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Optimization Model and Solution Algorithm for the Design of Connected and Compact Nature Reserves

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Abstract

Ecosystem conservation is fundamental to guarantee the survival of endangered species and to preserve other ecological functions important for human systems (e.g., water). Planning land conservation increasingly requires a landscape approach to mitigate the negative impacts of spatial threats such as urbanization, agricultural development, and climate change. In this context, landscape connectivity and compactness are vital characteristics for the effective functionality of conservation areas. Connectivity allows species to travel across landscapes, facilitating the flow of genes across populations from different protected areas. Compactness measures the spatial dispersion of protected sites, which can be used to mitigate risk factors associated with species leaving and re-entering the reserve. This research describes an optimization model for the design of conservation areas, while inducing connectivity and compactness. We use the Reock's index, a metric of compactness that maximizes the ratio of area of the selected patches to the area of their smallest circumscribing circle. Our model includes budget and minimum protected area constraints to reflect realistic financial and ecological requirements. The initial nonlinear model is reformulated into a mixed-integer linear program, which is solved using an adaptation of the Newton's method for problems with integer variables. We characterize an optimal solution and derive cuts to improve the model performance. We illustrate our results using real life landscapes with irregular patches.

Keywords
Mixed-Integer Fractional Programming, Newton's Method, Compactness, Conservation Planning

1. Introduction

Conservation planning aims to preserve the ecological and socio-economic value of the targeted ecosystems and their species. It requires a landscape approach, integrating cross-site information to recommend government and non-government organizations the best sites to protect, restore, or manage. This approach is essential when accounting for rapid changes in the environment due to urbanization, agricultural development, and climate change. Spatial characteristics of urban landscapes such as connectivity, habitat quality, patch size, compactness, and corridor size [1–4] help in comparing different conservation plans. Spatial requirements make landscape decisions challenging, for instance inducing connectivity and compactness in the selected areas. Connectivity allows species to travel across selected areas whereas compactness measures how close the selected sites are to each other. Compactness quantifies the spread of the conserved areas and allows decision makers to identify landscape configurations that, although connected, are undesirable. For instance, low-diameter (bulky) circular areas are preferred over long and thin areas, because the selected reserve will be closely packed and not spread over a wide area. Multiple models based on mathematical programming and spatial algorithms have been proposed in the literature to induce connectivity in conservation planning problems [5, 6]. The models in the literature are mainly developed for grid landscapes, and very few studies focus on landscapes consisting of irregularly shaped areas. Even though some of the models in the literature claim to be extensible to irregular landscapes, the simultaneous use of compactness and connectivity is new for such landscapes. Depending on the landscape discretization, as stated in [7], a circle or a square are considered to be the most compact structures. This paper provides a spatial optimization model and solution algorithm that supports connected and compact landscape conservation designs for irregular landscapes using a relatively unexplored compactness metric known as the Reock's index [8].

2. Related Work

The literature on spatial optimization models for conservation planning that include landscape connectivity and compactness features is relatively recent. Connectivity can be defined as the existence of a path from every patch to every other patch in the selected landscape. Connectivity ensures the safe movement of animals from one patch to another within the reserve, facilitating the migration of species due to changes in weather, seasons, and other ecological processes. Connectivity and compactness have been implemented using various techniques for a single connected reserve. Önal and Briers [5] discuss a graph-based method of modeling conservation decisions that minimizes the distance between selected patches. Önal et al. [9] model connectivity using constraints that enforce the selection of patches to form a path between selected patches. Carvajal et al. [6] discuss the importance of connectivity (contiguity) in landscapes and how integer programming can be used to achieve it using a set of (ring) constraints. These constraints enforce the connectivity in a reserve design by expanding the area of the disconnected reserves until they meet a minimum area requirement. Nalle et al. [10] imposes connectivity and compactness by minimizing a weighted objective function of the distances between all the selected patches, subtracting the parcel adjacencies (number of parcels with at least one neighbor selected). Jafari and Hearne [11] propose a method based on network flows to impose connectivity in multiple reserves.

The efficient design of a reserve not only depends on how well it is connected, but also on other spatial attributes. Young [7] discuss that there is no universal consensus on the definition of compactness as it can be integrated in multiple ways. Using visual inspection, any landscape that is close to a regular shape (e.g., circle, square) is more compact compared to irregular shapes. Young [7]'s 8 metrics of compactness, include visual inspection, Schwartzberg test, length-width test, Taylor's test, moment of inertia test, perimeter test, Boyce-Clark test, and Reock's test. Önal and Briers [12] explore the reserve boundary as a metric of compactness. The smaller the value of the boundary, the more compact is the landscape. Jafari and Hearne [11] and Lin et al. [13] define compactness as the perimeter of the reserve. They calculate the perimeter by adding the perimeter of the selected patches and subtracting the length of the shared edges between selected patches. The objective is to minimize the perimeter to produce a compact reserve. Cabeza et al. [3] define the ratio of the boundary length of the selected reserve to the area of the selected reserve as a metric of compactness. Clemens et al. [14] induce compactness using the concept of core and buffer patches, where a core patch can only be selected if all its surrounding (8 neighbors for a grid landscape) buffer patches are selected. McDonnell et al. [15] define compactness as the sum of the boundary length and the area of the selected reserve. Wang and Önal [16] define a metric of compactness based on the total sum of the distances between the selected patches and the center of the reserve. The center patch is also decided by the model and is the patch with the smallest sum of distances to all the other patches in the reserve. In the studied grid landscape, the distance between two patches is the minimum number of patches used to connect them, which makes the structural distance between two adjacent patches to be equal to zero. Weerasena et al. [17] induce compactness by minimizing the boundary length of the reserves and also minimize the total pairwise patch distances within a reserve. Billionnet [18] uses three metrics of compactness, the diameter of the reserve, the ratio of the perimeter and the area of the reserve, and the total pairwise distances between patches in the reserve. Compactness is enforced using an at-most constraint, where the pairwise patch distances are forced to be less than a given value. Ravishankar [19] defines a metric of compactness that minimizes the total number of patches having only one of their neighbors in the reserve, *i.e.*, leaves, which is applicable to landscapes with grid parcels but computationally expensive to extend to landscapes with irregular patches.

3. Model Formulation

We develop a nonlinear formulation to solve a connected and compact reserve design problem using the Reock's metric of compactness. The model uses general irregular candidate patches and a circle as a benchmark shape (*i.e.*, most compact shape). The proposed parametric method in Section 3.4 generalizes this assumption to any regular benchmark shape. The objective of the model is to find a reserve whose shape is close to that of its circumscribing circle, while satisfying budget, operational, ecological and spatial constraints. We use the Reock's metric, because it can induce compact landscapes [7] while balancing the reserve size and shape. We conjecture that this metric has been overlooked in the spatial optimization literature given the complexity (non-convexity) of the resulting mathematical models and their relaxations.

3.1 Nonlinear Model

We define P as the set of all patches available and n as the total number of patches in the landscape ($i.e.$, $n = |P|$). Set N_i contains the neighboring patches of patch i, *i.e.*, all patches that share at least one edge with i. For each patch i, V_i is the set of vertices, a_i is the area, and c_i is the purchase or restoration cost. Parameter α is the minimum area required to be selected and b is the available budget. The xy-coordinates of vertex v of patch i are assumed to be known and given by $\mathbf{x}_i^v \in \mathbb{R}^2$. The decision variables include vector $\mathbf{x}$, which denotes the (continuous) xy-coordinates of the center of the circle enclosing the selected patches. Variable r is the radius of the circumscribing circle and z is an auxiliary variable used in the proposed linear approximation of the circle's area. For each patch i, the binary variable t_i denotes whether patch i is selected. The nonlinear formulation for the landscape conservation problem is presented in (1) - (7).

$$\max \quad \frac{\sum_{i\in P} a_i t_i}{\pi r^2} \tag{1}$$

$$s.t. \quad ||\mathbf{x} - \mathbf{x}_i^v|| \leq r + M(1 - t_i), \quad \forall i \in P, v \in V_i, \tag{2}$$

$$\sum_{i\in P} c_i t_i \leq b, \tag{3}$$

$$\sum_{i\in P} a_i t_i \geq \alpha, \tag{4}$$

$$t_i \in \{0,1\}, \forall i \in P, \tag{5}$$

$$\mathbf{x} \in \mathbb{R}^2, \tag{6}$$

$$r \geq 0 \tag{7}$$

The objective function in (1) maximizes the ratio of the total area of the selected patches to the area of the circumscribing circle, i.e., the Reock's metric of compactness. Constraints (2) ensure that the circle circumscribes the selected patches ($i.e.$, $i \in P : t_i = 1$), where $||\cdot||$ refers to the Euclidean norm. Note that these constraints are relaxed for the vertices of the non-selected patches because of the big-M value, which in this case is equal to the radius of the smallest circle enclosing the entire landscape. Constraint (3) imposes a budget limit, which ensures that the total cost of the selected patches is no more than the budget allotted. Constraint (4) ensures that the sum of areas of the selected patches meets the minimum required area. Constraints (5), (6), and (7) are variable type constraints.

3.2 Linearized model

The nonlinear model (1)-(7) is expensive to solve, and cannot be efficiently solved using commercial nonlinear solvers for realistic landscapes. To overcome this challenge, we construct a linearized model to approximate model (1)-(7). We define R_0 as a finite set of radius samples. This set is used to avoid the nonlinearity in the denominator of the objective function. To linearize the Euclidean distance in Constraints (2), we use the method suggested in [20]. We define k unit vectors to discretize the unit circle and whose angles with respect to the horizontal line range in the continuous domain $[0, 2\pi]$. We define $U = \{1, ..., k\}$, $k \geq 3$ as the set of these unit vectors. The xy-coordinates of unit vector $\mathbf{v}_l$, $l \in U$, are given by

$$\mathbf{v}_l = \begin{pmatrix} \cos\left(\dfrac{2(l-1)\pi}{k}\right) \\ \sin\left(\dfrac{2(l-1)\pi}{k}\right) \end{pmatrix} \in \mathbb{R}^2. \tag{8}$$

Given the definition in (8), we have that $||\mathbf{v}_l|| = 1, \forall l \in U$. Constraints (9) are the (approximated) linear version of Constraints (2) using a set of unit vectors. Note that (9) provides a relaxation because $(\mathbf{x}^0 - \mathbf{x}_i^v)^T \mathbf{v}_l \leq ||\mathbf{x}^0 - \mathbf{x}_i^v||$ by Cauchy-Schwarz and the fact that $||\mathbf{v}_l|| = 1, \forall l$. The quality of the approximation in (9) also improves as k increases. Constraint (10) is the first order Taylor approximation of the circle's area, which ensures that the auxiliary variable z is a lower bound of the circumscribing circle's area, but close enough for a large sample of radii R_0.

$$(\mathbf{x}^0 - \mathbf{x}_i^v)^T \mathbf{v}_l \leq r + M(1 - t_i), \quad \forall l \in U, i \in P, v \in V_i \tag{9}$$

$$z \geq 2\pi r_0 r - \pi r_0^2, \quad \forall r_0 \in R_0 \tag{10}$$

Using Constraints (9)-(10) the objective function can be reformulated as

$$\max \quad \frac{\sum_{i\in P} a_i t_i}{z}. \tag{11}$$

3.3 Additional constraints and Strengthening

Consider a feasible solution to the model in Section 3.2 given by $(r^*, \mathbf{t}^*, \mathbf{x}^*)$ and let $S^* = \{i \in P : t_i^* = 1\}$ be the corresponding feasible set of selected patches. Let W be a set of all connected components in the graph induced by S^*, where each node is a patch and an arc exists only between adjacent patches. If $|W| > 1$, then the solution obtained is infeasible because the reserve is not connected. To prevent disconnected components, we impose Constraint (12), an extension of the ring inequalities proposed by Carvajal et al. [6] forces the the model to pick at least one patch from $N(S^*)$, the neighbors of S^*, if the patches in S^* are selected. We impose (12) as a cut during the Branch and Bound exploration whenever an integer solution with a set of connected components $W : |W| > 1$ is obtained.

$$\sum_{i \in S^*} t_i \leq \sum_{j \in N(S^*)} t_j + |S^*| - 1 \tag{12}$$

Consider ρ^* to be an incumbent objective value. Also define the maximum protected area as $\tilde{\alpha}$, which is the optimal solution of a knapsack problem with the objective of maximizing the total area subject to a budget b. Then, $\tilde{R} = \sqrt{\frac{\tilde{\alpha}}{\pi \rho^*}}$ is a valid upper bound on r. Constraints (13) are added as a cut in the Branch and Bound exploration every time an incumbent ρ^* is obtained.

$$r \leq \tilde{R} \tag{13}$$

Additionally, Constraints (14) enforce that no two patches whose distance is greater than twice the maximum radius ($\tilde{R}$) can be selected together. These constraints are added as cuts in the Branch and Bound exploration whenever a new incumbent is obtained.

$$t_i + t_j \leq 1, \quad \forall i, j \in P : i \neq j, \quad d_{ij} > 2\tilde{R} \tag{14}$$

Constraints (13) and (14) together are used to strengthen the linear relaxation of the model, whereas Constraints (12) are needed for feasibility.

3.4 Parameterized Algorithm

To convert the objective function in (11) to the form in (15), we use the parametric algorithm proposed by Dinkelbach [21]. Consider a feasible solution $\mathbf{x}_m$ to the problem in (1) - (7), with Constraints (2) replaced by (9), along with Constraints (10), the objective in (1) replaced with objective in (15), and the additional and strengthening Constraints (12)-(14) (added on the go in a Branch-and-cut fashion). Define $\mathbf{Y}_m = (r_m, \mathbf{t}_m, \mathbf{x}_m, z_m)$ as the optimal solutions of this model in iteration m. The algorithm sequentially solves parametric models for $m = 0, 1, \ldots,$ by updating parameter $q_{m+1} = \frac{N(\mathbf{Y}_m)}{D(\mathbf{Y}_m)}$, where $N(\mathbf{Y}_m) = \sum_{i \in P} a_i t_i$ and $D(\mathbf{Y}_m) = z$. The algorithm is initialized using $\mathbf{Y}_0$ from solving parametric model for $q_m = 0, m = 0$ and stops for an iteration when $\max\{N(\mathbf{Y}_m) - q_m D(\mathbf{Y}_m)\} = 0$ with optimal compactness, $q^* = q_m$.

$$\max \quad \sum_{i \in P} a_i t_i - q_m z \tag{15}$$

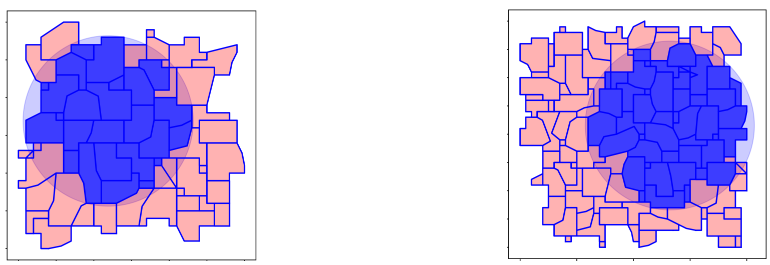
4. Results

We implemented the proposed solution approach in a computer with Intel Xeon E5-2680 v4 CPU running at 2.4 GHz, 16 GB of RAM and CPLEX 22.1.0.0 (E_1). Because of license availability we solve the nonlinear model using a computer with Intel i7 CPU running at 2.6 GHz, 16 GB RAM, Windows 11 and Gurobi 10.0.1 (E_2). We solve the formulations on one landscape (FLG9A) obtained from the Forest Management Optimization Site (FMOS) repository [6]. The area of the patch polygons range between 1-30 square units. We generate the cost of each patch using the distribution $c_i \sim 20a_i + \mathcal{N}(0, 100)$, where $\mathcal{N}$ is a Gaussian noise with mean zero and standard deviation 10. This strategy induces a cost that is proportional to the area but with some variability. We compare our model with a nonlinear version of the model that is solved using the Algorithm in 3.4 but uses constraints (2) instead of (9).

Table 1 shows the solution times for the different instances of FLG9A. The first column denotes the total number of patches in the landscape. The area (α) is represented as the percentage of the total area of the landscape. The budget

Table 1: Solution times for the default, enhanced and nonlinear models

			E_1			E_2		
# Patches	α (%)	b (%)	T_d (sec)	T_e (sec)	C_l	T_l (sec)	T_{nl} (sec)	C_{nl}
57	8	10	6.43	4.02	0.6366	4.94	23.23	0.6366
	8	20	54.67	126.44	0.7132	11.61	182.23	0.7132
	8	30	239.82	65.15	0.7539	9.68	260.41	0.7539
	30	35	51.31	27.75	0.7539	10.09	148.84	0.7539
	30	45	64.95	72.61	0.7548	16.33	354.44	0.7548
	30	60	36.46	49.92	0.7765	13.04	TL	0.7765
113	8	10	94.33	60.91	0.6262	25.5	134.05	0.6262
	8	20	738.93	548.61	0.7699	235.69	TL	0.7648
	8	30	1640.08	495.03	0.7901	565.43	TL	0.7901
	30	35	657.73	514.41	0.7901	243.88	2254.05	0.7901
	30	45	1412.83	687.03	0.7979	360.73	TL	0.7816
	30	60	739.66	626.42	0.8127	562.67	TL	0.7801



(a) Compactness of 75.48%

(b) Compactness of 79.79%

Figure 1: Solutions for FLG9A instances

(b) is represented as the percentage of the total cost of selecting all the patches in the landscape. Column T_d depicts the time taken by the default model (*i.e.*, without the strengthening constraints) to optimality, whereas column T_e depicts the time taken by the enhanced model (*i.e.*, with Constraints (13) and (14)) to achieve optimality. C_l depicts the optimal Reock's index of the linearized model. Column T_l depicts the time taken by the linearized model in experiments (E_2) to achieve optimality. Column T_{nl} is the time taken by the nonlinear model in experiments (E_2). We use "TL" to denote that the solution time reaches its limit of 7200s. Column C_{nl} depicts the optimal Reock's index (or best value within 7200 secs) to the nonlinear model in experiments (E_2). From Table 1, we observe that the linearized approach with enhancements outperforms the default models. We also observe that the linearized models significantly outperform the nonlinear formulation. Figure 1 depicts optimal solutions for instances with a minimum area requirement of 30% and a maximum budget of 45%. Figure 1a depicts the solution for an instance with 57 patches and Figure 1b depicts the solution on a landscape with 113 patches.

5. Conclusion

In this paper, we propose a mixed-integer linear model to reformulate the nonlinear mixed integer program maximizing the Reock's metric of compactness. The landscape conservation problem includes realistic spatial and operational constraints. The proposed formulation and logical and structural constraints to strengthen the reformulations's linear relaxation, reduce the solution times of the nonlinear model by at least one order of magnitude in most instances. We demonstrate the model performance on a real instance with irregular patches. Future work can be focused on developing more cuts, heuristics and a warm-start procedure, among other strengthening strategies for the solution of larger instances.

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