A Quantum-inspired Metaheuristic Algorithm for Beampattern Synthesis in Multi-User MIMO

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***Abstract*—This paper introduces a novel methodology for array synthesis in multi-user MIMO systems. Utilizing binary variables to represent antenna excitation states, we encode desired beampattern properties, such as a narrow main lobe, low peak sidelobe level, and minimized inter-user interference, into a quadratic objective function. The optimal array excitation profile is then obtained through a quantum-inspired simulated bifurcation algorithm. Large-scale numerical experiments are performed to validate the performance of our method.**

I. Introduction

Multi-user Multi-input Multi-output (MU-MIMO) technology plays a pivotal role in the evolution of wireless communication, especially in the context of 5G and beyond. In MU-MIMO, a base station (BS) equipped with many antennas can concurrently serve multiple users by distributing data across parallel streams, a technique known as spatial multiplexing [1]. From the perspective of antenna engineering, an ideal beam pattern for a BS antenna array should provide high gain signals to the intended user equipment, narrow main lobes to enhance direction-of-arrival (DOA) estimation, and specified null/notch to mitigate interference [2]. The associated optimization problem is typically NP-hard due to the nonconvex and nonsmooth min-max objective function.

In our paper, we introduce a quantum-inspired metaheuristic algorithm for the efficient synthesis of optimal radiation patterns in MU-MIMO systems. The basic problem statement is illustrated in Fig. 1. We consider a large planar phase array with uniform half-wavelength element spacing. The objective is to select subsets of the array to steer individual beams toward specific users while minimizing signal towards others, thereby reducing inter-user interference. It is noteworthy that the optimized solution inherently yields non-overlapping groups of sparse arrays. As these sparse arrays share similar aperture sizes, narrow main lobes are naturally produced.

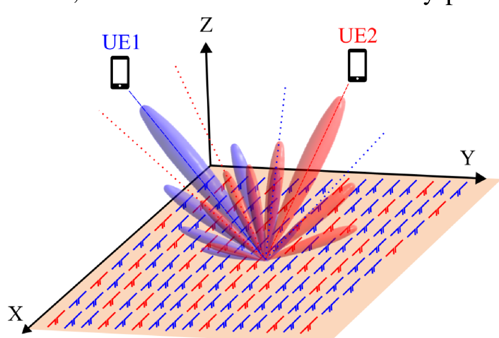

Fig. 1. BS antenna array performs beamforming towards the intended user equipment (UE) while null steering towards other UEs, with red and blue antenna elements creating respective radiation patterns.

II. Methodology

Consider the problem statement depicted in Fig. 1, the algorithm starts by defining the radiated electric field (E-field) $\mathbf{E}$ of the antenna array with M antenna elements, which is written as a weighted sum of radiation vectors, $\mathbf{G}_m(\theta, \phi)$, from individual antenna elements. The goal is to select a group of array elements beamforming towards UE1, and the other group to serve UE2. Here, we introduce an unknown binary variable a_m and then express the element-wise radiated E-field in one of two excitation states ($e^{j\phi_{1,m}}$ or $e^{j\phi_{2,m}}$). The total radiated field is given as:

$$\mathbf{E}(\theta, \phi) = \sum_{m=1}^{M} (a_m e^{j\phi_{1,m}} + (1 - a_m) e^{j\phi_{2,m}}) \mathbf{G}_m(\theta, \phi) \quad (1)$$

Next, we convert the complex electric field data into a power representation by calculating the squared norm. The desired radiation profile, e.g. beamforming and nullforming, is then recasted into a quadratic argmin function $\mathcal{E}$ of the excitation coefficients and binary variables ($a_1, \cdots, a_M$).

However, minimizing such objective function $\mathcal{E}$ poses an NP-hard computational challenge [3]. To tackle this, we introduce a modified simulated bifurcation (SB) algorithm, drawing inspiration from quantum adiabatic optimization and spin dynamics. SB is a rapid, parallelizable approach for Ising problems, reminiscent of a computational strategy from quantum mechanics where a system gradually transitions to seek the lowest-energy configuration of a Hamiltonian.

The original SB [4] begins by embedding the quadratic function $\mathcal{E}$ into a nonlinear Hamiltonian, with optimization variables initially assigned around zero. As the pumping parameter in the nonlinear Hamiltonian gradually increases over time, the system evolves, leading to bifurcation phenomena. This results in the emergence of two stable branches at +1 or -1, as depicted in Fig. 2 for a small problem with 100 spins. In our context, if particle m is at position 1, it indicates that antenna m exclusively serves UE1 and minimizes energy towards UE2.

Our modified SB algorithm (SB-TAR), in addition, incorporates the "trap and randomize" (TAR) protocol to address concerns like "retarded bifurcation", characterized by asynchronous convergence [5]. This is evident in Fig. 2, where each particle or node bifurcates at different times. During the SB time-stepping process, TAR is regularly performed to locate and round off "trapped nodes" near +1 or -1, while randomizing the other particles between -1 and +1. This minimizes solution oscillations and speeds up convergence to the global solution.

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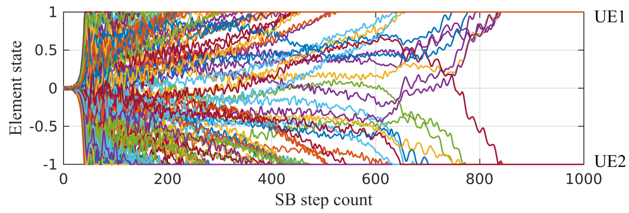

Fig. 2. Simulated Bifurcation: Antenna states bifurcate asynchronously into +1 or -1, with +1 indicating service for UE1 and -1 for UE2.

III. Numerical Experiment

In our experiment, we consider a massive MIMO setup with two UEs positioned at distinct angles: $(\theta, \phi) = (-20^\circ, 0^\circ)$ and $(40^\circ, 0^\circ)$. The transmitting planar antenna array consists of 20-by-50 antenna elements spaced half wavelengths apart.

We formulate the objective function $\mathcal{E}$ to achieve a balanced distribution of radiated power to both UEs while minimizing inter-user interference. This function incorporates the radiated electric fields ($\mathbf{E}_1$ and $\mathbf{E}_2$) directed towards each UE, as well as the differential electric field ($\mathbf{E}_d = \mathbf{E}_1 - \mathbf{E}_2$), ensuring an equal electric field strength at both UEs. Mathematically, the function is defined as $\mathcal{E} = \mathbf{E}_1^*\mathbf{E}_1 + \mathbf{E}_2^*\mathbf{E}_2 + P \cdot \Re[\mathbf{E}_d^*\mathbf{E}_d^*]$. At the ground state of $\mathcal{E}$, the first two terms minimize the inter-user interference around $(-20^\circ, 0^\circ)$ and $(40^\circ, 0^\circ)$, while the third term balances power between the two beams using a penalty constant P, biasing the multi-objective optimization between energy balancing and minimization.

We have considered five distinct array setups to evaluate their effectiveness. Firstly, the sub-array approach divides the aperture into two sub-panel clusters of equal size. Secondly, the alternating pattern involves grouping antenna elements for the two UEs in a periodic alternating manner. Thirdly, the random pattern selects antenna elements for UE1 and UE2 randomly, ensuring an equal number for each UE. Finally, we employ both SB-TAR and discrete Particle Swarm Optimization (d-PSO) [6] algorithms to achieve the desired radiation pattern configuration by minimizing the objective function $\mathcal{E}$.

Table I summarizes the gain of the five array configurations. The sub-array, alternating, and random arrays exhibit identical gain towards the intended UE. In contrast, SB-TAR and d-PSO show a slight offset gain towards each UE due to the soft enforcement of power balancing through the differential electric field $\mathbf{E}_d$. Despite this offset, the gain remains close to the original value of 28.75 dB. Notably, SB-TAR demonstrates the lowest mean interference to both UEs at -5.85dB, in contrast to the random array at 3.5dB and d-PSO at -1.38dB. The d-PSO solution seems to be stuck in a local minimum, leading to suboptimal interference levels for both UEs compared to the solution provided by SB-TAR.

TABLE I
Gain and interference (IF) to both UEs in dB

Setup	UE1 gain	UE2 gain	UE1 IF	UE2 IF	Mean IF
Sub-array	28.75	28.75	-0.85	-0.85	-0.85
Alternating	28.75	28.75	26.58	26.58	26.58
Random	28.75	28.75	5.70	0.57	3.50
SB-TAR	28.87	28.6	-12.88	-2.01	-5.85
d-PSO	28.86	28.63	2.07	-7.20	-1.38

In addition to evaluating the quality of the optimization solution, we can also compare the computational time required by both d-PSO and SB-TAR. SB-TAR achieves the argmin of the objective function in 8.03 seconds, while d-PSO takes 309.5 seconds to accomplish the same task. This demonstrates that SB-TAR exhibits greater time efficiency compared to traditional heuristic optimization methods such as d-PSO.

In Fig. 3, we present the radiation patterns resulting from five array configurations, with the focus on the group of subarrays serving UE2. The pattern from the alternating arrangement reveals a grating lobe issue occurring at $(\theta, \phi) = (-20^\circ, 0^\circ)$, indicating significant interference inherent in periodically distributed antenna arrays. Moreover, the sub-array configuration, while lacking grating lobes, exhibits a broader beamwidth due to its smaller effective area. In contrast, the random, d-PSO and SB-TAR distributions display narrower beamwidths, with SB-TAR offering the additional benefit of lower interference towards UE1 at $(\theta, \phi) = (-20^\circ, 0^\circ)$.

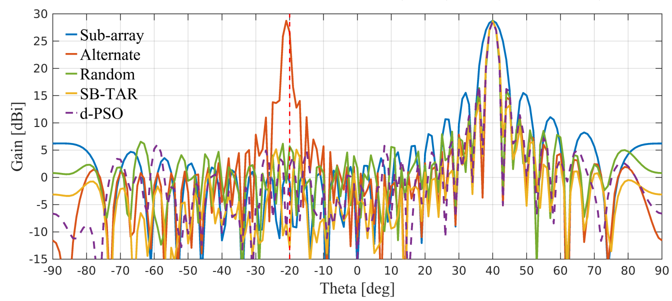

Fig. 3. Radiation patterns from the group of antenna elements that serve UE2 placed at $(\theta, \phi) = (40^\circ, 0^\circ)$.

IV. Conclusion

As widely recognized in our community, optimizing large-scale antenna arrays for beamforming presents challenges due to high computational complexity, especially with sparse or random arrays. Our introduced computational methodology, based on non-linear dynamics and bifurcation theory, offers a solution. It can be seamlessly extended to support multiple User Equipments (UEs) using one-hot encoding with binary variables. This approach can enable joint optimization of antenna positions and array excitation vectors, enhancing efficiency and performance in wireless communication systems.

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