



PERSPECTIVE

Information scrambling —A quantum thermodynamic perspective

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Perspective

Information scrambling — A quantum thermodynamic perspective

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Abstract – Recent advances in quantum information science have shed light on the intricate dynamics of quantum many-body systems, for which quantum information scrambling is a perfect example. Motivated by considerations of the thermodynamics of quantum information, this perspective aims at synthesizing key findings from several pivotal studies and exploring various aspects of quantum scrambling. We consider quantifiers such as the out-of-time-ordered correlator (OTOC) and the quantum mutual information, their connections to thermodynamics, and their role in understanding chaotic *vs.* integrable quantum systems. With a focus on representative examples, we cover a range of topics, including the thermodynamics of quantum information scrambling, and the scrambling dynamics in quantum gravity models such as the Sachdev-Ye-Kitaev (SYK) model. Examining these diverse approaches enables us to highlight the multifaceted nature of quantum information scrambling and its significance in understanding the fundamental aspects of quantum many-body dynamics at the intersection of quantum mechanics and thermodynamics.

perspective

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Quantum information scrambling, a dynamical phenomenon observed in complex quantum many-body systems, has garnered significant interest for its profound implications in various fields of modern research [1–4]. At its core, scrambling is driven by the dispersion of initially localized information throughout a quantum system, leading to intricate patterns of entanglement and correlations. This concept not only challenges our understanding of quantum dynamics but also provides an informative window into the elusive nature of quantum chaos [3] and thermalization [5].

The study of scrambling intersects with several critical areas of theoretical and experimental physics. For instance, in the realm of black-hole physics, it offers insights into the information paradox and the nature of Hawking radiation [6,7]. In contrast, understanding scrambling in the context of quantum computing is essential for developing robust systems resistant to decoherence.

Motivated by thermodynamic considerations about processing information, this perspective aims at delving into the intricate details of quantum information scrambling, exploring its theoretical foundations, experimental verifications, and the multitude of its implications in modern

physics. However, similar to all perspectives, the following exposition cannot serve as a comprehensive review, but only provides an introduction into the vast literature on the topic.

From black holes to quantum information. – The exploration of quantum information scrambling has unveiled profound connections between the enigmatic nature of black holes and the fundamental principles of quantum physics. Black holes, traditionally studied within the realm of general relativity, have emerged as pivotal objects in understanding the quantum-mechanical behavior of information in extreme gravitational fields. The seminal work by Hawking and others has led to the *information paradox* in black holes, propelling the study of scrambling as a possible resolution [8].

Typically, the paradox is phrased as a question of whether or how information, that crosses the event horizon, is “destroyed”. From the point of view of thermodynamics this question appears somewhat awkward, as it has been recognized that information is a thermodynamic resource that needs to be treated in complete analogy to heat and work [6]. In other words, “destroying information” is thermodynamically equivalent to “destroying energy”. Yet, it is a core principle of physics that energy

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cannot be created or destroyed, but only transformed into different forms.

Motivated by this insight, Hayden and Preskill [9] proposed a *thermodynamically consistent approach* to resolving the black-hole paradox. Their idea is that any information that crosses the event horizon of a singularity in space time, is rapidly and chaotically “scrambled” across this horizon. Eventually, the infalling information will be transformed into heat and re-emitted as Hawking radiation [10]. They further argued that if the black hole was a purely classical system, the event horizon would effectively act as an information mirror. This means that any infalling information would rapidly be recoverable from the outgoing radiation.

A more reasonable treatment requires considering the quantum entanglement between Hawking radiation and external observers. In other words, Hayden and Preskill [9] assert that black holes should be described as quantum communication channels. Consequently, *quantum information scrambling* appears as *the* thermodynamically motivated and consistent resolution of the black-hole information paradox. Thus, one might expect that information scrambling was quickly picked up by the quantum thermodynamics community. Yet, ref. [9] was met with more immediate impact in cosmology [10] and quantum information theory [11–13].

Additionally, theoretical advancements in string theory and the holographic principle suggest that the dynamics of black holes can be modeled by lower-dimensional quantum systems with high degrees of scrambling. This is exemplified by the Sachdev-Ye-Kitaev (SYK) model, a solvable model that exhibits maximal scrambling in the large- N limit, where N is the total number of Majorana fermions in the system. More specifically, the SYK model is described by the Hamiltonian,

$$H_{\text{SYK}} = (i)^{\frac{q}{2}} \sum_{1 \leq i_1 < i_2 < \dots < i_q \leq N} J_{i_1 i_2 \dots i_q} \psi_{i_1} \psi_{i_2} \dots \psi_{i_q}, \quad (1)$$

where $J_{i_1 i_2 \dots i_q}$ are real independent random variables with values drawn from a Gaussian distribution with mean $\langle J_{i_1 \dots i_q} \rangle = 0$ and variance $\langle J_{i_1 \dots i_q}^2 \rangle = J^2(q-1)!/N^{q-1}$. The parameter J (in the variance) sets the scale of the Hamiltonian. Further, ψ_i are Majorana field operators for $i \in \{1, \dots, N\}$, and q is the number of fermions interacting at a time. The chaotic dynamics of this model mimic the hypothesized behavior of quantum information in the vicinity of a black hole’s event horizon.

Quantifying scrambling: the OTOC. – The natural question arises as to how to best quantify and characterize information scrambling and quantum chaos. In classical Hamiltonian dynamics, chaos can be identified from the exponential growth of the Poisson bracket [14]. Hence, it may not be a surprise that the most prominent tool to diagnose scrambling of quantum information is a closely related quantity —the out-of-time-ordered correlator (OTOC) [5].

The OTOC is a four-point correlation function that measures the growth of operators, V and W , in the Heisenberg picture, defined as

$$F(t) = \langle W^\dagger(t) V^\dagger W(t) V \rangle, \quad (2)$$

where $W(t) \equiv \exp i/\hbar H t W \exp -i/\hbar H t$, and H is the Hamiltonian of the system of interest. The average, *i.e.*, expectation value, is taken over the initial state of the system. It is often convenient to also analyze the expectation value of the squared commutator [5,15] in cases where V and W are unitary,

$$C(t) \equiv \langle [W^\dagger(t), V]^2 \rangle = 2(1 - \Re(F(t))), \quad (3)$$

where $\Re(F(t))$ denotes the real part of the OTOC (2).

For many-body quantum systems, W and V have to be chosen as meaningful and accessible observables depending on the complexity of the system. However, the quantum-classical analogy becomes much more stringent if we set $W = x$ as position and $V = p$ as momentum. In this case, $C(t)$ (3) is nothing but the quantum version of the average Poisson bracket. Hence, quantum information scrambling is evidenced by any growth of $C(t)$, whereas an exponential scaling of $C(t)$ as a function of time signifies quantum chaotic dynamics.

The OTOC was not actually defined for information scrambling, but rather first appeared in the context of spin echos [16]. However, the OTOC has proven itself to be uniquely suited to analyze the growth of correlations in many-body systems [17–26].

Experimental verification of scrambling. – Moreover, the OTOC (2) is an experimentally accessible quantity, which has been exploited to demonstrate information scrambling. In particular, using the OTOC aspects of scrambling dynamics were elucidated in ion traps [27–30]. In addition, information scrambling has also been demonstrated on Google’s Sycamore chip [31,32], IBM’s Q Experience [33], a ladder-type superconducting [34] and superconducting qutrits [35] quantum processors, and in NMR [36]. Moreover, experiments have been proposed for macroscopic spins [37], quantum many-body spin models [38,39], and neutral atom arrays [40].

Additionally, recent years have seen a surge in experimental efforts exploring fundamental concepts of quantum thermodynamics. To date, experiments have been reported employing, *e.g.*, optomechanical systems, nuclear magnetic resonance, nitrogen vacancy centers, superconducting systems, and many more. For a comprehensive exposition we refer to a recent review article [41]. However, owing to their exquisite controllability, ion traps are arguably the most important platform for quantum thermodynamics, with which many of the first experiments have been realized [42–46].

Therefore, for the present perspective the experiment by Landsman *et al.* [28] is of particular importance. It had

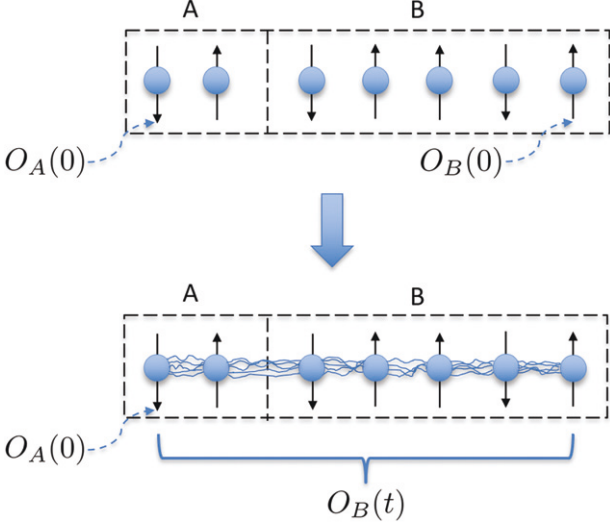


Fig. 1: Depiction of the underlying substructure illustrating the support for the local operators O_A and O_B , within a specific spin chain. Following the application of a scrambling unitary, O_B evolves into a non-local operator, expanding into the support of A , thereby initiating the decay of the OTOC (2). Note that the way we divide the total Hilbert space depends on the specific problem at hand.

been shown by Yoshida and Yao [47] that the Hayden-Preskill protocol [9] for scrambling in black holes can be mapped onto a problem of quantum teleportation. In fact, maximally scrambling dynamics is necessary for successful teleportation. Therefore, the teleportation fidelity provides an independent diagnostics for scrambling. Landsman *et al.* [28] implemented the corresponding quantum circuit implementable in their seven-qubit quantum computer, which is realized in a crystal of trapped $^{171}\text{Yb}^+$. This experiment provided convincing evidence of information scrambling throughout the quantum computer, even in the presence of decoherence. However, it also highlighted the shortcomings of the OTOC as a sole identifier of scrambling, which made the quest for other quantifiers instrumental.

Other quantifiers of scrambling. — To date, the OTOC has remained the most important quantifier of scrambling in experiments. Yet, also several other metrics have been developed to capture different aspects of this complex phenomenon. These quantifiers provide additional insights into the dynamics of entanglement, chaos, and information dispersal [48–59].

The main conceptual problem of using the OTOC as a scrambling quantifier stems from the seemingly arbitrary choice of operators V and W . For bipartite quantum systems, $A \otimes B$, it is most convenient to consider observables that initially only have support on either A or B , and hence $V = O_A$ and $W = O_B$. In fig. 1 we illustrate the ensuing scrambling of information.

To further alleviate the arbitrariness, one typically considers a Haar average over all possible observables with

such support. Interestingly, for quantum systems comprised of qubits, such as the experimental system analyzed in ref. [28], the Haar average is equivalent to an average over the Pauli group for each operator. Note, however, that in practice, an experimentalist has only access to a set of measurements they can implement depending on the physical setup. Hence, average OTOCs are mostly of theoretical interest, as they do not describe what can be implemented in an experiment. Therefore, constructing alternative measures of scrambling appears highly desirable.

Mutual information. As a staple of information theory, the mutual information seems to be the natural quantifier of scrambling in many-body systems. It is given by

$$\mathcal{I}(t) = \mathcal{S}_A(t) + \mathcal{S}_B(t) - \mathcal{S}_S(t), \quad (4)$$

where $\mathcal{S}_i = -\text{tr } \rho_i \ln(\rho_i)$ represents the von Neumann entropy of system i with the density matrix ρ_i .

In ref. [55], we showed that

$$\mathcal{I}(t) \geq \bar{\mathcal{O}}(0) - \bar{\mathcal{O}}(t), \quad (5)$$

where $\bar{\mathcal{O}}(t) \equiv \langle O_A O_B(t) O_A O_B(t) \rangle_{\text{avg}}$ is the Haar average of the OTOC over both operators O_A and O_B . Equation (5) holds true for all times $t > 0$, and shows that the decay of the OTOC $\bar{\mathcal{O}}(t)$ is upper bounded by the growth of mutual information.

Interestingly, eq. (5) provides a mathematically rigorous statement of the intuitive understanding of scrambling, which is often equated with the augmentation of bipartite correlations between subsystems A and B . Namely, the decay of the OTOC is governed by the growth of correlations in a complex quantum many-body system.

Even more importantly, the quantum mutual information is directly related to stochastic thermodynamic quantities, which offers a lucid thermodynamic description of scrambling. More specifically, we showed [55] that

$$\dot{\mathcal{I}} \leq \mathcal{A} \dot{\mathcal{S}}_A + \mathcal{B} \dot{\mathcal{S}}_B + \mathcal{C} |\dot{\mathcal{S}}_E|, \quad (6)$$

where $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ represent discrete analogs of the Frieden integral, as referenced in [60,61], and are dependent solely on the spatial geometry of the problem. Additionally, $\dot{\mathcal{S}}_A$ and $\dot{\mathcal{S}}_B$ denote the stochastic irreversible entropy production within subsystems A and B , as discussed in [62]. Lastly, $\dot{\mathcal{S}}_E$ refers to the flow of information between A and B .

Quantum work statistics. In fact, understanding the thermodynamics of scrambling is under active investigation in the literature [63–70].

One of the hallmark results of modern thermodynamics is the Jarzynski equality [71,72], $\langle \exp(-\beta W) \rangle = \exp(-\beta \Delta F)$, which expresses the difference in equilibrium free energy, ΔF , as an average over non-equilibrium processes. Here, W is the thermodynamic work, and β is the inverse temperature. In the quantum case, many different notions of work have been proposed, of which the two-time

measurement scheme [72] is the most prominent. In this case, the average of the exponentiated work becomes a two-time correlation function. Motivated by this insight, Yunger Halpern derived an identity for the OTOC (2) that formally resembles the Jarzynski equality [63].

A more genuinely thermodynamic perspective was provided by Campisi and Goold [64]. By expressing one of the OTOC (2) operators V in exponential form, $V = \exp iuO$ with an appropriate Hermitian operator O and real number u . Reference [64] proposed the wing-flap protocol: i) prepare the system in some state ρ ; ii) measure O ; iii) evolve the system with H for a time $t = \tau$; iv) apply the wing-flap perturbation W ; v) evolve the system with $-H$ for a time $t = \tau$; vi) measure O .

The projective measurements of O yield eigenvalues O_n and O_m , correspondingly collapsing the system into eigenstates $|n\rangle$ and $|m\rangle$. It is easy to see [64] that the characteristic function, that is the Fourier transform of the probability density function of $O_m - O_n$ is identical to the OTOC.

This close relationship between the quantum work statistics and the OTOC becomes even more stringent, once one recognizes that for sudden changes the characteristic function becomes a Loschmidt echo [73]. More generally, in ref. [54], it was proven that the *thermal Haar average* of the OTOC (for infinite temperature) is equal to the Haar averaged Loschmidt echo. Namely, we have

$$\overline{F_{\beta=0}(t)} \approx \left| \overline{\langle e^{i(H_B + V_\alpha)t} e^{-i(H_B + V_{\alpha'})t} \rangle_{\beta=0}} \right|^2, \quad (7)$$

where V_α are noise operators, and the average is over all noise realizations, and H_B is the self-Hamiltonian on the support B .

Scrambling in open systems. – Analyzing quantum information scrambling in open systems presents formidable challenges compared to closed systems. In open quantum systems, interactions with the environment play a crucial role in the dynamics. This interaction often leads to decoherence, which can either facilitate or hinder the scrambling process. A number of studies tackled this question [47, 74–81] from various perspectives.

One of the key models used to study scrambling in open systems is the Lindblad master equation, which describes the time evolution of the density matrix ρ of an open quantum system,

$$\frac{d\rho}{dt} = -i[H, \rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2} \{L_k^\dagger L_k, \rho\} \right), \quad (8)$$

where H is the Hamiltonian of the system, and L_k are the Lindblad operators that model the system's interaction with its environment. The first term on the right-hand side represents the unitary evolution, while the sum accounts for dissipation into the environment.

Scrambling in open systems can be characterized by the decay of quantum correlations and coherences. For example, the behavior of the OTOC in an open system can

reveal how environmental interactions affect the rate and nature of scrambling. Additionally, the entanglement dynamics in open systems, as quantified by entanglement measures like concurrence or entanglement entropy, provide valuable insights into the scrambling process under the influence of external noise and decoherence.

In ref. [75] we analyzed the effects of decoherence in both energy and computational bases. In fig. 2, we present the Haar averaged OTOC (2) for the SYK model (1) for different coupling intensities between $\mathcal{S}$ and $\mathcal{E}$, specifically at zero ($\hbar\gamma/J = 0$), weak ($\hbar\gamma/J \ll 1$), and moderate ($\hbar\gamma/J \simeq 1$) levels. Here, the parameter γ is the strength of the interaction with the environment. This figure shows that the OTOC does not distinguish noise and scrambling.

However, regarding the mutual information, we note that maximal scrambling is indicated by a maximum of the mutual information, which for qubit models simply becomes $\mathcal{I} = 2 \ln(2)$; therefore, any environmental impact causes $\mathcal{I}$ to decrease below $2 \ln(2)$ over extended periods. Hence, it appears natural to consider the dynamics of $\mathcal{I}$. To this end, we showed in ref. [75] that in open systems the change of the von Neumann entropy can be separated into three terms,

$$\mathcal{I}(\mathcal{S} : \mathcal{E}) + \Delta S_{\text{ex}} + D(\rho_{\mathcal{E}} || \rho_{\mathcal{E}}^{\text{eq}}) = \Delta S_{\mathcal{S}}. \quad (9)$$

This equation expresses that any departure from ideal unitary scrambling can be attributed to three factors: i) $\mathcal{I}(\mathcal{S} : \mathcal{E})$, the development of correlations between $\mathcal{S}$ and $\mathcal{E}$, ii) ΔS_{ex} , the thermal interchange between $\mathcal{S}$ and $\mathcal{E}$, and iii) $D(\rho_{\mathcal{E}} || \rho_{\mathcal{E}}^{\text{eq}})$, the deviation of $\mathcal{E}$ from thermal equilibrium.

In figs. 3 and 4 this insight is further illustrated for the SYK model as a chaotic model (fig. 3) and the Lipkin-Meshkov-Glick (LMG) model (fig. 4) as a fully integrable model, whose Hamiltonian reads

$$H_{\text{LMG}} = -\frac{J}{N} \sum_{i < j}^N (\sigma_x^i \sigma_x^j + \sigma_y^i \sigma_y^j) - \sum_{i=1}^N \sigma_z^i. \quad (10)$$

In ref. [75], we further found that the mutual information exhibits universal behavior. Any deviation from the monotonic growth of the mutual information points to some interaction with the environment, given that the unitary dynamics are indeed scrambling. This interaction might be in the form of pure decoherence (destruction of coherences + no dissipation), or destruction of coherences accompanied with dissipation.

Additionally, it is important to mention that a specific modification of the OTOC was defined in ref. [76]. This modification, dubbed “open bipartite OTOC”, essentially captures the operator entanglement of two operators that start (at $t = 0$) in different supports. More specifically, this paper adopts the Heisenberg picture, where the evolution of observables is modeled by a completely positive (CP) map, $\mathcal{E}$. A quantum channel $\mathcal{E}$ is considered unital

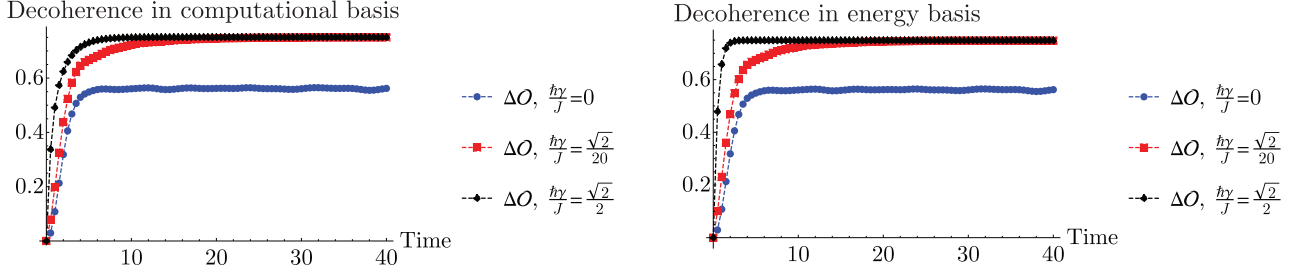


Fig. 2: Plots of the growth of the average OTOC $\bar{\mathcal{O}}(0) - \bar{\mathcal{O}}(t)$ as a function of time for the SYK model with $N = 12$ and an initial “all-up” state: $|0 \dots 0\rangle$. Plots were generated after averaging over 10^2 realizations.

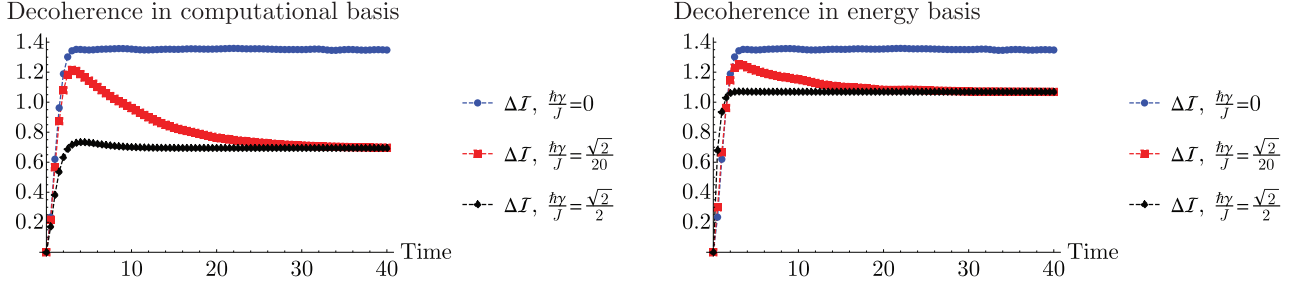


Fig. 3: Plots of the evolution of the change of mutual information (4) for the SYK model (1) with $N = 12$, for an initial “all-up” state. Plots were generated after averaging over 10^2 realizations.

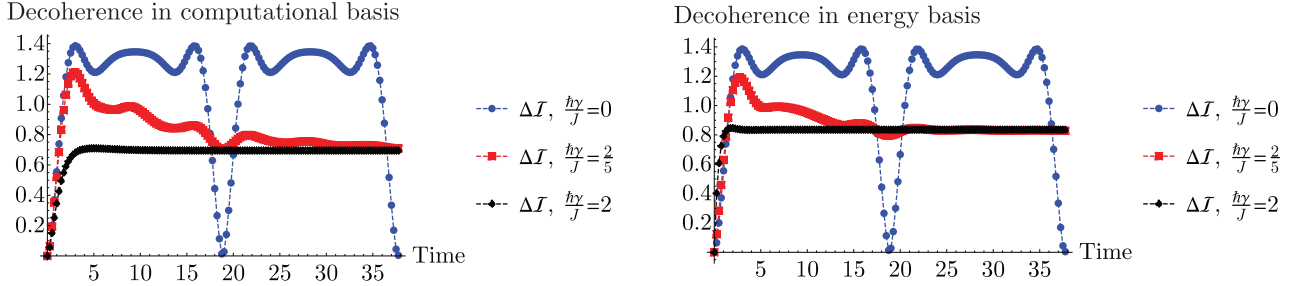


Fig. 4: Plots of the evolution of the change in mutual information (4) for the LMG model with $N = 6$ and an initial Néel state.

if it leaves the maximally mixed state, represented by $\frac{\mathbb{I}}{d}$, unchanged. Such channels include unitary evolution, projective measurements without postselection, and dephasing channels. A quantum channel is trace-preserving if its adjoint $\mathcal{E}^\dagger$ is unital. For simplicity, the paper assumes $\mathcal{E}^\dagger$ to be unital, implying $\mathcal{E}$ is a quantum channel.

The open bipartite out-of-time-order correlator (OTOC) is defined as [76]

$$F(t) = \frac{1}{2d} \mathbb{E}_{V_A, W_B} \|\mathcal{E}(V_A), W_B\|_2^2, \quad (11)$$

where the operators are averaged using the Haar measure of unitaries discussed above. From this form, this modified OTOC captures bipartite correlations between the supports A and B .

The numerical results were identical to what the quantum mutual information captures, which is not surprising given the parallel between the mutual information and the

OTOC, as shown in the previous section of this perspective, and detailed in refs. [55,75,82].

Applications and future directions. — We close this perspective with a few considerations on future directions. In fact, the study of quantum information scrambling has far-reaching implications and applications, extending across various domains of physics and opening new avenues for future research, all of which are typically considered in quantum thermodynamics [83]. In fact, as our understanding of scrambling, from a quantum thermodynamic perspective, deepens, its potential impact on several fields becomes increasingly evident [84–91].

Quantum computing and error correction. In the realm of quantum computing, scrambling plays a crucial role in understanding and mitigating quantum errors. Quantum error correction codes can be analyzed through the lens of scrambling, where the ability of a system to disperse local errors throughout its many degrees of freedom

can be harnessed for more robust error correction schemes. For example, the surface code, a widely studied error correction code, can be analyzed using scrambling metrics to optimize its fault-tolerance,

$$\text{Surface Code Hamiltonian: } H_{\text{SC}} = - \sum_s A_s - \sum_p B_p, \quad (12)$$

where A_s and B_p are stabilizer operators corresponding to the vertices (stars) and plaquettes of the lattice, respectively. This hints at direct applications of information scrambling in cybersecurity for quantum computers [92].

Many-body localization. Another exciting avenue is the exploration of many-body localization (MBL) in the context of scrambling. MBL systems, characterized by the absence of thermalization, present a unique platform to study how scrambling dynamics are altered in the presence of disorder and localization,

$$H_{\text{MBL}} = \sum_i h_i n_i + \sum_{i < j} J_{ij} n_i n_j, \quad (13)$$

where n_i are the occupation numbers, h_i represents on-site potentials, and J_{ij} are interaction strengths [93].

Quantum metrology. Quantum scrambling also finds applications in quantum metrology, enhancing the precision of quantum measurements. The sensitivity of quantum systems to initial conditions, as quantified by scrambling metrics, can be exploited to improve the accuracy of quantum sensors and clocks [94].

Future theoretical and experimental challenges. Looking ahead, both theoretical and experimental challenges remain in fully understanding and harnessing quantum scrambling. Experimentally, realizing systems that can coherently demonstrate scrambling dynamics over extended periods is a key challenge. Theoretically, developing a unified framework that incorporates scrambling in diverse quantum systems remains an active area of research.

In conclusion, the applications and future directions of quantum information scrambling span a wide range of fields, from quantum computing and black-hole physics to condensed matter and quantum metrology. As research in this area continues to evolve, it holds the promise of revealing deeper truths about the quantum world and unlocking new technological capabilities in quantum science.

Concluding remarks. — As we conclude this perspective on quantum information scrambling, it is evident that this phenomenon has become crucial in our understanding of quantum mechanics, with broad implications across various fields of physics. From providing insights into the fundamental nature of black holes to shaping the development of quantum computing, scrambling has emerged as a key concept in both theoretical and experimental physics.

This perspective highlights how quantum information scrambling is key in our effort to understand the quantum world from a *thermodynamic* viewpoint. Its study tests

our current knowledge and leads to new opportunities in technology and basic quantum science.

* * *

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REFERENCES

- [1] BERRY M., *Phys. Scr.*, **40** (1989) 335.
- [2] GOLDFRIEND T. and KURCHAN J., *Phys. Rev. E*, **102** (2020) 022201.
- [3] XU T., SCAFFIDI T. and CAO X., *Phys. Rev. Lett.*, **124** (2020) 140602.
- [4] XU S. and SWINGLE B., arXiv preprint, arXiv:2202.07060 (2023).
- [5] SWINGLE B., *Nat. Phys.*, **14** (2018) 988.
- [6] CALMET X. and HSU S. D. H., *EPL*, **139** (2022) 49001.
- [7] ANTONINI S., MARTYN J. and NAMBIAR G., *The Black Hole Information Paradox* (2018), <https://api.semanticscholar.org/CorpusID:145817748>.
- [8] DEFFNER S. and JARZYNSKI C., *Phys. Rev. X*, **3** (2013) 041003.
- [9] HAYDEN P. and PRESKILL J., *J. High Energy Phys.*, **2007** (2007) 120.
- [10] ALMHEIRI A. *et al.*, *Rev. Mod. Phys.*, **93** (2021) 035002.
- [11] BAO N. and KIKUCHI Y., *J. High Energy Phys.*, **2021** (2021) 17.
- [12] CHEN B., CZECH B. and WANG Z.-Z., *Rep. Prog. Phys.*, **85** (2022) 046001.
- [13] FISHER M. P. *et al.*, *Annu. Rev. Condens. Matter Phys.*, **14** (2023) 335.
- [14] GOLDSTEIN H., POOLE C. and SAFKO J., *Classical Mechanics* (American Association of Physics Teachers) 2002.
- [15] ROBERTS D. A. and SWINGLE B., *Phys. Rev. Lett.*, **117** (2016) 091602.
- [16] HAHN E. L., *Phys. Rev.*, **80** (1950) 580.
- [17] SWINGLE B. *et al.*, *Phys. Rev. A*, **94** (2016) 040302.
- [18] HASHIMOTO K., MURATA K. and YOSHII R., *J. High Energy Phys.*, **2017** (2017) 138.
- [19] SWINGLE B. and CHOWDHURY D., *Phys. Rev. B*, **95** (2017) 060201.
- [20] SWINGLE B. and YUNGER HALPERN N., *Phys. Rev. A*, **97** (2018) 062113.
- [21] KHEMANI V., VISHWANATH A. and HUSE D. A., *Phys. Rev. X*, **8** (2018) 031057.
- [22] XU S. and SWINGLE B., *Nat. Phys.*, **16** (2020) 199.
- [23] AKUTAGAWA T. *et al.*, *J. High Energy Phys.*, **2020** (2020) 13.
- [24] BELYANSKY R. *et al.*, *Phys. Rev. Lett.*, **125** (2020) 130601.
- [25] GARCIA R. J., ZHOU Y. and JAFFE A., *Phys. Rev. Res.*, **3** (2021) 033155.
- [26] BRAUMÜLLER J. *et al.*, *Nat. Phys.*, **18** (2022) 172.
- [27] GÄRTTNER M. *et al.*, *Nat. Phys.*, **13** (2017) 781.

- [28] LANDSMAN K. A. *et al.*, *Nature*, **567** (2019) 61.
- [29] JOSHI M. K. *et al.*, *Phys. Rev. Lett.*, **124** (2020) 240505.
- [30] GREEN A. M. *et al.*, *Phys. Rev. Lett.*, **128** (2022) 140601.
- [31] MI X. *et al.*, *Science*, **374** (2021) 1479.
- [32] JAFFERIS D. *et al.*, *Nature*, **612** (2022) 51.
- [33] HARRIS J., YAN B. and SINITSYN N. A., *Phys. Rev. Lett.*, **129** (2022) 050602.
- [34] ZHU Q. *et al.*, *Phys. Rev. Lett.*, **128** (2022) 160502.
- [35] BLOK M. S. *et al.*, *Phys. Rev. X*, **11** (2021) 021010.
- [36] LI J. *et al.*, *Phys. Rev. X*, **7** (2017) 031011.
- [37] BLOCHER P. D. *et al.*, *Phys. Rev. A*, **106** (2022) 042429.
- [38] JOSHI L. K. *et al.*, *Phys. Rev. X*, **12** (2022) 011018.
- [39] SUNDAR B. *et al.*, *New J. Phys.*, **24** (2022) 023037.
- [40] HASHIZUME T. *et al.*, *Phys. Rev. Lett.*, **126** (2021) 200603.
- [41] MYERS N. M., ABAH O. and DEFFNER S., *AVS Quantum Sci.*, **4** (2022) 027101.
- [42] HUBER G. *et al.*, *Phys. Rev. Lett.*, **101** (2008) 070403.
- [43] ABAH O. *et al.*, *Phys. Rev. Lett.*, **109** (2012) 203006.
- [44] AN S. *et al.*, *Nat. Phys.*, **11** (2015) 193.
- [45] ROSSNAGEL J. *et al.*, *Science*, **352** (2016) 325.
- [46] SMITH A. *et al.*, *New J. Phys.*, **20** (2018) 013008.
- [47] YOSHIDA B. and YAO N. Y., *Phys. Rev. X*, **9** (2019) 011006.
- [48] YUNGER HALPERN N., SWINGLE B. and DRESSEL J., *Phys. Rev. A*, **97** (2018) 042105.
- [49] IYODA E. and SAGAWA T., *Phys. Rev. A*, **97** (2018) 042330.
- [50] SESHADRI A., MADHOK V. and LAKSHMINARAYAN A., *Phys. Rev. E*, **98** (2018) 052205.
- [51] VERMERSCH B., ELBEN A., SIEBERER L. M., YAO N. Y. and ZOLLER P., *Phys. Rev. X*, **9** (2019) 021061.
- [52] GONZÁLEZ ALONSO J. R., YUNGER HALPERN N. and DRESSEL J., *Phys. Rev. Lett.*, **122** (2019) 040404.
- [53] YUNGER HALPERN N., BARTOLOTTA A. and POLLACK J., *Commun. Phys.*, **2** (2019) 92.
- [54] YAN B., CINCIO L. and ZUREK W. H., *Phys. Rev. Lett.*, **124** (2020) 160603.
- [55] TOUIL A. and DEFFNER S., *Quantum Sci. Technol.*, **5** (2020) 035005.
- [56] CAO Z., XU Z. and DEL CAMPO A., *Phys. Rev. Res.*, **4** (2022) 033093.
- [57] ZONNIOS M., LEVINSSEN J., PARISH M. M., POLLOCK F. A. and MODI K., *Phys. Rev. Lett.*, **128** (2022) 150601.
- [58] BHATTACHARYA A., CHEMISSANY W., HAQUE S. S. and YAN B., *Eur. Phys. J. C*, **82** (2022) 87.
- [59] OMANAKUTTAN S., CHINNI K., BLOCHER P. D. and POGGI P. M., *Phys. Rev. A*, **107** (2023) 032418.
- [60] NIKOLOV B. and FRIEDEN B. R., *Phys. Rev. E*, **49** (1994) 4815.
- [61] BRODY D. and MEISTER B., *Phys. Lett. A*, **204** (1995) 93.
- [62] ESPOSITO M. and VAN DEN BROECK C., *Phys. Rev. E*, **82** (2010) 011143.
- [63] YUNGER HALPERN N., *Phys. Rev. A*, **95** (2017) 012120.
- [64] CAMPISI M. and GOOLD J., *Phys. Rev. E*, **95** (2017) 062127.
- [65] CHENU A., EGUSQUIZA I. L., MOLINA-VILAPLANA J. and DEL CAMPO A., *Sci. Rep.*, **8** (2018) 12634.
- [66] KNAP M., *Phys. Rev. B*, **98** (2018) 184416.
- [67] CHENU A., MOLINA-VILAPLANA J. and DEL CAMPO A., *Quantum*, **3** (2019) 127.
- [68] STYLIARIS G., ANAND N. and ZANARDI P., *Phys. Rev. Lett.*, **126** (2021) 030601.
- [69] MIKKELSEN M., FOGARTY T. and BUSCH T., *Phys. Rev. Lett.*, **128** (2022) 070605.
- [70] OHANESJAN V., CHEIPESH Y., GNEZDILOV N. V., PAVLOV A. I. and SCHALM K., *J. High Energy Phys.*, **2023** (2023) 237.
- [71] JARZYNSKI C., *Phys. Rev. Lett.*, **78** (1997) 2690.
- [72] TALKNER P., LUTZ E. and HÄNGGI P., *Phys. Rev. E*, **75** (2007) 050102.
- [73] SILVA A., *Phys. Rev. Lett.*, **101** (2008) 120603.
- [74] XU Z., GARCÍA-PINTOS L. P., CHENU A. and DEL CAMPO A., *Phys. Rev. Lett.*, **122** (2019) 014103.
- [75] TOUIL A. and DEFFNER S., *PRX Quantum*, **2** (2021) 010306.
- [76] ZANARDI P. and ANAND N., *Phys. Rev. A*, **103** (2021) 062214.
- [77] XU Z., CHENU A., PROSEN T. and DEL CAMPO A., *Phys. Rev. B*, **103** (2021) 064309.
- [78] DOMÍNGUEZ F. D., RODRÍGUEZ M. C., KAISER R., SUTER D. and ÁLVAREZ G. A., *Phys. Rev. A*, **104** (2021) 012402.
- [79] CORNELIUS J., XU Z., SAXENA A., CHENU A. and DEL CAMPO A., *Phys. Rev. Lett.*, **128** (2022) 190402.
- [80] HAN L.-P., ZOU J., LI H. and SHAO B., *Entropy*, **24** (2022) 1532.
- [81] ANDREADAKIS F., ANAND N. and ZANARDI P., *Phys. Rev. A*, **107** (2023) 042217.
- [82] TOUIL A., *From black holes to quantum computers: The universality of information dynamics* (Order No. 30427157). Available from ProQuest Dissertations & Theses Global. (2835353251). Retrieved from <http://search.proquest.com.ezproxy.lib.uh.edu/dissertations-theses/black-holes-quantum-computers-universality/docview/2835353251/se-2> (2023).
- [83] DEFFNER S. and CAMPBELL S., *Quantum Thermodynamics* (Morgan & Claypool Publishers) 2019.
- [84] DAĞ C. B., SUN K. and DUAN L.-M., *Phys. Rev. Lett.*, **123** (2019) 140602.
- [85] DAĞ C. B., DUAN L.-M. and SUN K., *Phys. Rev. B*, **101** (2020) 104415.
- [86] CARAVELLI F., COULTER-DE WIT G., GARCÍA-PINTOS L. P. and HAMMA A., *Phys. Rev. Res.*, **2** (2020) 023095.
- [87] YUAN D., ZHANG S.-Y., WANG Y., DUAN L.-M. and DENG D.-L., *Phys. Rev. Res.*, **4** (2022) 023095.
- [88] GARCIA R. J., BU K. and JAFFE A., *J. High Energy Phys.*, **2022** (2022) 27.
- [89] BIN Q., WAN L.-L., NORI F., WU Y. and LÜ X.-Y., *Phys. Rev. B*, **107** (2023) L020202.
- [90] LI Z. *et al.*, *Science*, **380** (2023) 1381.
- [91] GARCIA R. J., BU K. and JAFFE A., *Proc. Natl. Acad. Sci. U.S.A.*, **120** (2023) e2217031120.
- [92] RAHEMAN F., *Future Internet*, **14** (2022) 335.
- [93] CHEN Y., arXiv preprint, arXiv:1608.02765 (2016).
- [94] YAN B. and SINITSYN N. A., *Phys. Rev. Lett.*, **125** (2020) 040605.