ECO-TRAJECTORY PLANNING FOR CONNECTED AND AUTONOMOUS VEHICLES WITH THE HEURISTIC EXPLICIT MODEL PREDICTIVE CONTROL

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ABSTRACT

The trajectory planning problem (TPP) for Connected and Autonomous Vehicles (CAVs) holds increasing significance in the research of next-generation transportation systems, yet it poses two major challenges that significantly limit its practical applicability. The first one is low computational efficiency, especially under specific cases within TPP, such as the Eco-trajectory Planning Problem (EPP), due to the non-linear nature of formulations. Another one concerns the inability to account for signal information and leading vehicles' movements, either a Human driven vehicle or another CAV. To tackle these concerns, this paper proposes a heuristic explicit predictive model control (heMPC) framework containing two modules: offline and online. The offline module constructs an optimal eco-trajectory batch by solving a sequence of simplified optimization problems for minimizing fuel consumption, considering various initial and terminal system states. Each candidate trajectory in the batch yields the lowest fuel consumption subject to a specific travel time from the vehicle entry to the departure from the network. The online module contains a heuristic trajectory planning algorithm that selects the trajectories that can ensure the CAV would not violate signals or follow to closely to the leading vehicles. This batch-based selection method significantly enhances computational efficiency attributed to the small-sized solution set for the optimization procedures. The case study with a wide range of MPRs demonstrates significantly reduced computation time without a noticeable loss of optimality. The proposed framework can effectively enhance the applicability of TPP in real-world mixed traffic scenarios and possesses substantial potential for incorporating more complicated interrelations between vehicles.

Keywords: Heuristic explicit model predictive control (heMPC), Connected and automated vehicles (CAV), Eco-driving, offline computing, heuristic trajectory planning

INTRODUCTION

The rapid advancement of technology in vehicle detection, short-range communication, and computing has paved the way for the development and implementation of connected and autonomous vehicles (CAVs). These vehicles have the potential to revolutionize transportation by offering increased safety, efficiency, and convenience. Among all studies contributing to the application of CAV, developing algorithms controlling the motion (acceleration, speed, lane change, etc.) of CAVs to achieve various objectives is one of the fundamental yet challenging tasks in the operation of CAVs. For example, emission minimization is often treated as part of the control objectives, which inevitably increases computational complexity due to the non-linear relations between the kinematic variables and fuel consumption. Under a mixed traffic environment with both human-driven vehicles (HDVs) and CAVs, avoiding crashes is often more complicated because of the need to account for the driving behavior of HDVs and the interactions between those two vehicle types. Naturally, a control problem considering various such practical concerns, the trajectory planning problem (TPP), forms a well-established research direction recently receiving considerable attention.

In recent decades, various families of solutions have been investigated, starting from adopting nonlinear programming models (NLP) and moving toward optimal control (OC) methods. For example, Yang et al., (*1*) developed a heuristic discrete feedback control algorithm to compute the advisory speed limit based on Newell's car-following model. They evaluated the effectiveness of fuel consumption savings under various Minimum Performance Requirements (MPRs) while accounting for the Vehicle-to-Vehicle (V2V) communication delays. Building upon V2I (Vehicle-to-Infrastructure) communication, Ubiergo et al., (*2*) proposed an advisory speed limit method based on Gipp's car-following model, which formulated the impact of queue length and quantified the effectiveness on emission reduction, fuel consumption savings, and travel delays through simulation experiments. Yang et al., (*3*) developed the eco-cooperative adaptive cruise control (Eco-CACC) problem as an optimal control problem, which is also a heuristic trajectory planning problem. In this study, the influence of queues has been considered, which is equivalent to the safe distance constraints in other studies to some extent. This developed algorithm is essentially a direct-solving approach considering the queue and traffic signal information, while its computational efficiency has not been specifically discussed. Similar to (*3*), (*4*) formulated a two-stage optimal control model for minimizing fuel consumption applicable to both isolated intersections and arterial roads. A pseudospectral solution method (PM) is used to solve the two-stage model, and it takes 1.6s to get an optimal eco-trajectory for a vehicle in a real case example. Despite the achievement of these methods in hypothetical traffic networks, most NLP-based TPP solutions suffer from insufficient computational efficiency that limits their applicability to real-world scenarios, even with the help of some heuristic paramours shooting algorithms ((*5*), (*6*),(*7*), (*8*), (*9*), (*10*), (*11*), (*12*)) which compressed the computation time to less than 5 seconds per vehicle. Recognizing such a limitation of NLP-based methods, some Model Predictive Control solutions ((*13*),(*14*),(*15*),(*16*),(*17*),(*18*),(*19*)) adopted algorithms and techniques from control theory and significantly improved computational efficiency. Nevertheless, the computing time would greatly increase with the number of input vehicles in the network. Such a deficiency limited those TPP solutions at a theoretical level, or at most, the planning stage without the feasibility of a real-time application.

Another constraint of applying the existing methods to practical use is the common negligence of the diverse driving behaviors and signal information under a mixed-flow environment. Not accounting such real-time information in TPP would incur crash risks when a front vehicle is relatively slower. An ideal solution to TPP should allow CAVs to respond to a leading HDV differently based on the perceived driving behavior and be capable of taking advantage of the advanced signal phase and timing information to guide the entire mixed traffic flow to achieve various pre-specified objectives.

To enhance the applicability of TPP in real-world scenarios by addressing both computational efficiency issues and potential concerns in mixed-flow traffic, this study develops a new TPP solution to minimize emission by applying the idea of explicit model predictive control (eMPC) that can avoid time-consuming online optimization processes through pre-computing processes (*20*). Innovatively, the developed method contains an offline module, which builds an optimal eco-trajectory batch that serves as a lookup table in traditional eMPC, and an online module to select an optimal trajectory from a very limited set of candidate trajectories. The proposed model with such a two-stage design will have the following features:

- replacing the time-consuming optimization process with a straightforward trajectory selection process;
- possessing significantly higher computational efficiency despite the highly nonlinear nature of the emission minimization;
- yielding trajectories customized for each CAV based on the real-time observed driving behavior of the front vehicle to avoid crashes effectively;
- being responsive to perceived traffic signal phase and timing information; and
- resulting in low emission impact.

Ultimately, these advancements can contribute to the widespread adoption of autonomous vehicles, paving the way for a safer and more sustainable future of transportation.

STRUCTURE AND ASSUMPTION OF EXPLICIT PREDICTIVE CONTROL FRAMEWORK

Structure of explicit predictive control framework

Figure 1 depicts the structure of the proposed model where the offline module generates an optimal eco-trajectory batch comprising several sets of most fuel-efficient trajectory solutions, each set corresponding to a pair of initial and terminal states defined by the vehicle speeds, location and time upon its entry and departure from the study intersection. Those candidate trajectory solutions will be produced from an optimization problem to minimize fuel consumption and subject to various fundamental constraints on vehicle dynamics. These candidate trajectories will form an optimal eco-trajectory batch and serve as the solution set of the online module, which assigns each CAV one trajectory that satisfies all real-time constraints, including its entry time, initial speed, received traffic signal information and safe following distance with the leading vehicle.

To achieve the desired computational efficiency and aforementioned objectives, below innovative designs are integrated into the model:

- Compared to the conventional trajectory planning optimization, the optimization problem in the offline module is simplified by ignoring the traffic signal and safe following distance constraints yet only including fuel-consumption-related formulations and basic kinematic constraints so that a set of fuel-efficient trajectories can be generated without considering any time-varying conditions (i.e., signal information and behavior of other vehicles). Such a design relaxes the need to generate duplicate trajectories that share the same travel time, acceleration, and deceleration patterns but differ only on the starting times.

- In the online module, the most fuel-efficient trajectory will be first translated to match the entry time of a subject vehicle. Note that this step does not change the resulting fuel consumption of a trajectory and allows the model to examine the feasibility of the trajectory with respect to time-sensitive constraints.
- The translated trajectory will then be examined by the traffic signal and safe following distance constraints, which ensure that all vehicles will only pass the stop line within an effective green time and that the crashes can be avoided.
- If either constraint denies a trajectory, it will be temporarily removed from the solution set for the subject vehicle, the most fuel-efficient trajectory from the remaining batch will be then translated and examined until a trajectory is found to satisfy both constraints and assigned to the subject CAV.
- The safe following distance will be maintained between the subject and the front vehicles, referring to either a predicted or planned trajectory of the front vehicles depending on whether it is an HDV or a CAV.

Followed by a brief review of the traditional formulation for the eco-trajectory planning problem, all formulations and algorithms utilized in the offline and online modules of the proposed heMPC framework will be discussed in detail.

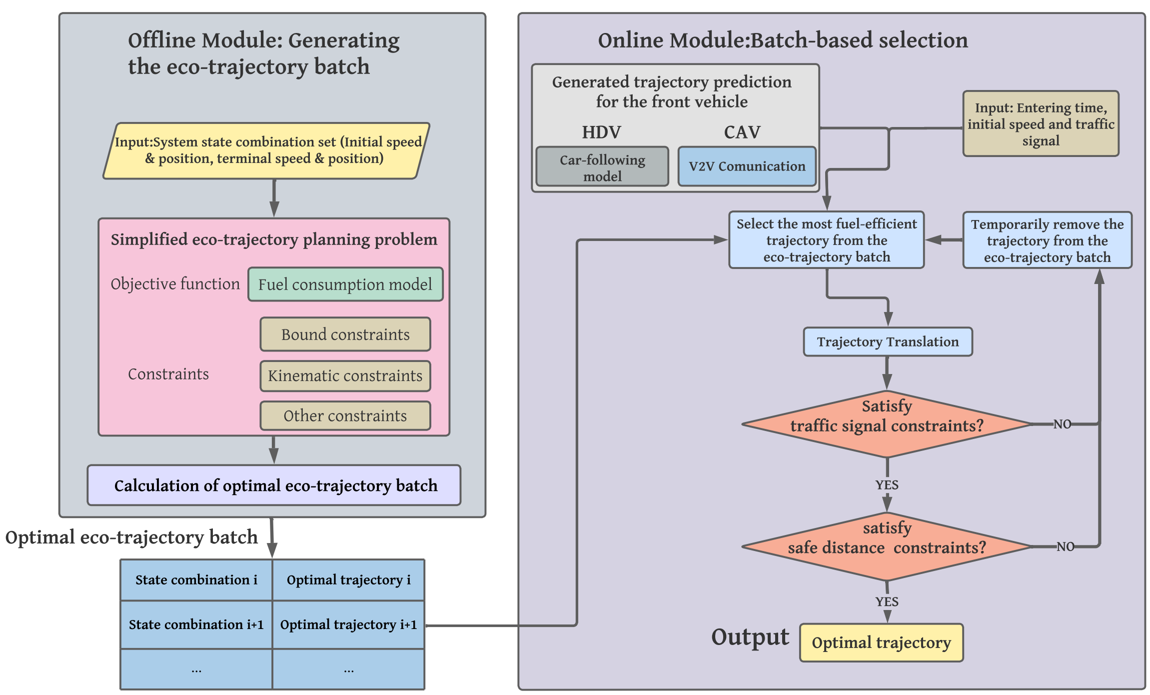


FIGURE 1: Heuristic explicit model predictive control framework

Assumption of explicit predictive control framework

The assumptions for the proposed heMPC framework are presented below, and other specific assumptions used in certain comparison experiments or algorithms will be clarified in relevant sections.

1. Every vehicle will travel through the intersection within φ cycles, where φ is an integer.
2. The fuel consumption model used in the proposed system should be a continuous or piecewise continuous function.
3. All HDVs strictly follow a car-following model so that their trajectory is fully predictable.
4. The communication delays between vehicles are ignored.

HEURISTIC ECO-TRAJECTORY PLANNING

This section introduces the logic and formulations of each function or algorithm in the heMPC framework shown in Figure 1. The general form of the Eco-trajectory planning problem (EPP) closely related to the generation of the optimal eco-trajectory batch is modeled in detail first, followed by an introduction of the simplified version exclusively for the proposed heMPC. The generation of candidate trajectory batch, translation processes and heuristic eco-trajectory planning will then be presented. At last, the optimality analysis is conducted to verify the worst case of the solution of the proposed heMPC framework. The critical notations adopted in the proposed study are shown in Table 1.

TABLE 1: Sets, variables, and parameters

Symbols	Descriptions
B	Set of vehicles, B ={ b_0, b_1, b_2, b_3, ..., b_n }
t_e^b,t_d^b	Entry and departure time of vehicle b
B_{HDV},B_{CAV}	Set of HDV and CAV
l_v	Length of vehicle
L_s	Distance from entry point to the stop line
S_{CAV},S_{HDV}	Minimum safe distance of CAV and HDV
v_b^I	Initial speed of vehicles
v_t^b	Speed of vehicle b at time t
a_t^b	Acceleration or deceleration of vehicle b at time t
v_{max},v_{min}	Maximum and minimum speed of vehicles
d_E	Emergency braking deceleration of vehicles
a_{max},d_{max}	Maximum acceleration and deceleration of vehicles
θ	Length of time intervals
C	Cycle lengths
G_i,Y_i,R_i	Duration of ith green phase, yellow phase, red phase
x_t^b	Traveling distance of vehicle b at time t
xp_t^b	Predicted traveling distance of vehicle b at discrete time t
vp_t^b	Predicted speed of vehicle of vehicle b at discrete time t
Π^b	The trajectory of vehicle b, Π^b is a sequence, Π^b = {x_t^b : t $\in$ τ }
Π	optimal trajectory batch, index by i

Review of a traditional solution to Eco-trajectory planning problem

The offline module of the proposed heMPC aims to generate candidate trajectories for CAVs entering an intersection, as shown in Figure 1. Considering the high non-linearity of the fuel consumption functions, this study, enlightened by (*21*), adopts a discrete-time modeling approach to optimize the motion of CAVs. Specifically, the modeling time horizon will be divided into a set of sufficiently short equal-length time intervals expressed as $\tau = \{t_0, t_1, ..., \mathrm{T}\}$. The decision variable x_t^b, as shown in Figure 2, is to represent the cumulative traveling distance of vehicle b until time interval t ($t \in \tau$) from the origin. By assuming that CAV will make a uniform motion within each time interval, the generated time-space curve can be regarded as a feasible trajectory. To facilitate the formulations embedded in the proposed heMPC, this section will elaborate on the general concept and formulations of solutions to traditional Eco-trajectory planning problems (TEPP) using such a discrete-time modeling approach. Generally, a TEPP will have border, kinematic, traffic signal, and safe distance constraints.

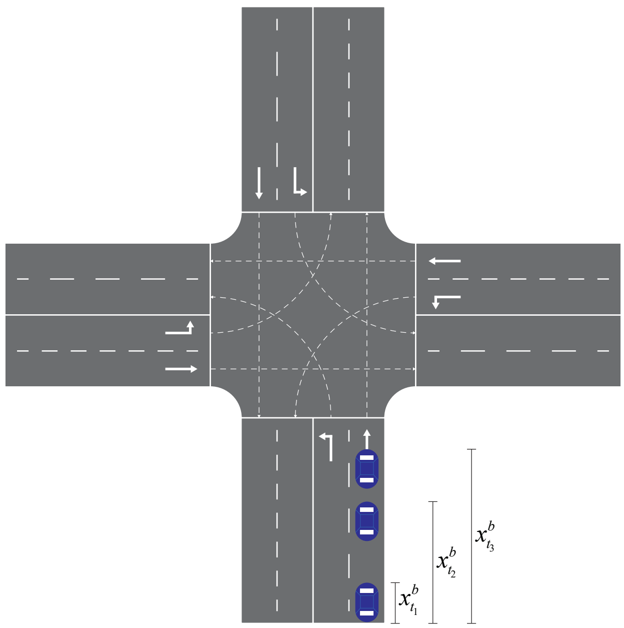


FIGURE 2: An example of discrete-time-based modeling

For a given CAV b, its entry time t_e^b and departure time t_e^d must fall within the set τ, which is ensured with border constraints, as expressed below.

$$x^b_{t^b_e} = 0 \ , t \in \tau, b \in B \tag{1}$$

$$x^b_T \geq L_s \ , b \in B \tag{2}$$

$$x^b_t - L_s \leq M \times \omega^b_t \quad , t \in \tau, b \in B \tag{3}$$

$$x^b_t - L_s \geq M \times (\omega^b_t - 1) \quad , t \in \tau, b \in B \tag{4}$$

$$\textbf{If} \quad \omega^b_t == 0 \quad \textbf{Then} \quad t^b_d \geq t + \theta \quad , t \in \tau, b \in B \tag{5}$$

$$\textbf{If} \quad \omega^b_t == 1 \quad \textbf{Then} \quad t^b_d \leq t \quad , t \in \tau, b \in B \tag{6}$$

$$x^b_{t^b_e+\theta} - x^b_{t^b_e} \geq v^D_b \times \theta \ , b \in B \tag{7}$$

$$x_{t_d^b} = L_s, b \in B \tag{8}$$

Equation 1 illustrates that at the entry time $x^b_{t^b_e}$, the CAV b has just entered the intersection, resulting in a zero traveling distance within the intersection. Equation 2 denotes that CAV b must depart the intersection before the final second of the modeling time horizon to ensure valid modeling. Equations 3 through 6 illustrate whether the vehicle b has departed the study network at time t. If not, i.e., $\omega_t^b = 0$, the CAV b can depart no earlier than time $t + \theta$. Additionally, as depicted in Equations 7 and 8, when CAV b exits the intersection, its speed at time t_d^b should be greater than or equal to a specified speed threshold v_D, and it must precisely cross the stop line at the departure time t_d^b, represented by $x_{t_d^b} = L_s$.

Kinematic constraints are used to ensure the basic kinematic rules, which generally incorporate velocity limitation (as expressed in Equation 9,10), deceleration and acceleration limitation (as expressed in Equation 11,12), and primary kinematic constraints (as expressed in Equation 13,14).

$$v_t^b \geq v_{min} \ , t \in \tau, b \in B \tag{9}$$

$$v_t^b \leq v_{max} \ , t \in \tau, b \in B \tag{10}$$

$$a_t^b \geq d_{max} \ , t \in \tau, b \in B \tag{11}$$

$$a_t^b \leq a_{max} \ , t \in \tau, b \in B \tag{12}$$

$$v_{t+\theta}^b = v_t^b + a_t^b \times \theta \ , t \in \tau, t \neq T, b \in B \tag{13}$$

$$x_{t+\theta}^b = x_t^b + v_t^b \times \theta \ , t \in \tau, t \neq T, b \in B \tag{14}$$

Traffic signal constraints ensure all vehicles travel through the stop line during the effective green time, which can be expressed as below.

$$\textbf{If } (Y_i^e - t) \times v_t^b + x_t^b \leq L_s \textbf{ Then } x_{t+0.5C}^b \leq L_s \quad t \in Y_i, b \in B_{CAV} \tag{15}$$

$$\textbf{If } x_t^b \leq L_s \textbf{ Then } x_{i\times C}^b \leq L_s \quad t \in R_i, b \in B_{CAV} \tag{16}$$

Equation 15 ensures that vehicles not able to reach the stop line by the end of the yellow phase will take the following green phase, and Equation 16 applies to those arriving during the red phase.

Safe distance constraint is to be applied between each pair of adjacent vehicles traveling on the same path, whose distance should always be greater then or equal to $S_{CAV} + l_v$, where S_{CAV} is the minimum car-following gap, l_v is the vehicle's length, as expressed below

$$x_t^{b_{i-1}} - x_t^{b_i} - S_{CAV} - l_v \geq 0 \ \ b_{i-1}, b_i \in B_{CAV}, t_e^{b_{i-1}} \leq t_e^{b_i}, t \in \tau \tag{17}$$

Offline module: generating optimal eco-trajectory batch

As shown in Figure 1, the core purpose of the offline module is to generate an optimal eco-trajectory batch that contains the most fuel-efficient trajectory for each possible length of the CAV travel time through the study network. The feasibility and effect of such a task are verified below:

- Assuming that all studied vehicles will finish their trip in the studies network by the end of φth cycle, the travel time of any vehicle would not exceed φC - R_φ. By adopting the discrete-time framework, the number of possible values for travel times from vehicle entry to the stop line is finite. Therefore, a finite number of optimization problems will be solved, each corresponding to a travel time value. The optimal solution, indicating a trajectory with the highest fuel efficiency, will be added to the batch

- The fuel consumption does not vary with the vehicle entry time and solely depends on the vehicle travel time. Therefore, generating duplicate trajectories that differ only on the vehicle entry time is unnecessary. All candidate trajectories generated in this module are assumed to start from t=0.
- Those real-time constraints, i.e., traffic signal and safe following distance constraints, are not required in the offline module since they can be addressed in the online module simply by specifying a proper travel time.

To achieve the objective of the off-line module, Algorithm 1 is developed, where each iteration is to generate a trajectory yielding the optimal fuel-efficient eco-trajectory with a specific traveling time ξ. The fuel consumption function, $F(v,a)$, shall be dependent on instantaneous speed and acceleration, and the VT-micro emission model ((*22*)) is adopted in this study. It is worth noting that for this optimization problem $\rho\{\Phi(0,v_b^0,x_b^0),\Phi(\xi,v_b^\xi,x_b^\xi)\}$, where $\Phi(0,v_b^0,x_b^0)$ is the initial state and $\Phi(\xi,v_b^\xi,x_b^\xi)$ is the terminal state, Theorem 1 holds.

Theorem 1 : If fuel consumption function F(v,a) is continues. Then for any generating optimization problem $\rho\{\Phi(0,v_b^0,x_b^0),\Phi(\xi,v_b^\xi,x_b^\xi)\}$, which is feasible, then it must have at least one optimal solution. Proof can be found in (*23*).

Because there is a minimum time required for a CAV to travel through the intersection, the feasibility of the generated trajectory will be examined, as shown in Row 7 of Algorithm 1. Note that the energy consumption of the optimal eco-trajectory with a relatively shorter travel time is not necessarily low than that of a longer one. Therefore, Row 11 is added to sort Π in ascending order of energy consumption. Figure 3 illustrates an optimal trajectory batch obtained from the offline module, where each line indicates the most fuel-efficient trajectory with a specific travel time, where the blue line denotes the trajectory with the lowest fuel consumption.

Algorithm 1: Generating optimal trajectory batch

Input: Input: Set $\Pi = \varnothing$, arbitrary vehicle b, integer parameter φ, **p**
1 **for** ξ ***in*** *range [0, φC - R_φ]* **do**
2 Generating optimization problem $\rho\{\Phi(0,v_b^0,x_b^0),\Phi(\xi,v_b^\xi,x_b^\xi)\}$:
3 Objective function : Min:$\sum_b^B \sum_0^\xi F(v_t^b,a_t^b)\theta$
4 Add border constraints : Equation 1 - 8
5 Add kinematic constraints : Equation 9 - 14
6 Add traveling time constraints : $x_\xi^b = L_s$
7 **if** ρ *is infeasible* **then**
8 Continue
9 **else**
10 Add optimal eco-trajectory of ρ to Π
11 Sort Π in ascending order of energy consumption
Output: Π

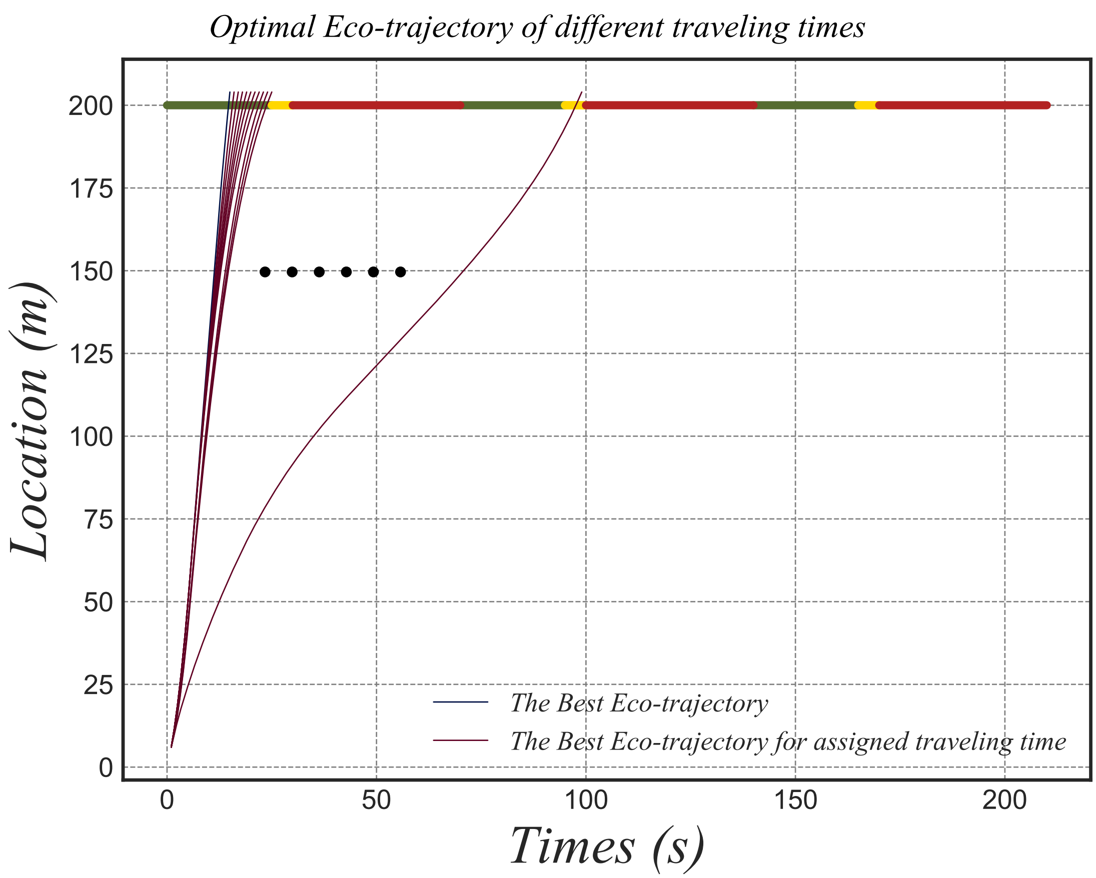


FIGURE 3: An example of optimal trajectory batch

Online module

Upon the entry of a vehicle into the studied network, ideally, the time-varying speed and accelerations of the blue trajectory in Figure 3 can be applied to that vehicle for it to travel through the stop line with the lowest fuel consumption. However, it might not be feasible because it would violate the signal or follow the leading vehicle too closely. Such infeasibility can be examined in two steps:

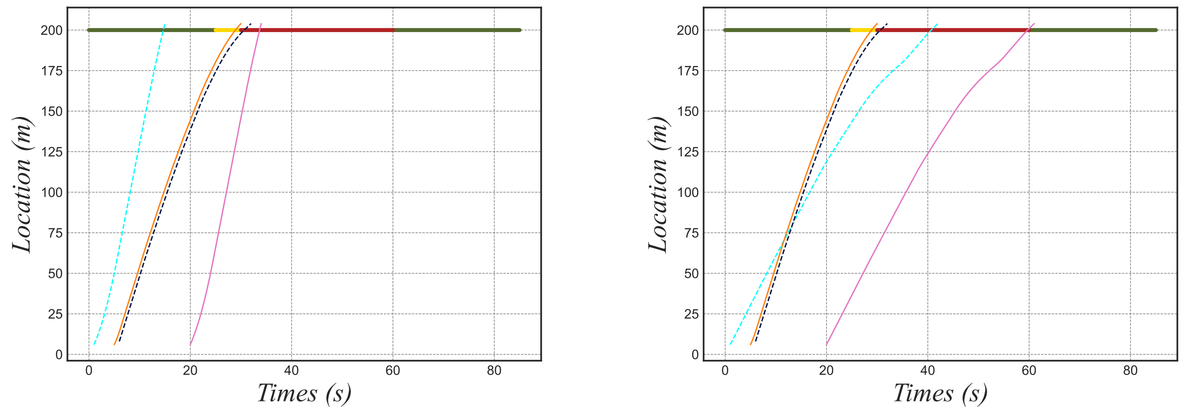
1. translates the selected trajectory in a way such that it starts at the vehicle entry time (See Trajectory translation in Figure 1);
2. check whether the signal constraint and safety following distance constraints are violated

If a translated trajectory violates any of the constraints, the next trajectory in the batch, with slightly higher fuel consumption, will be selected and examined until one trajectory is deemed feasible. Those two steps are detailed below, followed by a summary of the entire procedure.

Figure 4a presents an example of an infeasible translated trajectory, whereas Figure 4b illustrates a feasible example. It is evident that the translated trajectory in Figure 4a adheres to the safe distance constraints but violates the traffic signal constraints. Consequently, an alternative trajectory with a different travel time (i.e., a different arrival time at the stop line) should be chosen from the optimal trajectory batch Π and translated to commence at the entry time of the subject vehicle. As depicted in Figure 4b, a new trajectory selected and translated from Π satisfies both the safe distance constraints and traffic signal constraints, indicating that the iteration can be terminated, as the current translated trajectory is the most energy-efficient and feasible among all in the batch.

The heMPC framework operates within a heuristic trajectory planning scenario where all HDVs (Human-Driven Vehicles) are interconnected and strictly adhere to specific car-following models ((*24*) is adopted in this study), while all CAVs strictly follow predefined trajectories. Additionally, roadside devices have the capability to detect the entry times and speeds of all vehicles. Hence, when planning an eco-trajectory for a particular vehicle, complete knowledge of its front vehicle's trajectory is possessed, represented as the known sequence x_t^b: t $\in \tau$. Given that only the front vehicle's movement needs consideration when planning an eco-trajectory for a given vehicle, this heuristic eco-trajectory planning algorithm is decentralized. Consequently, the model plans eco-trajectories vehicle by vehicle without requiring a computation process involving all vehicles simultaneously, resulting in remarkable computational efficiency. Once a CAV enters the intersection, its entire trajectory is immediately planned and remains fixed, impervious to external influences, ensuring smooth traffic flow and optimizing trajectory optimization.

The above procedure can be summarized by algorithm 2 where the indicator variable ι denotes whether the selected trajectory from the optimal trajectory batch Π satisfies both the safe distance and traffic signal constraints. In the first loop, algorithm 2 will select trajectory from the optimal trajectory batch Π one by one. In the second loop, algorithm 2 will translate the selected trajectory to the entry time of the current vehicle b_i and check the safe distance and traffic signal constraints. Because all generated trajectories in Π start from discrete-time 0, therefore, the algorithm needs to translate trajectory for $t_e^{b_i}$ unit based on the time scale. From the 4th to the 7th row, the algorithm will determine when the current vehicle will leave the intersection if it follows the selected trajectory. And suppose this departure time does not lie in the green or yellow phases, which means the selected trajectory will violate signal constraints. We will set the indicator variable ι to false and select the next trajectory from the sorted optimal trajectory batch



(a) An example of infeasible translating trajectory (b) An example of feasible translating trajectory

FIGURE 4: Translation process in trajectory planning

Π. The 8th row to the 9th row indicates that if the current vehicle is the first vehicle, it will never violate the safe distance constraints because it does not even have a leading vehicle; otherwise, the safe distance constraint will be checked in the 10th row whatever its front vehicle is a CAV or HDV. The 13th row to the 16th row in algorithm 2 ensures the deceleration of CAV will not exceed emergency braking deceleration. From the 17th to the 27th row, algorithm 2 will apply the selected trajectory to the subject vehicle.

In this paper, the predicted HDVs' trajectory is generated by Gipps' car-following model[1], and readers can replace it with any other car-following model.

Optimality analysis of heuristic trajectory planning algorithm

In this section, some potential non-optimal scenarios will be shown and discussed. It is worth noting that discrete modeling itself is bound to cause some optimality losses. The non-optimal scenarios analyzed in this section ignore this optimality loss caused by the framework itself.

In the optimal trajectory batch Π, only the optimal eco-trajectory of specific discrete traveling times has been computed, which means that for every possible traveling time, there is only one candidate trajectory in the batch, and this trajectory is optimal. Suppose that a trajectory Π_{t_i} is selected from the optimal trajectory batch Π with a traveling time t_i. After the translation process, if the translated trajectory violets safe distance or traffic signal constraints, then another trajectory must be selected. However, a non-optimal trajectory with the same traveling time t_i may not violet safe distance or traffic signal constraints. As shown in Figure 5, CAV b and its front vehicle b', a trajectory is planned for vehicle b, the front vehicle's trajectory are know for vehicle b through V2V communication, therefore, $t_e^{b'}$ and $t_d^{b'}$ are known, so does t_e^b. $t_e^{b'} = 20$s, $t_d^{b'} = 70$s and $t_e^b = 25$s. The light purple dash trajectory is the safe distance boundary of the front vehicle b'. By feasibility examination, the optimal eco-trajectories with traveling time from discrete times interval [0, 54]

[1]the detailed algorithms used to predict the HDVs' trajectory can be found in (*23*)

Algorithm 2: Online module of the proposed heMPC framework: selection of the most fuel-efficient feasible trajectory upon the entry of each CAV

Input: $b_i, b_i \in B$; t_0, $t_0 \in \tau$;Π

ι = **True**
for π ***in*** Π **do**
  for i ***in*** $range(t_e^{b_i}, size(\pi))$ **do**
    if $\pi_{i-t_e^{b_i}} > L_s$ **then**
      if i ***not in*** $G \cup Y$ **then**
        ι = **False**
        break
    if $b_i == b_0$ **then**
      break
    if $\pi_{i-t_e^{b_i}} > xp_i^{b_{i-1}} - (l_v + S_{HDV})$ **then**
      ι = **False**
      break
  if t *!= 0* **then**
    if $\pi_{t-t_e^{b_i}} - xp_{t-1}^{b_i} > vp_{t-1}^{b_i} + d_E \times \theta$ **then**
      ι = **False**
      break
  if ι *is* ***True*** **then**
    if $t_0 == 0$ **then**
      $xp_t^{b_i} = 0$
      $vp_t^{b_i} = v_{b_i}^I$
      for t ***in*** $range(t_0 + \theta, T - \theta)$ **do**
        $xp_t^{b_i} = xp_{t-\theta}^{b_i} + vp_{t-\theta}^{b_i} \times \theta$
        $vp_t^{b_i} = \pi_{i-t_e^{b_i}+\theta} - xp_t^{b_i}$
    else
      for t ***in*** $range(t_0, T - \theta)$ **do**
        $xp_t^{b_i} = xp_{t-\theta}^{b_i} + vp_{t-\theta}^{b_i} \times \theta$
        $vp_t^{b_i} = \pi_{i-t_e^{b_i}+\theta} - xp_t^{b_i}$
  else
    $\iota = $ **True**

are all infeasible because they violate the safe distance constraints. Moreover, the light green trajectory with a traveling time of 55s is finally selected after planning. However, a red trajectory that does not violate the safe distance constraints may exist, and its traveling time is 50s. Because this is not the optimal eco-trajectory[2] with traveling time 50s, which means it is not from Π. Therefore, the energy consumption of the red trajectory must be larger than the golden trajectory. However, it may still be better than the light green one because they have different traveling times. Therefore, considering this potential non-optimal scenario, theorem 3 is expressed as follows.

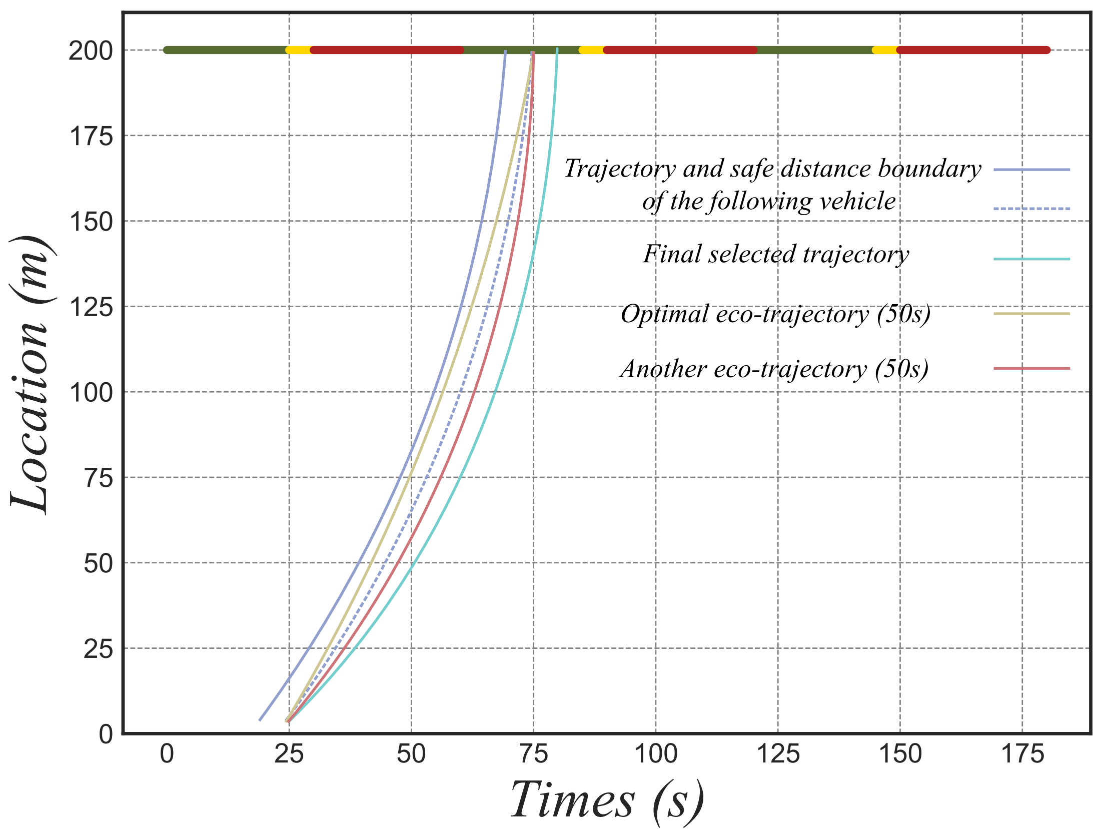


FIGURE 5: An example of the potential non-optimal case under heuristic trajectory planning

Theorem 3 : Under heuristic trajectory planning, for current CAV b and its front vehicle $\dot{b}$ ($\dot{b} \in B_{CAV}$ or B_{HDV}), they trajectory are $\Pi^{\dot{b}}$ and Π^b, respectively. Π^b is obtained by the heuristic planning algorithm, $t_d^{\dot{b}}$ and t_d^b are known. **Inf**$\{M_{E_{\Pi^b}}\} = 0$ and **sup**$\{M_{E_{\Pi^b}}\} = \frac{|E_{\Pi^b} - E_{\Pi^*}|}{E_{\Pi^b}}$, $E_{\Pi^*} = \arg\min_{E_{\Pi_i}} \{E_{\Pi_i} : \Pi_i \in \Pi, i \in [t_d^{\dot{b}} - t_e^b + \theta, t_d^b - t_e^b - \theta], i \notin [0, T_{min}) \cup \{[R_1^b - t_e^b, R_1^e - t_e^b], [R_2^b - t_e^b, R_2^e - t_e^b], \ldots, [R_j^b - t_e^b, R_j^e - t_e^b]\}\}$, $M_{E_{\Pi^b}}$ is the mixed integer programming gap of E_{Π^b} between the current

[2]From theorem 1, the optimal solution must exist but may not be unique. Therefore, this trajectory can also be the optimal eco-trajectory. However, we only add one optimal eco-trajectory of specific traveling time to Π. Therefore, this trajectory is not from Π even in this situation.

solution and the upper bound of the optimal solution value. T_{min} is the minimal traveling time to travel through the intersection. R_j^b and R_j^e are the beginning and ending time of the jth red phase.

Proof. Only an optimal eco-trajectory with a certain discrete time t will be planned by trajectory eco-planning algorithm, $t \in \tau$. Therefore, it is possible for a non-optimal eco-trajectory or another optimal eco-trajectory with a certain discrete time t that does not violate the safe distance constraints but an optimal eco-trajectory with a certain discrete time t does. Suppose we do not select a trajectory with a certain discrete time for vehicle b from the optimal eco-trajectory batch Π. In that case, it is possible to find one more trajectory with a traveling time between $[t_d^b - t_e^b + \theta, t_d^b - t_e^b - \theta]$ that satisfies safe distance constraints and traffic signal constraints. It is also worth noting that CAV cannot travel through the intersection with a traveling time within $[0, T_{min})$ due to speed and acceleration limits and $\{[R_1^b - t_e^b, R_1^e - t_e^b], [R_2^b - t_e^b, R_2^e - t_e^b], ..., [R_j^b - t_e^b, R_j^e - t_e^b]\}$ due to traffic signal constraint. And energy consumption of this trajectory can be infinitely close or equal to $E_{\Pi^*} = \arg\min_{E_{\Pi_i}} \{E_{\Pi_i} : \Pi_i \in \Pi, i \in [t_d^b - t_e^b + \theta, t_d^b - t_e^b - \theta], i \notin [0, T_{min}) \cup \{[R_1^b - t_e^b, R_1^b - t_e^b], [R_2^b - t_e^b, R_2^b - t_e^b], ..., [R_j^b - t_e^b, R_j^b - t_e^b]\}\}$.

NUMERICAL STUDY

This section will test the computational efficiency, energy consumption, and optimality of heuristic trajectory planning under randomly mixed traffic flow with different market penetration rates (MPR). All key parameters used for experiments on an isolated intersection shown in Figure 2 are listed in Table 2, where a_{max} and d_{max} refer to the maximum acceleration and deceleration during normal driving, d_E is the emergency braking deceleration. Each cycle's phase order and time are fixed, 25 seconds, 5 seconds, and 30 seconds for the green, yellow, and red phases. Six levels of MPR ranging between 50% and 100% with 10% intervals are applied. Also, for each MPR, five-group experiments are conducted to eliminate occasionality.

All experiments are conducted on a Thinkpad PC running Windows 11 Pro with a 12th Gen Intel(R) Core(TM) i7-12800H 2.40 GHz processor and 32GB of RAM. All models are implemented using Python, and the simplified EPPs in building the optimal eco-trajectory batch are solved by Gurobi.

Testing computational quality and efficiency for heuristic trajectory planning

Figure 6 demonstrated the optimal trajectory batch with an initial speed of vehicles equal to 6 m/s. In Figure 7, the energy consumption of different trajectories is demonstrated from this batch. The trajectory with a traveling time of 15 seconds is the most energy-efficient in batch Π, which is also the leftmost one in Figure 6.

TABLE 2: Experiment parameters

Parameters	Value	Parameters	Value
v_{min}	0 m/s	v_{max}	16 /s
a_{max}	2 m/s^2	d_{max}	- 2 m/s^2
v_I	6 m/s	L_s	200 m
l_v	4 m	S_{HDV}	1 m
d_E	- 6 m/s^2	C	60 s

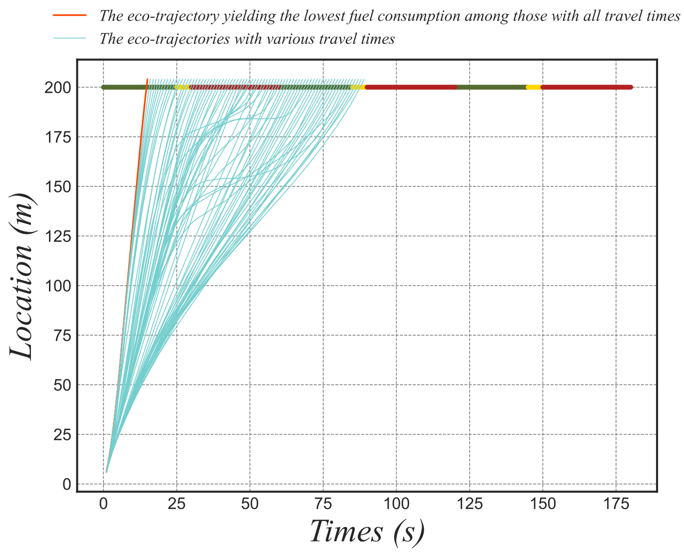


FIGURE 6: optimal trajectory batch with an initial speed of vehicles equal to 6 m/*s*

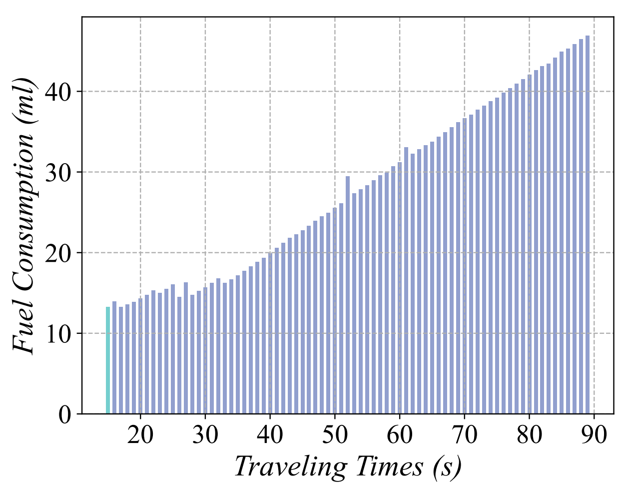


FIGURE 7: Fuel consumption pattern of optimal trajectory batch with an initial speed of vehicles equal to 6 m/*s*

In Figures 8 to 11, the trajectories obtained through heuristic trajectory planning under different MPRs have been depicted. For comparative analysis, the benchmark assumes strict adherence to the classic Gipps car-following model by all HDVs and CAVs. Additionally, Figures 12 to 15 display the benchmark trajectories under varying MPRs.

Upon examining Figures 8 to 15, it becomes evident that the first vehicle unable to travel through the intersection within the current effective green phase tends to decelerate in advance. They maintain a slower but steady speed to navigate the intersection, thereby avoiding any stop during the red phase in heuristic trajectory planning. Conversely, in the benchmark, the vehicle prefers to maintain its original speed and suddenly decelerate just before the stop line. Considering the car-following behavior, as a result, the benchmark exhibits a relatively larger average deceleration and acceleration for all vehicles that are unable to cross the intersection within the current effective green phase. This, in turn, leads to higher fuel consumption and less smooth trajectories.

These findings highlight three key aspects. Firstly, heuristic trajectory planning significantly smoothens the trajectory under different MPRs. Moreover, as shown in Figure 17, fuel consumption decreases by 42.25%, 45.31%, 48.58%, and 46.68% under different MPRs ranging from 50% to 100%, respectively. Notably, heuristic trajectory planning yields greater energy savings at higher MPRs.

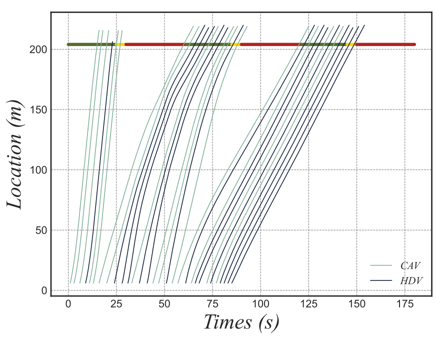


FIGURE 8: *PMR* = 50%

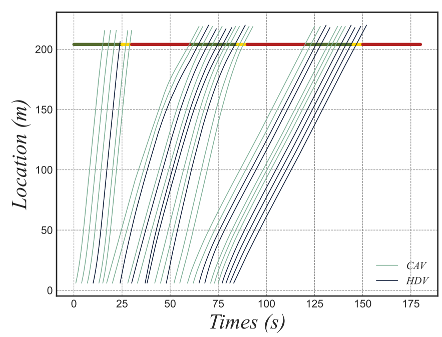


FIGURE 9: *PMR* = 60%

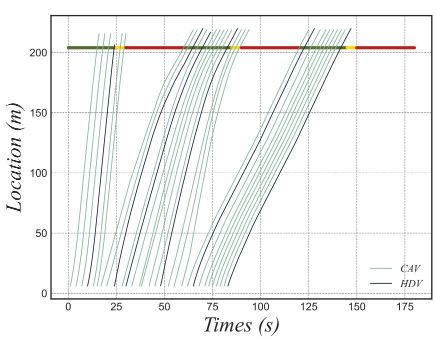


FIGURE 10: *PMR* = 80%

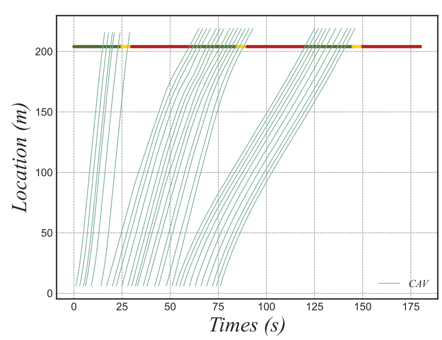


FIGURE 11: *PMR* = 100%

Secondly, Figure 16 demonstrates that the average processing time for heuristic planning to generate a trajectory for a single CAV remains below 1 ms across all MPRs. This level of computational efficiency is exceptionally high compared to almost all current methods. The efficiency stems from the fact that heuristic planning in our heuristic framework does not require the repeated calculation of eco-trajectories for multiple CAVs. Instead, heuristic planning identifies the best feasible solution from a solution set, with most of the optimization and calculation processes performed during the pre-computing phase. The remarkably high computational efficiency of heuristic trajectory planning provides robust support for dynamic trajectory planning.

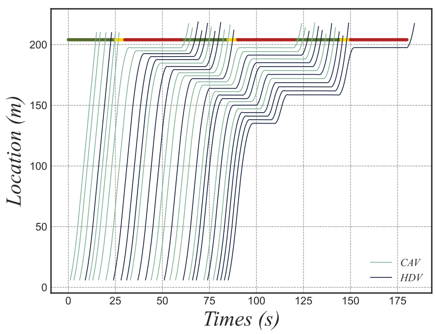


FIGURE 12: Benchmark:MPR = 50%

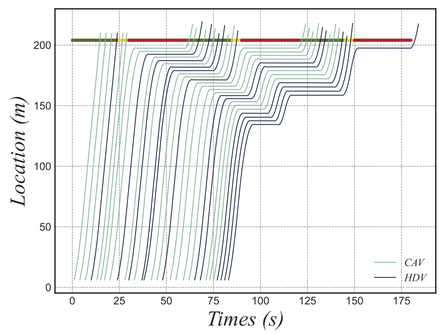


FIGURE 13: Benchmark:MPR = 60%

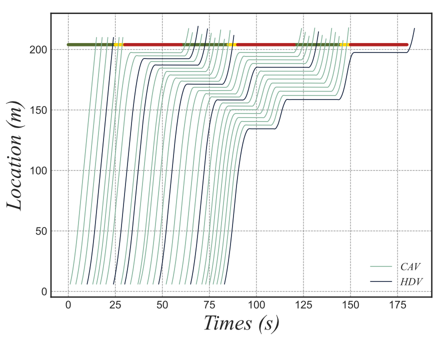


FIGURE 14: Benchmark:MPR = 80%

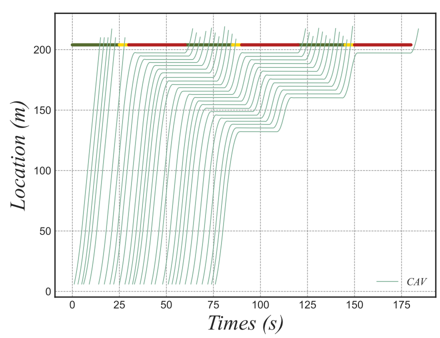


FIGURE 15: Benchmark:MPR = 100%

Thirdly, Figure 17 demonstrates that the difference upper bound between the final solutions and theoretical optimal solutions is remarkably small. Here, the difference upper bound represents the absolute value of the fuel consumption discrepancy between the final solutions and theoretical optimal solutions. This difference upper bound is calculated based on Theorem 3, and it is important to note that exact theoretical optimal solutions are not obtained in our paper. Instead, a lower bound of optimal solutions is proven and provided. Consequently, the actual difference between the final and theoretical optimal solutions is even smaller than our experimental results suggest. The difference upper bound percentages under different MPRs are 5.10%, 3.00%, 1.42%, and 3.12%, respectively. Therefore, based on the findings, it can be concluded that the heMPC framework achieves both good computational quality and extremely high computational efficiency.

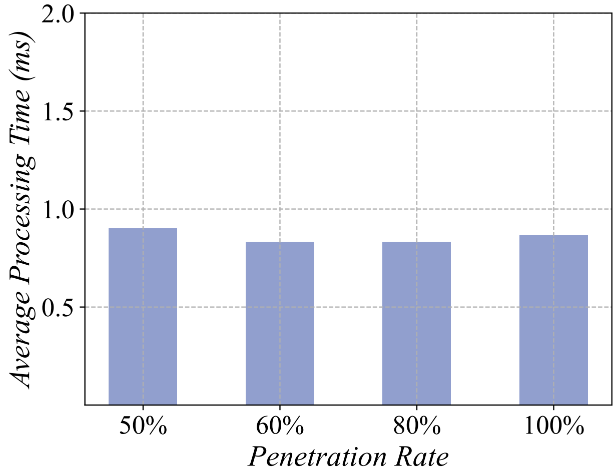


FIGURE 16: Average processing time of heuristic trajectory planning

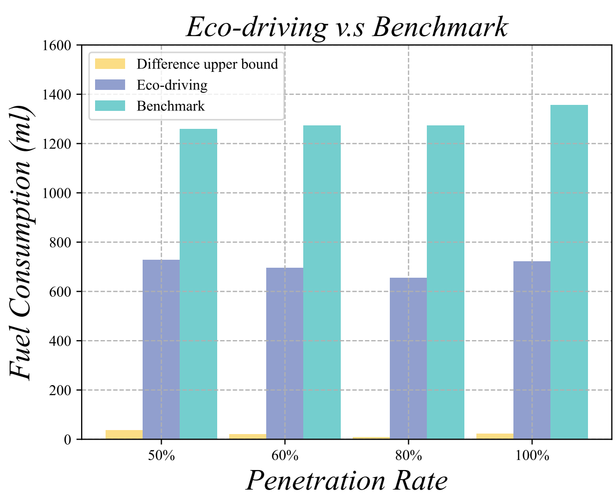


FIGURE 17: Energy consumption analysis

CONCLUSION

TPP is a critical aspect of the trajectory planning problem and holds significant importance in the context of the development and operation of CAVs. Nevertheless, computational efficiency issues under the objective of highly non-linear functions like minimizing fuel consumption and the insufficient capability of responding to signal control and diverse driving behavior in a mixed traffic flow remain major challenges that prevent relevant algorithms from being practically used. This paper proposes a heuristic explicit model predictive control (heMPC) framework that integrates heuristic trajectory planning techniques to tackle these challenges effectively.

The heMPC framework consists of both offline and online modules for expediting the optimization process. In the offline module, an optimal eco-trajectory batch is constructed by solving a sequence of optimization problems, considering various initial and terminal system states. Each candidate trajectory in the batch yields the lowest fuel consumption subject to a specific travel time from the vehicle entry to the departure from the network. Unlike existing TPP studies that rely on an online optimization process from an infinite solution set, the online module of the proposed framework investigates prespecified trajectories in the batch and selects the ones that can ensure the CAV would not violate signals or follow to closely to the leading vehicles. This batch-based selection method significantly enhances computational efficiency.

The case study with a wide range of MRP shows that. in heuristic trajectory planning, computational times for different MPR scenarios consistently remain below 1 ms. This approach yields significant fuel savings of over 40% compared to the benchmark, with a difference upper bound of less than 5.10% compared to theoretically optimal solutions.

The heMPC framework presented in this paper is designed for isolated intersection scenarios with the primary focus of enhancing computational efficiency and can be extended with respect to many aspects. For example, including lane-changing behavior introduces complexities in constructing the eco-trajectory batch and deserves careful investigation. Investigating the integration of deep learning and nonlinear programming (physics-informed neural network) and exploring the combination of reinforcement learning and optimal control techniques would also be beneficial. These approaches hold promise for further enhancing the capabilities and effectiveness of the heMPC framework in addressing challenges associated with lane-changing behavior.

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AUTHOR CONTRIBUTIONS

The authors confirm their contribution to the paper as follows: Concepts and methodology: Yuanzheng Lei, Xianfeng Terry Yang. Experiment design and analysis: Yuanzheng Lei, Yao Cheng, Xianfeng Terry Yang. Writing and revision: Yuanzheng Lei, Yao Cheng. All authors reviewed the results and approved the final version of the manuscript.

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