Modeling of 2D ADI-FDTD for THz Region Using a Tridiagonal Matrix Inversion Approach

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***Abstract*—This study investigates a Transverse Electric (TE) mode using the 2D Alternating Direction Implicit (ADI) FDTD method. A 200×200 grid is designed with a PML boundary so that waves in the boundary don't reflect back or scatter outside of the grid. ADI-FDTD is an unconditionally stable method that isn't restricted by CFL conditions. We present a matrix inversion approach that uses the LU decomposition method to generate the implicit electric and magnetic field equations. Around 50-60% of computation time reduction with small errors was observed with a 3THz electric wave. This time efficiency is dependent on the courant number (CN) and the effect of changing CN is discussed in this paper.**

***Index Terms*—computation time, implicit, matrix inversion, TE mode, time domain.**

I. Introduction

Finite-Difference Time-Domian (FDTD) is a tool to analyze electromagnetic (EM) waves and devices using differential equations as the building blocks of the model. FDTDs are particularly popular for optical and microwave applications [1], [2]. The development of FDTD models has been done since 1990s and they were very efficient for low frequency simulations. But due to Courant-Friedrich-Levy (CFL) stability condition, the simulation becomes unstable when higher frequency is applied [3]. This condition limits the cell size and step size, which leads to a large overall computation time.

To counter this problem, Alternating Direction Implicit (ADI) FDTD method was proposed and developed [3], [4]. Due to the implicit nature of the field equations, the CFL restriction is lifted and the time step size can be increased without sacrificing the integrity of the outcomes. Increased step size reduces the overall computation time by a significant margin. It also enables high frequency simulations and facilitates the simulations of THz devices and applications. Based on the different boundary conditions, different methods have been tried to construct this ADI-FDTD model.

In this paper, a perfectly matched layer (PML) is proposed as a virtual absorbing boundary across the 2D grid. The PML is applied using non-zero conductivity terms and a grating was done to facilitate soft absorption in the boundary. The field equations were found using LU decomposition and matrix inversion method. Courant Numbers (CN) are introduced to increase the time efficiency while keeping the errors minimal. A significant reduction in computation time was observed with low simulation error. A comparison of different cases is provided in the Result Analysis section.

II. Geometry and Simulation Modeling

The field update equations of ADI-FDTD in this study are designed for TE mode in the z direction. The H_{zx} and H_{zy} components produce H_z field but the separation is done to apply the perfectly matched layer(PML). The time step Δt is calculated from the following equation,

$$\Delta t = CN \times \frac{1}{c\sqrt{(dx^{-2}) + (dy^{-2})}}. \tag{1}$$

Here, CN is the courant number which is less than 1 for Basic FDTD because of the CFL restriction. ADI-FDTD doesn't require the CFL condition for stability and $CN > 1$ for those cases. The governing equation to find all the field components for TE mode is following [5],

$$(I - \frac{\Delta t}{2}[A])(I - \frac{\Delta t}{2}[B])u^{n+1} = (I + \frac{\Delta t}{2}[A])(I + \frac{\Delta t}{2}[B])u^n + \Delta t v^n. \tag{2}$$

Here, I is the identity matrix and,

$$[A] = \begin{bmatrix} -\frac{\sigma_y}{2\epsilon} & 0 & \frac{1}{\epsilon}\frac{\delta}{\delta y} & \frac{1}{\epsilon}\frac{\delta}{\delta y} \\ 0 & -\frac{\sigma_x}{2\epsilon} & 0 & 0 \\ 0 & 0 & -\frac{\sigma_x *}{2\mu} & 0 \\ \frac{1}{\mu}\frac{\delta}{\delta y} & 0 & 0 & -\frac{\sigma_y *}{2\mu} \end{bmatrix} and, u = \begin{bmatrix} E_x \\ E_y \\ H_{zx} \\ H_{zy} \end{bmatrix}.$$

$$[B] = \begin{bmatrix} -\frac{\sigma_y}{2\epsilon} & 0 & 0 & 0 \\ 0 & -\frac{\sigma_x}{2\epsilon} & \frac{1}{\epsilon}\frac{\delta}{\delta x} & \frac{1}{\epsilon}\frac{\delta}{\delta x} \\ 0 & \frac{1}{\mu}\frac{\delta}{\delta x} & -\frac{\sigma_x *}{2\mu} & 0 \\ 0 & 0 & 0 & -\frac{\sigma_y *}{2\mu} \end{bmatrix} and, v = \begin{bmatrix} -\frac{J_{ex}}{\epsilon} \\ -\frac{J_{ey}}{\epsilon} \\ 0 \\ 0 \end{bmatrix}.$$

The conductivity terms inside the [A] and [B] matrices are used for wave propagation and boundary conditions. J_{ex} and J_{ey} are just the current terms in their corresponding directions. This equation is separated into the following two different equations to apply matrix inversion,

$$(I - \frac{\Delta t}{2}[A])u^{tmp} = (I - \frac{\Delta t}{2}[B])u^n + \frac{\Delta t}{2}v^n. \tag{3}$$

$$(I - \frac{\Delta t}{2}[B])u^{n+1} = (I - \frac{\Delta t}{2}[A])u^{tmp} + \frac{\Delta t}{2}v^n. \tag{4}$$

Equation (4) will provide intermediate field components (u^{tmp}) and those will be used to find the field components in the next cell (u^{n+1}). The term 'implicit' is justified as all the updates in the field equations are interrelated and outcomes show an alternating relation.

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III. Result Analysis

The current ADI-FDTD model is designed in a 500×500 grid where each of the grid lengths is 2μm. It is excited with the following 3THz sinusoidal electric wave,

$$E_x = A_0 sin(2\pi f n \Delta t) \tag{5}$$

where Δt is the time step size, n is step number and f is the operation frequency. Fig. 1 demonstrates the magnetic field's (H_z) propagation through the grid. This wave propagation's validity is verified as the wavelength (λ) from the calculations matches the wavelength (λ) of the simulation (Table I).

TABLE I . Verification of The Simulation Model

Each Grid Size	Distance between Two Adjacent Maxima	λ from Calculation	λ from Simulation
2μm	50 grids	0.1mm	0.1mm
1μm	100 grids	0.1mm	0.1mm

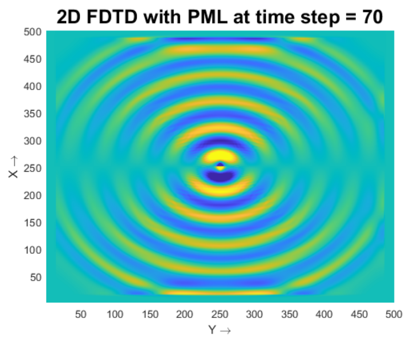

Fig. 1 . Propagation of the wave through the grid

It is an important step to compare the ADI-FDTD method with the regular FDTD to check the feasibility of the newly built solver. All the cases are excited with an electric sinusoidal wave to analyze the qualitative behavior of each of the cases. In both x- and y-axis, each of the sides of the surrounding PML layers has 30 μm of thickness. Due to the PML grating, conductivity varies from 22361 S/m to 50000 S/m from the nearest to the farthest grid. The basic FDTD is restricted by the CFL condition and it shows around 10.6881 sec to complete the simulation. As the implicit equation freed the solver from this CFL restriction, all the ADI cases show significant time reduction in the calculation. Table II and Fig. 2 properly demonstrates these outcomes.

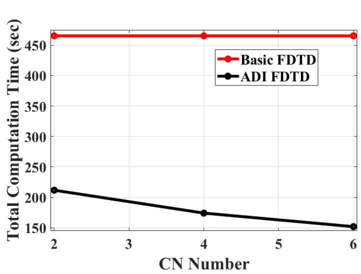

Fig. 2 . Computation time for Regular and ADI FDTD

TABLE II . Comparison Between Basic and ADI FDTD

Model Type	CN Number	Δt (sec)	Computation time
Basic FDTD	-	4.6669×10^{-15}	465.299228 sec
ADI-FDTD	2	9.4281×10^{-15}	211.811224 sec
ADI-FDTD	4	1.8856×10^{-14}	174.216286 sec
ADI-FDTD	6	2.8284×10^{-14}	151.984211 sec

In Fig. 3, wave propagation in the x-direction of the grid is displayed. Introduction of CN introduces small errors too. Taking the basic FDTD as reference, Fig. 3 demonstrates the deviation as we add and increase the courant number for the ADI cases. Although the errors are minimal, there is a need for a tradeoff between faster computation and the errors associated with it.

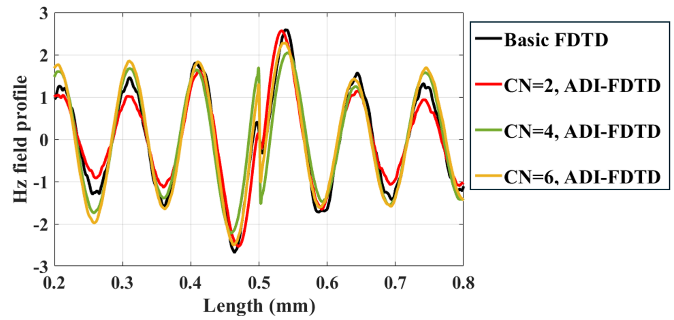

Fig. 3 . Comparison of wave Propagation for different FDTD Cases

IV. Conclusion

The ADI-FDTD model discussed in this paper is a skeleton of a solver with a significant reduction of computation time. This study demonstrates the propagation of TE mode (H_z) from a point source to a space grid. The wave applied is in the THz region and this grid can be used to simulate any kind of THz device as well as low-frequency devices. Increasing the CN number shows less computation time and for CN=6, the simulation time is 151.984211 sec. instead of the 465.299228 sec. of the basic FDTD system. All the ADI cases show 50-60% time efficiency and the time difference will become more prominent when more time steps and space are introduced. But this time efficiency comes with errors, so an optimization of CN and errors is also required to get the best outcome.

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