

Combinatorial Inference on the Optimal Assortment in the Multinomial Logit Model

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Assortment optimization has received active explorations in the past few decades due to its practical importance. Despite the extensive literature dealing with optimization algorithms and latent score estimation, uncertainty quantification for the optimal assortment still needs to be explored and is of great practical significance. Instead of estimating and recovering the complete optimal offer set, decision-makers may only be interested in testing whether a given property holds true for the optimal assortment, such as whether they should include several products of interest in the optimal set, or how many categories of products the optimal set should include. This paper proposes a novel inferential framework for testing such properties. We consider the widely adopted multinomial logit (MNL) model, where we assume that each customer will purchase any item within the offer set of products with a probability proportional to the underlying preference scores associated with the product. For a full assortment of products, our objective is to conduct a hypothesis test concerning a general optimal assortment property, given by:

$$H_0 : S^* \in \mathbf{S}_0 \text{ versus } H_1 : S^* \notin \mathbf{S}_0,$$

where S^* denotes the optimal offer set, and $\mathbf{S}_0$ is a set of offer sets satisfying the property of interest. We reduce inferring a general optimal assortment property to quantifying the uncertainty associated with the sign change point detection of the marginal revenue gaps defined as $\Delta_k = \sum_{i=1}^k r_i u_i^* - (\sum_{i=0}^k u_i^*) r_k$, $k = 1, \dots, n$, where $r_1 \geq \dots \geq r_n$ are the revenue parameters associated with the products. By plugging in the Newton-debiased maximum likelihood estimator (MLE) for the latent preference scores, we obtain the marginal revenue gap estimators $\hat{\Delta}_k$'s and show their asymptotic normality. Furthermore, we construct a maximum statistic via the gap estimators to detect the sign change point:

$$T = \max_{k \in [n]} (\hat{\mathbf{b}}_k^2 / \hat{\mathcal{L}})^{-1/2} \hat{\Delta}_k - \Delta_k,$$

where $\hat{\mathbf{b}}_k^2$ is a consistent estimator for the asymptotic variance of $\hat{\Delta}_k$. By approximating the distribution of the maximum statistic with multiplier bootstrap techniques, we propose a valid testing procedure. We also conduct numerical experiments to assess the performance of our method.

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CCS Concepts: • **Computing methodologies**; • **Machine learning**; • **Mathematics of computing**; • **Probability and statistics**; • **Applied computing** → **Operations research**.

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