ATTITUDES TOWARD MATHEMATICS AND GRAPHS INFLUENCE GRAPH REASONING AND SELECTION

Heather Lynn Johnson, Courtney Donovan, Robert Knurek, Kristin Whitmore, and Livvia Bechtold

University of Colorado Denver, USA

We report on a mixed methods study in which we investigated college algebra students' attitudes toward mathematics and graphs in connection to their graph reasoning and graph selection. Students (n=599) completed a fully online survey of their attitudes toward math and graphs in conjunction with a fully online measure of their graph reasoning and selection for dynamic situations. Using structural equation modelling, we explored how students' attitudes might link to their graph reasoning and/or graph selection. We found that more positive attitudes toward mathematics and graphs linked to more quantitative forms of graph reasoning and more accuracy in graph selection.

INTRODUCTION

There is a complex relationship between students' attitudes toward mathematics and their mathematical thinking; it is essential that researchers engage in methods to embrace this complexity (Goldin et al., 2016). Drawing on DiMartino and Zan's (2010, 2011) model, we adopt a multidimensional view of attitudes toward mathematics, encompassing emotional disposition, perceived competence, and view of the subject. To theorize graph reasoning, we draw on the framework from Johnson et al. (2020), which puts forward four forms of reasoning: covariation, variation, motion, and iconic. To draw connections between students' attitudes and their graph reasoning and selection, we use structural equation modelling (SEM). SEM is a high-level statistical technique, in which researchers can demonstrate efficacy of theory-based models that relate different research-based constructs (Kline, 2023).

Our population comprises college algebra students (n=599), across three different U.S. postsecondary institutions. College algebra is a credit bearing course that often serves as a prerequisite to courses such as calculus, and functions and graphs are central to the course content. We investigate the following research question: To what extent does students' attitudes toward mathematics and graphs relate to the forms of their graph reasoning and/or the accuracy of their graph selection?

THEORIZING STUDENTS' ATTITUDES TOWARD MATHEMATICS

DiMartino and Zan (2010, 2011) grounded their perspective on attitude in students' written narratives about their experiences, resulting in three interrelated dimensions: emotional disposition, perceived competence, and view of the subject. Emotional disposition referred to students' like, dislike, or indifference toward mathematics. Students' perceived competence referred to students' perceptions of their mathematical capabilities. Students' view of the subject referred to what mathematics meant for

2024. In T. Evans, O. Marmur, J. Hunter, G. Leach, & J. Jhagroo (Eds.). *Proceedings of the 47th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 105-112). PME.

students. Notably, Di Martino and Zan's perspective emerged from a goal to embrace complexities in students' attitudes, to push back against positive/negative dichotomies in investigations of students' attitudes.

THEORIZING STUDENTS' GRAPH REASONING

The four-form graph reasoning framework from Johnson et al. (2020) distinguished between students' quantitative-based forms of graph reasoning (covariation, variation) and students' physical-based forms of graph reasoning (motion, iconic). The framework was developed to explain students' reasoning when interpreting and sketching graphs representing relationships between attributes in dynamic situations (e.g., a turning Ferris wheel). The covariation and variation constructs were rooted in Thompson's theory of quantitative reasoning (Thompson, 1994; Thompson & Carlson, 2017). In Thompson's theory, a quantity referred to a person's conception of some attribute as being possible to measure. For example, a person could separate the attribute of height from an object itself and conceive of how they might measure the height, even if they did not engage in any actual measuring. With covariation, Johnson et al. (2020) referred to students' reasoning about relationships between attributes, with at least a loose connection between their directions of change (e.g., height increases and decreases, while distance increases). With variation, they referred to students' reasoning about directions of change in a single attribute (e.g., height increases and decreases). With motion, they referred to students' reasoning about observable movements (e.g., Bell & Janvier, 1981; Kerslake, 1977) in the situation (e.g., a graph should show the path of the cart turning around the Ferris wheel). With iconic, they referred to students' reasoning about observable features (e.g., Clement, 1989; Leinhardt, 1990) in the situation (e.g., the Ferris wheel is curved, so my graph should be curved).

METHODS

Our research design is a fully mixed, sequential, quantitative dominant status research design (Leech & Onwuegbuzie, 2009), with qualitative analysis preceding quantitative analysis. For data collection, we employed two fully online instruments, a survey of students' attitudes toward math and graphs (see Bechtold et al., 2022) and a measure of graph reasoning and selection for dynamic situations (MGSRDS) (Donovan et al., accepted; Johnson et al., in press). Both instruments were optimized for access on computers, tablets, and mobile phones. Students (n=599) completed the attitude survey and the MGSRDS concurrently, near the end of their college algebra course. Data collection occurred over three semesters (spring 21, fall 21, spring 22).

Design of the attitude survey

To design our survey of students' attitudes toward mathematics and graphs (Table 1), we drew on Di Martino and Zan's (2010, 2011) conceptualization of attitudes toward math. This survey was an adaptation of a survey that Pepin (2011) administered, including three questions (Q1-Q3). Because we were investigating students' graph reasoning in conjunction with students' attitudes, we decided to also include items specifically connected to graphs (Q4, Q5). To allow students to express multifaceted responses, students were not forced to choose between like/dislike (emotional disposition) or can/cannot (perceived competence). Students responded to the survey with a text entry.

Item
Q1. I like/dislike mathematics because ____
Q2. I can/cannot do mathematics because _____
Q3. Mathematics is ____
Q4. I like/dislike graphs because ____
Q5. I can/cannot make sense of graphs because _____

Table 1: Survey of students' attitudes toward math and graphs

Design of the MGSRDS

The MGSRDS contains six items, with dynamic situation including a turning Ferris wheel, a person walking to a tree a back, a fishbowl filling with water, a cone growing and shrinking, a toy car moving along a square track, and two insects walking back and forth from home. Items appear in random order, and each item has two screens. On the first screen, there is a video animation of a dynamic situation (e.g., a turning Ferris wheel), written description of the attributes in the situation (e.g., In this situation, we will focus on the Ferris wheel cart's height from the ground and total distance travelled.), and a check for understanding. On the second screen, there are written instructions (e.g., Select the graph that best represents a relationship between the Ferris wheel cart's height from the ground and the distance travelled, for one revolution of the Ferris wheel.), and the video repeats. Then there are four graph choices representing relationships between attributes in the situation, and a text box for students to explain their graph choice. We have demonstrated validity for the MGSRDS (Donovan et al., accepted). For more on the design of MGSRDS items, see Johnson et al. (in press).

Coding the attitude survey

We used an interpretive approach to qualitative analysis to address complexities in students' attitudes toward mathematics and graphs. We coded students' attitudes along four categories: positive, mixed, negative, and detached (Table 2). The codes arose from our analysis of students' text responses (see Bechtold et al., 2022; Gardner et al., 2019). To code, we used a mix of machine and human coding. Our team hired a consultant to train a machine learning program based on our coding scheme. For responses receiving less than 70% confidence with machine coding, we brought in human coders (this tended to be about 30% of responses). With human coders, we used consensus coding (Olson et al., 2016); two people coded independently, then met to calibrate their codes, necessitating 100% agreement. After qualitative coding, we transformed the descriptive codes into numerical codes for statistical analysis. Larger values indicated more positive attitudes (0-detached, 1-negative, 2-mixed, 3-positive).

Code	Description	Sample Response
Positive	Like/can	I can do mathematics because I've always been fluent in the 'language' of mathematics.
mixed	Combination of positive/negative	I am inbetween sometimes I do understand graphs and other times I can get very confused.
negative	Dislike/cannot	I dislike graphs because I tend to forget what the rules are to how functions and equations are placed.
detached	Separation from oneself	It's all just following the formulas step by step.

Table 2: Attitude codes, descriptions, and sample responses

Qualitative analysis: coding the MGSRDS

We coded students' graph reasoning based on the four-form graph reasoning framework from Johnson et al. (2020): covariation (COV), variation (VAR), motion (MO), iconic (IC). To account for written responses that indicated limited evidence (LE) of reasoning, we added LE as a fifth code. Table 3 shows codes, descriptions, and sample responses. For the graph reasoning coding, we used only human coders. Again, we used consensus coding, which necessitated 100% intercoder agreement. After qualitative coding, we transformed the descriptive codes into numerical codes for statistical analysis. The values (0-LE, 1-IC, 2-MO, 3-VAR, 4-COV) indicated a hierarchy of graph reasoning (Donovan et al., accepted), with the largest values indicating quantitative graph reasoning (3-VAR, 4-COV).

Code	Description	Sample Response
COV	relationships between directions of change in attributes	Total distance keeps increasing but the height increases then decreases
VAR	directions of change in a single attribute	The height increases, then decreases, and finally increases again
MO	Physical movement of objects in a situation	Shows the motion of the ferris wheel
IC	Physical features of a situation	If you connect the line, it becomes a circle just like the route it made
LE	Limited evidence	Just seems like the answer

Table 3: Graph reasoning codes, descriptions, and sample responses

We coded students' graph selection using a spreadsheet. To guard against bias, we separated students' written explanations of their graph choice from their graph selections. The design of the MGSRDS included graph choices that were correct, partially correct, and incorrect. The partially correct graph choices accurately represented the direction of change in each attribute but did not accurately represent values of each attribute (for more, see Johnson et al., in press). Again, we transformed the descriptive codes into numerical codes for statistical analysis. Larger values indicated more positive attitudes (0=incorrect, 1=partially correct, 2=correct).

Quantitative analysis: SEM

SEM is a statistical technique that examines relationship patterns between variables that are modelled latently (Kline, 2023). Latent variables are preferred because they allow the variance between items to be examined instead of combining items into a mean score; An additional benefit of SEM is the ability to model multiple pathways with multiple dependent variables being tested simultaneously. To use SEM, researchers first develop a theory-based model. Then they determine whether data patterns fit their model. If there is unsatisfactory model fit, researchers modify and/or re-evaluate (Kline, 2023). Because of its complexity, SEM requires larger sample sizes than techniques such as multiple regression models.

The model for this study (Figure 1) includes two independent variables: attitudes towards mathematics and attitudes towards graphs. The independent variables predict two dependent variables, graph selection and graph reasoning. The four directional arrows in Figure 1 show this. To operationalize the constructs of attitudes toward mathematics and attitudes toward graphs, we used students' responses to questions 1, 2, 4, and 5 from the attitude survey (see Table 1). We did this because there was parallel structure in the design of the questions; one question about mathematics and graphs related to the dimensions of emotional disposition (Q1, Q4) and perceived competence (Q2, Q5), respectively. To operationalize the constructs of graph selection and graph reasoning, we used students' text responses explaining their graph reasoning and their graph choices for each of the six MGSRDS items.

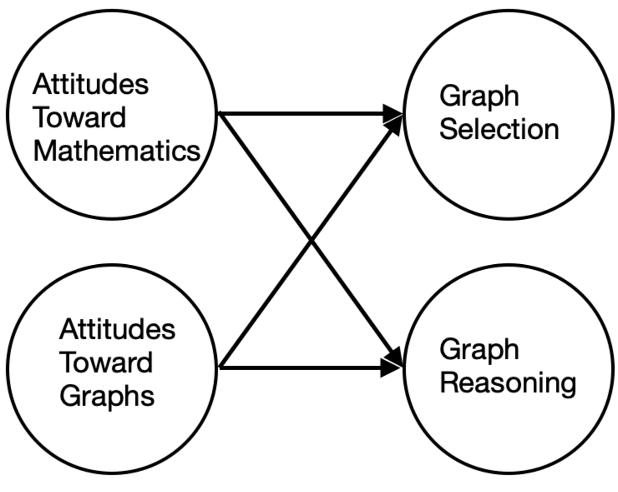

Figure 1: Conceptual model

We use three statistics to assess model fit: the chi-square goodness of fit, the Comparative Fit Index (CFI), the Root Mean Square Error of Approximation (RMSEA. The CFI addresses relative fit, assessing the model in comparison to a null, baseline model comprised of uncorrelated variables. CFI values of 0.90 and above provide sufficient evidence of good fit (Bentler & Bonett, 1980). The RMSEA addresses absolute fit, comparing a model is from an ideal. RMSEA values of 0.08 and below are considered acceptable fit (Browne & Cudeck, 1992). After assessing model fit, then we examine whether the items contributing to latent variables provide evidence of good fit (e.g., whether the six MGSRDS items contribute to the graph reasoning and graph selection constructs, and whether the four attitude survey items contribute to the attitude toward mathematics and attitudes toward graphs constructs). Standard regression weights of 0.30 or above are expected, with higher values indicating stronger contributions (Leech et al., 2014). If some items contribute at values lower than 0.30, they still may be included if they are significant and removing them does not improve the fit of the model.

RESULTS

The conceptual model shown in Figure 1 demonstrated good fit, $\chi 2$ (99) = 154.93, $p < 0.001$, CFI = 0.96, RMSEA = 0.03. All items significantly contributed ($p < 0.01$) to the respective latent variable pathways. For graph reasoning, all six MGSRDS items contributed at values greater than 0.30 (values ranged from 0.58 to 0.77). For graph selection, four MGSRDS items contributed at values greater than 0.30 (values ranged from 0.33 to 0.48). The other two MGSRDS items contributed at values of 0.29 and 0.15. Removing these two items did not improve the model, thus we kept them. For attitudes toward mathematics and attitudes toward graphs, the four attitude survey questions (Q1, Q2, Q4, Q5, see Table 1) contributed at values greater than 0.30 (values ranged from 0.36 to 0.72). Hence, there was statistical support for our model.

All latent variable predictive pathways shown in the conceptual model (Figure 1) are significant. Like standardized regression weights, higher values indicate stronger relationships. Attitudes toward mathematics influences graph selection ($\beta = 0.80$, $p < 0.001$) and graph reasoning ($\beta = 0.64$, $p < 0.001$). Attitudes toward graphs influences graph selection ($\beta = 0.44$, $p = 0.006$) and graph reasoning ($\beta = 0.37$, $p = 0.002$).

DISCUSSION

To begin, we asked: To what extent does students' attitudes toward mathematics and graphs relate to the forms of their graph reasoning and/or the accuracy of their graph selection? We found students' attitudes towards mathematics and attitudes toward graphs to influence their graph reasoning and graph selection. While all relationships were statistically significant, our results demonstrated that students' attitudes toward mathematics more strongly influenced their graph reasoning and graph selection than did students' attitudes toward graphs. Furthermore, the relationship between attitudes toward mathematics was stronger for graph selection than for graph reasoning. This also held for attitudes toward graphs. In all cases, more positive attitudes linked to more quantitative forms of graph reasoning and to more accuracy in graph selection. Furthermore, our results pointed to the interrelationships between the dimensions of emotional disposition and perceived competence within the constructs of attitudes toward mathematics and attitudes toward graphs, underscoring the complexity of the attitude construct posited by Di Martino and Zan (2010, 2011).

To contextualize our results, we discuss some limitations. First, we use students' written responses as proxies for their attitudes toward mathematics, their attitudes toward graphs, and their graph reasoning. Hence, there may be fuller aspects of these constructs not revealed by students' written responses. Second, while students completed the MGSRDS and attitude survey as part of their course, they may have viewed these instruments as "add-ons," and thus may have felt less investment in their responses (see also Johnson et al., in press). Third, our analysis conceptualizes attitudes as comprising only two of the dimensions of attitudes toward mathematics put forward by Di Martino and Zan (emotional disposition and perceived competence). Hence, our design simplifies the construct somewhat.

In conclusion, Goldin et al. (2016) suggested directions for future research to include the development of new instruments and the investigation of adults' attitudes toward mathematics. Our study furthered these research directions. In future studies, researchers could use the instruments we have developed with different populations.

Acknowledgment. This research was funded by the U.S. National Science foundation under DUE-2013186, Division of Undergraduate Education.

References

Bechtold, L., Donovan, C., & Johnson, H. L. (2022). College algebra students' attitudes toward math and graphs: An exploratory factor analysis. In Lischka, A. E., Dyer, E. B., Jones, R. S., Lovett, J. N., Strayer, J., & Drown, S. (Eds.), *Proceedings of the 44th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1630-1634). MTSU. https://doi.org/10.51272/pmena.44.2022

Bell, A., & Janvier, C. (1981). The interpretation of graphs representing situations. *For the Learning of Mathematics*, *2*(1), 34–42. http://www.jstor.org/stable/40240746

Bentler, P. M., & Bonett, D. G. (1980). Significance tests and goodness of fit in the analysis of covariance structures. *Psychological Bulletin*, *88*(3), 588. https://doi.org/10.1037/0033-2909.88.3.588

Browne, M. W., & Cudeck, R. (1992). Alternative Ways of Assessing Model Fit. *Sociological Methods & Research*, *21*(2), 230–258. https://doi.org/10.1177/0049124192021002005

Clement, J. (1989). The concept of variation and misconceptions in cartesian graphing. *Focus on Learning Problems in Mathematics*, *11*(1-2), 77–87. https://eric.ed.gov/?id=EJ389508

Di Martino, P., & Zan, R. (2010). "Me and maths": Towards a definition of attitude grounded on students' narratives. *Journal of Mathematics Teacher Education*, *13*(1), 27–48. https://link.springer.com/article/10.1007/s10857-009-9134-z

Di Martino, P., & Zan, R. (2011). Attitude towards mathematics: a bridge between beliefs and emotions. *ZDM: The International Journal on Mathematics Education*, *43*(4), 471–482. https://doi.org/10.1007/s11858-011-0309-6

Donovan, C., Johnson, H. L., Knurek, R., Whitmore, K. A., & Bechtold, L. (accepted). Validating a measure of graph selection and graph reasoning for dynamic situations. *Journal of Mathematical Behavior.*

Gardner, A., Smith, A, & Johnson, H. L. (2019) Humanizing the coding of college algebra students' attitudes towards math. In Weinberg, A., Moore-Russo, D., Soto, H., & Wawro, M. (Eds.). *Proceedings of the 22nd Annual Conference on Research in Undergrad Math Ed* (pp. 1113-1114). OK. http://sigmaa.maa.org/rume/RUME22_Proceedings.pdf

Goldin, G. A., Hannula, M. S., Heyd-Metzuyanim, E., Jansen, A., Kaasila, R., Lutovac, S., Di Martino, P., Morselli, F., Middleton, J. A., Pantziara, M., & Zhang, Q. (2016). *Attitudes, beliefs, motivation and identity in mathematics education: An overview of the field and future directions*. Springer Nature.

Johnson, H. L., Donovan, C., Knurek, R., Whitmore, K. A., & Bechtold, L. (in press). Proposing and testing a model relating students' graph selection and graph reasoning for dynamic situations. *Educational Studies in Mathematics.*

Johnson, H. L., McClintock, E., & Gardner, A. (2020). Opportunities for reasoning: Digital task design to promote students' conceptions of graphs as representing relationships between quantities. *Digital Experiences in Mathematics Education*, *6*(3), 340–366. https://doi.org/10.1007/s40751-020-00061-9

Kerslake, D. (1977). The understanding of graphs. *Mathematics in School*, *6*(2), 22–25. http://www.jstor.org/stable/30212405

Kline, R. B. (2023). *Principles and Practice of Structural Equation Modeling*. Guilford.

Leech, N. L., Barrett, K. C., & Morgan, G. A. (2014). *IBM SPSS for intermediate statistics: Use and interpretation. (5th ed.)*. Routledge.

Leech, N. L., & Onwuegbuzie, A. J. (2009). A typology of mixed methods research designs. *Quality & Quantity*, *43*(2), 265–275. https://doi.org/10.1007/s11135-007-9105-3

Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research*, *60*(1), 1–64. https://doi.org/10.3102/00346543060001001

Olson, J., McAllister, C., Grinnell, L., Gehrke Walters, K., & Appunn, F. (2016). Applying constant comparative method with multiple investigators and inter-coder reliability. *The Qualitative Report*, *21*(1), 26–42. https://doi.org/10.46743/2160-3715/2016.2447

Thompson, P. W. (1994). The development of the concept of speed and its relationship to concepts of rate. In G. Harel & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 179–234). SUNY Press.

Thompson, P. W., & Carlson, M. P. (2017). Variation, covariation, and functions: Foundational ways of thinking mathematically. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 421–456). National Council of Teachers of Mathematics.