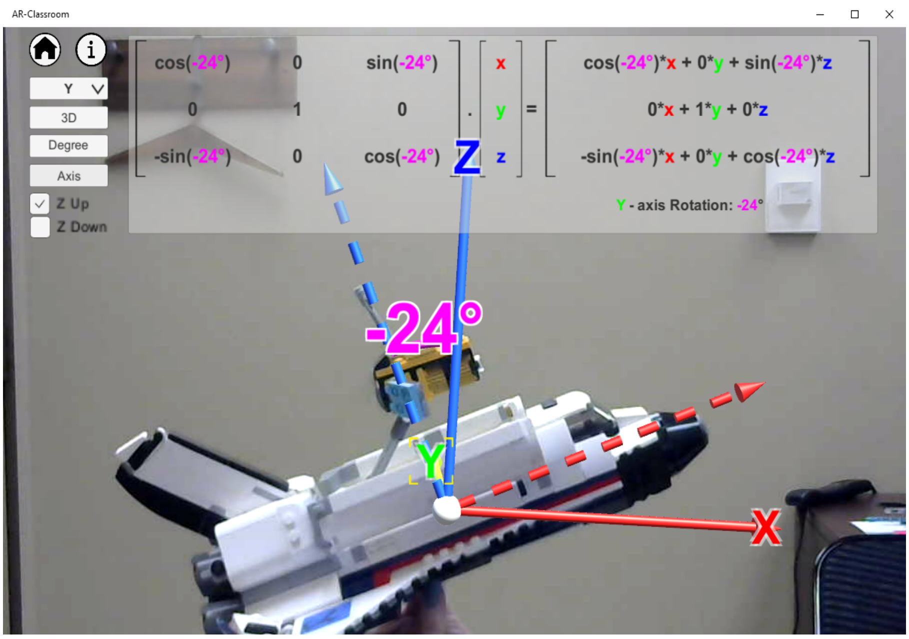


Fig. 2. The AR-Classroom's Physical workshop with a Y-axis rotation and angle in degrees.

1.4 Previous Research on the AR-Classroom

After the initial development of the AR-Classroom, several usability tests have been used to assess user-app interactions, app feature functionality, app ease of use, and the impact of iterative app improvements. First, a set of usability tests were run [19]. These tests included a benchmark usability test, recommendations for changes to the AR-Classroom based on the benchmarked usability, changes made to the AR-Classrsoom, an updated usability test, and a comparison between benchmarked usability and usability after the recommended changes were made. After changes were made, the usability of both workshops was improved as users could more easily set up the LEGO space shuttle, they more effectively used in-app instructions, and they more quickly accessed the app's features. Even with these improvements, salient issues in user-app interactions remained. Next, another round of changes were made to the AR-Classroom and a third usability test evaluated the cumulative impact of these changes [20]. The third usability test found that the changes made to the AR-Classroom increased metrics of user experience, ratings of ease of use, and better supported user's understanding of how the app functioned. The findings derived from previous research on the BRICKxAR/T and AR-Classroom apps informed the current learning efficacy experiment.

1.5 Learning Efficacy Experiment on the AR-Classroom

The present study investigated the AR-Classroom's efficacy in teaching undergraduate students the basics of matrix algebra using three learning conditions: using the AR-Classroom’s virtual workshop, using the AR-Classroom’s physical workshop, and completing active control activities that were designed not to train matrix algebra skills. The purpose of using two experimental conditions was to compare students' matrix algebra learning after using virtual or physical manipulatives. This paper will focus on quantitative differences between the experimental and active control groups. In contrast, another paper will focus on qualitative differences in learning from the AR-Classroom [21]. The current study asked two broad research questions about the AR-Classroom's efficacy in teaching the basics of matrix algebra:

1. How does matrix algebra skill and/or confidence change after using the AR-Classroom application, compared to an active control?
2. How does using physical versus virtual manipulatives in the AR-Classroom contribute to changes in matrix algebra skill and/or confidence?

2 Method

2.1 Participants

Sixty Texas A&M University undergraduates were recruited using the Department of Psychological & Brain Sciences’ research sign-up system (Table 1). These students received research course credit for participating in the experiment. Each participant was randomly assigned to interact with the AR-Classroom’s virtual workshop ($N = 20$) or physical workshop ($N = 20$), or they completed the active control activities ($N = 20$).

Table 1. Demographics and Previous Experience with Matrices by Learning Condition.

Heading level	Virtual Workshop	Physical Workshop	Active Control
Year of schooling	13 freshmen 5 sophomores 1 junior	12 freshmen 5 sophomores 2 juniors 1 senior	14 freshmen 4 sophomores 2 seniors
Mean Age	19.1 years old	19.0 years old	18.6 years old
Gender	12 men 8 women	8 men 12 women	5 men 15 women
Experience with 2D Matrices	19 had experience 1 did not	19 had experience 1 did not	17 had experience 3 did not
Experience with 3D Matrices	10 had experience 10 did not	14 had experience 6 did not	5 had experience 15 did not

2.2 Design

The efficacy experiment utilized a three between-subjects learning conditions (virtual, physical, active control) by two within-subjects test (pre and post) design.

2.3 Procedures

Participants in the virtual and physical conditions completed the following in a two-hour session: a pre-test, interacted with the AR-Classroom's virtual or physical workshop (based on their assigned condition), and a post-test. Participants in the active control condition completed the following in a two-hour session: a pre-test, completed assessments and activities about spatial thinking and analogies, and a post-test.

2.4 Pre-Test Materials

All three learning conditions followed the same pre-test: demographic and experience survey, the Purdue Spatial Visualization Test-Visualization of Rotations, and a matrix algebra test with confidence ratings.

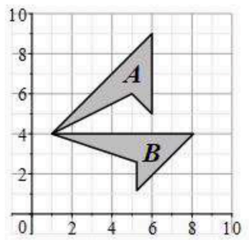
Demographics and Experience Survey. Participants answered multiple-choice questions about their year of schooling, age, gender, major, previous experience with 2D and 3D matrices, and level of experience (5-point scale from 1 = "not at all familiar" to 5 = "extremely familiar") with video games, 3D modelling, and AR.

Purdue Spatial Visualization Test-Visualization of Rotations (PSVT:R). The PSVT:R is a commonly used assessment of spatial visualization skills that is composed of 20 problems. In this assessment, participants see diagrams of 3D object in a starting orientation and a finishing orientation after an unknown rotation on one or more axes has been completed. The participant must identify a diagram of a second 3D object that has been rotated in the same manner as the first 3D object. They are presented with the second 3D object in a starting orientation and must select the correct finishing orientation from a set of five options. Performance on the PSVT:R is calculated as a percentage of correctly answered problems.

Matrix Algebra Test with Confidence Ratings. The matrix algebra test was composed of sixteen multiple-choice questions, each followed by a confidence rating. The questions were developed by a mathematics professor to cover simple to complex matrix algebra concepts. Low complexity questions required identifying transformations based on diagrams, and identifying the center, direction, vector, and/or angle of a transformation (Fig. 3). Moderate complexity questions required identifying a 3x3 rotation matrix that represents a given transformation or identifying the transformation from a given 3x3 rotation matrix (Fig. 4). High complex questions required identifying a 4x4 rotation matrix that represents a given translation and transformation, or the inverse (Fig. 5). The concepts in the matrix algebra test included rotations, which were the focus of the AR-Classroom's current capabilities, and also translations and scaling, which were only covered in the introductory video on matrix algebra. Performance on the matrix algebra test was calculated as a percentage of correctly answered questions.

After each of the 19 matrix algebra questions, participants were asked to rate their confidence in their answer. Participants rated their confidence on a 5-point scale from 1 = "Not at all confident (I guessed)" to 5 = "Completely confident (I know I answered correctly)". Average confidence ratings were calculated.

Problems 1-4 refer to the plot:



1. What transformation (motion) is applied to quadrilateral A to get quadrilateral B?
   a. translation
   b. rotation Correct Answer
   c. reflection
   d. dilation (scale)

Fig. 3. An example of a low complexity question from the matrix algebra test that requires identifying transformations based on a diagram.

12. What type of 3D transformation is performed by the matrix

$$M = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}?$$

a. Rotation about the x axis
b. Rotation about the y axis Correct Answer
c. Reflection through the plane $x\cos\theta + z\sin\theta = 0$
d. Reflection through the plane $x\cos\theta - z\sin\theta = 0$

Fig. 4. An example of a moderate complexity question from the matrix algebra test that requires identifying the transformation represented by a given 3x3 rotation matrix.

15. Using affine coordinates for 3D space, which matrix performs a rotation by θ about the y axis followed by a translation by $\vec{v} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$?

a. $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 2 & 3 & 4 & 1 \end{pmatrix}$

b. $\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & \cos\theta & -\sin\theta & 3 \\ 0 & \sin\theta & \cos\theta & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

c. $\begin{pmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 2 & 3 & 4 & 1 \end{pmatrix}$

d. $\begin{pmatrix} \cos\theta & 0 & -\sin\theta & 2 \\ 0 & 1 & 0 & 3 \\ \sin\theta & 0 & \cos\theta & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ Correct Answer

Fig. 5. An example of a high complexity question from the matrix algebra test that requires identifying the transformation represented by a given 3x3 rotation matrix.

2.5 AR-Classroom Interaction Materials

After completing the pre-test, participants watched introductory videos on matrix algebra and the AR-Classroom. The introductory video on matrix algebra covered key concepts and terminology to remind students of previous coursework in matrices or to provide foundational understanding if students lacked previous coursework in matrices. The introductory video on the AR-Classroom taught students how to set-up a LEGO space shuttle model for use with the AR-Classroom. After watching the two videos, a desktop computer with a webcam was used to run the AR-Classroom application.

To guide participants through using the AR-Classroom, they followed a rotation booklet with three questions (one question for each of X-, Y-, and Z-axes) on 90-degree counterclockwise rotations (Fig. 6) and three questions (one question for each axis) on 30-degree counterclockwise rotations (Fig. 7). Each question included two rotation matrices that students would fill out: one was filled out using the rotation booklet and the other was filled out using the AR-Classroom (Fig. 8).

Rotation #1 - Rotation by 90 degrees counterclockwise about the x-axis

1. We would like to understand the matrix which describes a rotation by 90° counterclockwise about the x-axis.

Step A: Fill in the matrix in the answer booklet using the LEGO model and instructions below

Let this matrix be $R = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$. If $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ is any point on the shuttle, then this rotation moves the point X to the point

$$RX = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} ax + by + cz \\ dx + ey + fz \\ gx + hy + iz \end{pmatrix}$$

We want to determine the numbers $a, b, c, \cdots, i$. Pick up the shuttle, hold it so you are looking directly at the nose (the x-axis). Then the y-axis (the left wing) is on your right and the z-axis (the antenna) is up. Rotate it counterclockwise by 90°.

(a) Notice the nose did not move, i.e. the point $(N, 0, 0)$ moved to the point $(N, 0, 0)$, or

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} N \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} N \\ 0 \\ 0 \end{pmatrix}$$

Using the rotation matrix at the top for reference, we can formulate:

$$a(N) + b(0) + c(0) = N \qquad (1)$$
$$d(N) + e(0) + f(0) = 0 \qquad (2)$$
$$\vdots$$

Use this to determine a, d, g.

Fig. 6. The first step of instructions for rotation #1 from the rotation booklet. These instructions guide students through using the LEGO space shuttle model to fill in an empty rotation matrix.

Rotation #6 - Rotation by 30 degrees counterclockwise about the z-axis

6. We would like to understand the matrix which describes a rotation by 30° counterclockwise about the z-axis.

Step A: Fill in the matrix in the answer booklet using the LEGO model and instructions below

Again let this matrix be $R = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$. Hold the shuttle so you are looking down from above at the antenna. Hold the shuttle so the x-axis (the nose) is on your right and the y-axis (the left wing) is forward. Rotate the shuttle by 30° counterclockwise about the z-axis. The antenna does not move, i.e. $(0, 0, A)$ moves to $(0, 0, A)$.

The nose moves to 30° from the x-axis toward the y-axis but stays in the xy-plane, i.e. $(N, 0, 0)$ moves to $(N \cos 30°, N \sin 30°, 0)$. Check the sin and cos and the signs against the figure.

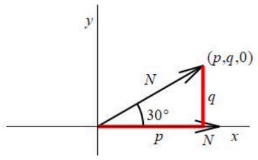


Fig. 7. The first step of instructions for rotation #6 from the rotation booklet. These instructions guide students through using the LEGO space shuttle model to fill in an empty rotation matrix.

Answer Sheet #2: Rotation by 90° counterclockwise about the y-axis.

Step A: Fill in Matrix R using the instructions in the Rotation Booklet and the Lego Shuttle:

Matrix R

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Step B: Fill in Matrix R' using the AR Classroom – Workshop 1 Virtual and the Lego Shuttle:

Hold the shuttle in front of the AR Classroom App and again rotate it **90° counterclockwise about the y-axis**. Remember to set the AR Classroom: Dimension = 3D, Angle = Degrees

Matrix R'

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

STEP C: Does the Matrix R' displayed by the AR Classroom agree with the Matrix R? If not, ask the experimenter for help.

Fig. 8. For each of the 6 questions, students filled out the rotation matrix on the top using the rotation booklet and filled out the rotation matrix on the bottom using the AR-Classroom.

Each set of three questions followed a similar progression. The first rotation question was thoroughly explained so that the participant understood how each of the 9 values in the matrix were found. The second rotation question was presented with a shortened explanation, requiring the student to use what they learned from the first rotation. The third rotation question contained the most basic explanation. For each of the six rotation questions, participants would read the rotation booklet and use the LEGO space shuttle to fill out a rotation matrix. Then the AR-Classroom with the LEGO space shuttle were used to allow the participants to see how the rotation was performed and to check if their rotation matrix was correct or not. In this way, the learning session was self-paced and allowed students to correct their own mistakes. An experimenter was available for questions throughout the learning session but tried to direct the participant to answer their own questions using the rotation booklet and AR-Classroom.

The primary difference between the physical and virtual learning conditions is in how participants interacted with LEGO space shuttle and the AR-Classroom. For both workshops, participants hold the LEGO space shuttle in front of the webcam and the AR-Classroom presents 3D visualizations of axes and angle of rotation superimposed onto the video of the participant holding the space shuttle. In the virtual workshop (Fig. 1), participants use a dropdown box to select the axis of rotation and then move a slider to change the angle of rotation. By moving the slider, they rotate the axes superimposed onto the stationary shuttle and the rotation matrix is updated with every movement of the slider. In the physical workshop (Fig. 2), participants use a dropdown box to select the axis of rotation and then physically rotate the shuttle along the axis of rotation. The rotation matrix is updated with the rotation of the shuttle along the selected axis of rotation. It should be noted that the AR-Classroom will present participants with a warning message if they physically rotate the shuttle along an axis that does not match the axis they selected using the dropdown box.

2.6 Active Control Materials

The active control group did not interact with the AR-Classroom. Instead, they spent an equivalent amount of time doing the following: watching the introductory video on matrix algebra, completing two spatial thinking tests, a worksheet that used analogies to teach how spatial thinking can help solve problems, and another version of the same two spatial thinking tests. While spatial thinking is involved in matrix algebra and in understanding the visualizations used in the AR-Classroom, completing the active control activities should not train participants in the skills and concepts tested by the matrix algebra test.

2.7 Post-Test Materials

All three learning conditions completed a post-test composed of the PSVT:R and matrix algebra test with confidence ratings.

3 Results

3.1 Pre-Test

Between-subjects ANOVAs were run to test if there were any differences between the learning conditions (Table 2). There were no significant differences (*p*s > .05) between the learning conditions in experience with video games, 3D modelling, and AR (ratings closer to 5 represent experience), PSVT:R accuracy, matrix algebra accuracy, and matrix algebra confidence (ratings closer to 5 represent confidence). Participants across all three learning conditions were the most experienced with video games but not experienced with 3D modelling or AR. Participants struggled with the PSVT:R and the matrix algebra test, and they were not confident in their matrix algebra understanding.

Table 2. Mean Pre-Test Scores by Learning Condition.

	Virtual Workshop	Physical Workshop	Active Control
Experience with video games	3.4	3.1	2.7
Experience with 3D modelling	1.6	1.7	1.7
Experience with Augmented Reality	1.7	1.7	1.5
PSVT:R Accuracy	63%	61%	64%
Matrix Algebra Accuracy	50%	53%	52%
Matrix Algebra Confidence	2.5	2.3	2.3

3.2 Post-Test

Between-subjects ANOVAs were run to test if there were any differences between the learning conditions (Table 3). There were no significant differences (*p*s > .05) between the learning conditions in PSVT:R accuracy, matrix algebra accuracy, and matrix algebra confidence. Participants across all three learning conditions improved slightly on the PSVT:R and the matrix algebra test, and their confidence improved. However, the learning conditions did not significantly differ from one another.

Table 3. Mean Post-Test Scores by Learning Condition.

	Virtual Workshop	Physical Workshop	Active Control
PSVT:R Accuracy	67%	71%	67%
Matrix Algebra Accuracy	61%	65%	60%
Matrix Algebra Confidence	3.4	3.5	3.2

3.3 Changes in from pre-test to post-test

Using ANCOVAs, we investigated group differences in post-test PSVT:R accuracy, matrix algebra accuracy, and matrix algebra confidence after controlling for variation in their respective pre-test performance. There was a significant difference in PSVT:R accuracy from pre-test to post-test, $F(1, 55) = 97.17$, $p < .001$, meaning that participant performance on the PSVT:R improved from the beginning to the end of the experiment. However, there were no significant group differences for accuracy on the matrix algebra test, $F(2, 55) = 1.34$, $p = .27$. This means that all three learning conditions improved equally, so the learning experience did not impact their spatial thinking skills. This is expected as we did not hypothesize that interacting with the AR-Classroom or completing the active control activities would improve participants' spatial thinking.

For matrix algebra accuracy, there was a significant improvement from pre-test to post-test, $F(1, 55) = 35.16$, $p < .001$. However, there were no significant group differences for accuracy on the matrix algebra test, $F(2, 55) = 0.67$, $p = .51$. While participants showed improved performance on the matrix algebra test, there were no group differences in post-test accuracy (Fig. 9). A similar pattern was found for matrix algebra confidence. There was a significant improvement in matrix algebra confidence from pre-test to post-test, $F(1, 53) = 26.31$, $p < .001$; however, there were no significant group differences, $F(2, 53) = 0.26$, $p = .78$. Unfortunately, we did not find evidence that following the rotation booklet and interacting with the AR-Classroom improved matrix algebra accuracy and confidence compared to the active control. This result conflicts with our hypothesis that the two workshop groups would improve more than the active control.

It could be that individual differences, such previous experience with matrix algebra or other participant characteristics and PSVT:R accuracy, might add help explain the lack of group differences. Therefore, we ran a series of ANCOVAs that included variables collected in the pre-test. The only significant predictor of matrix algebra accuracy was PSVT:R accuracy, $F(1, 54) = 27.13$, $p < .001$, and the only significant predictor of matrix algebra confidence was PSVT:R accuracy, $F(1, 52) = 23.01$, $p < .001$. As expected because of the tightly coupled relationship between spatial thinking and mathematical reasoning, spatial thinking skills are predictive of improvements in matrix algebra accuracy and confidence, regardless of learning condition.

To further evaluate the AR-Classroom's efficacy in teaching matrix algebra, we conducted a more detailed analysis on the matrix algebra test. The matrix algebra test was composed of three problem types: translations (which were not taught by the AR-Classroom), rotations (which were taught by the AR-Classroom), and translation with rotations (more challenging problems than what was taught by the AR-Classroom). We hypothesized that there would be no group differences for translation problems because none of the groups received training in translations beyond an explanation in the introduction to matrix algebra video. We hypothesized that there would be group differences for rotation and translation with rotation problems, such that the physical and virtual groups will outperform the active control group. This is because the two workshop groups received training in these concepts through completing the rotation booklet and using the AR-Classroom.

Using ANCOVAs, we investigated group differences in post-test matrix algebra accuracy after controlling for variation in pre-test matrix algebra accuracy. As expected, there were no significant group differences for translation problems, $F(2, 56) = 1.44$, $p = .24$ (Fig. 9). The result confirms our hypothesis that participants did not improve their understanding of translation problems because they received minimal instruction about translations. There was a marginal group difference for rotation problems, $F(2, 56) = 2.52$, $p = .09$ (Fig. 9). Using adjusted means, there was a trend that physical workshop participants were the most accurate ($M = 52\%$), virtual ($M = 44\%$) were moderately accurate, and control participants were the least accurate ($M = 43\%$). There was a marginal group difference for translation with rotation problems, $F(2, 55) = 2.68$, $p = .08$ (Fig. 9). Using adjusted means, there was a trend that virtual participants were the most accurate ($M = 58\%$), physical ($M = 41\%$) were moderately accurate, and control participants were the least accurate ($M = 34\%$). While these two findings were not statistically significant, they both are in alignment with our prediction that participants who completed the rotation booklet and interacted with the AR-Classroom would be more accurate on these problems compared to the active control group.

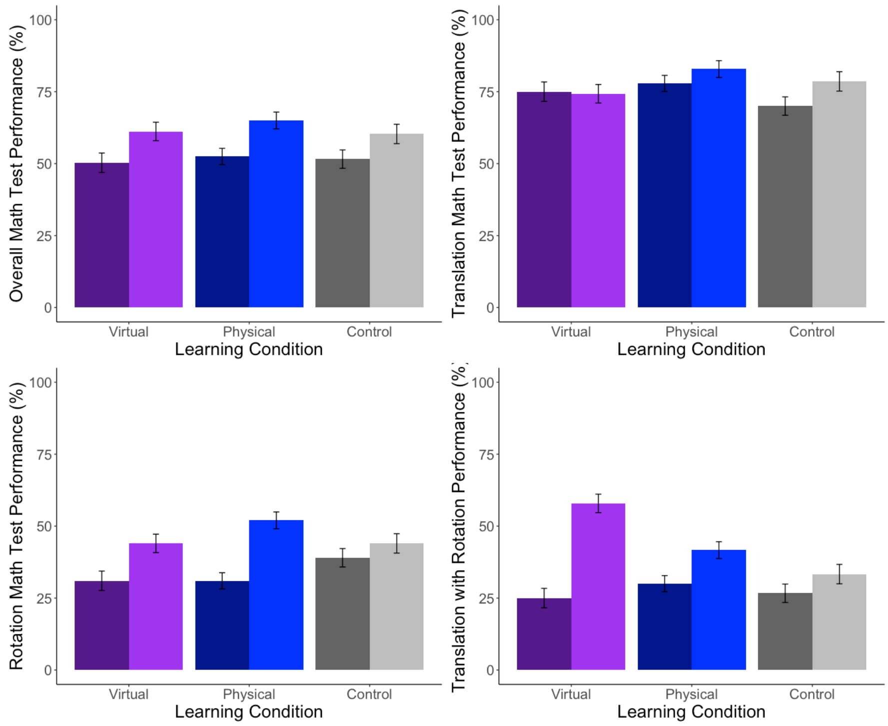


Fig. 9. Performance on the entire matrix algebra test (top left), only translation problems (top right), only rotation problems (bottom left), and translation with rotation problems (bottom right) for all three learning conditions split by pre-test (left bar; darker color) and post-test (right bar; lighter color).

4 Discussion

The efficacy of the AR-Classroom was evaluated using a learning experiment followed by quantitative analyses presented in the current paper and qualitative analyses presented in companion paper [21]. The findings from both papers suggest that the matrix algebra learning interventions delivered by AR-Classroom may be helpful and lead to improvements in mathematical skills. While performance on the matrix algebra test improved equally for all learning groups, there was a trend that students who used the physical workshop were more accurate on rotation problems and that students who used the virtual workshop were more accurate on translation with rotation problems. This suggests that the both workshops were supporting the learning of concepts covered in the rotation booklets and AR-Classroom. Unfortunately, it remains unclear if using physical versus virtual manipulatives resulted in different learning outcomes. Future work on the AR-Classroom will include integrating the rotation booklet activities into the application itself and continuing to improve user-app interactions. Future learning efficacy research on the AR-Classroom will include larger sample sizes to increase the power to detect meaningful differences between learning groups, balancing groups by gender and spatial thinking skills to reduce the impact of individual differences within the learning groups, and a more comprehensive matrix algebra test to more clearly identify changes in conceptual knowledge. In conclusion, the development of and research on the AR-Classroom and its predecessor, BRICKxAR/T has been guided by a data-informed and iterative approach. This approach has been fundamental in understanding how students interact with educational technologies along with how students engage with complex mathematical concepts using interactive materials and AR.

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