

Microwave-Induced Cooling in Double Quantum Dots: Achieving Millikelvin Temperatures to Reduce Thermal Noise around Spin Qubits

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Abstract

Spin qubits in gate-defined quantum dots are emerging as a leading technology due to their scalability and long coherence times. However, maintaining these qubits at ultra-low temperatures typically requires complex cryogenic systems. This paper proposes a novel gate-defined double quantum dot (DQD) cooling system using microwave-induced state depopulation and phonon filtering to achieve local temperatures below 10 mK at a bath temperature of 1 K. The system utilizes microwave-induced state depopulation and phonon filtering, combined with fast cyclic detuning of the quantum dot on-site energies and Rabi oscillations, to efficiently transfer thermal populations to the ground state, thereby surpassing natural thermal transition rates. The cooling cycle involves adjusting gate potentials to drive the system through adiabatic and diabatic transitions, complemented by microwave pulses resonant with specific energy level differences. This mechanism continuously pumps population from excited states into the ground state, effectively reducing the system's temperature. Numerical calculations demonstrate the feasibility of achieving these low local temperatures, with detailed analysis showing the sensitivity of cooling performance to detuning energy, magnetic field strength, and diabatic return time.

Introduction

Quantum computing promises to revolutionize various fields by solving problems that are intractable for classical computers. Among the different qubit technologies, spin qubits in gate-defined quantum dots (QDs) have emerged as a leading candidate due to their scalability and relatively long coherence times.¹ These qubits typically operate at sub-100 mK temperatures, which presents a significant challenge for large-scale integration of control electronics and other components.² There is a substantial technological interest in increasing their operational temperature to above 1 K, which would facilitate easier integration and reduce the complexity of cooling systems.³ Achieving this would not only simplify the infrastructure but also enhance the practicality and scalability of quantum computing systems.

Cooling is a critical aspect of quantum computing, particularly for maintaining qubit coherence and minimizing thermal noise. For gate-defined QD spin qubits, operating at higher temperatures while maintaining high fidelity is a significant challenge. Traditional methods such as dilution refrigerators are effective but cumbersome and expensive. Innovations in cooling technologies can thus have profound implications for the development and deployment of quantum computers.

Over the past several decades, various cooling technologies have been developed for different qubit systems. Sympathetic cooling for ion traps involves cooling one species of ions using laser cooling and then transferring the cooling effect to another species through Coulomb interactions.⁴ This technique has been extended to spatially separated traps using superconducting LC circuits, enabling efficient cooling over macroscopic distances.⁵ Sideband cooling for superconducting qubits has been demonstrated, where thermal populations are driven to higher excited states via sideband transitions, followed by relaxation to the ground state. This method has achieved effective temperatures as low as 3 mK for bath temperatures ranging from 30 to 400 mK.⁶ Algorithmic cooling leverages quantum algorithms to transfer entropy from some qubits to others or to the environment, effectively cooling the system. Algorithmic cooling has been shown to exceed Shannon's bound on data compression and has applications in initializing highly pure qubits for quantum computation.^{7,8}

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Recent advancements propose the use of few-qubit quantum refrigerators for cooling multi-qubit systems. These systems typically involve a central qubit coupled to several ancilla qubits, using short interactions to achieve cooling.^{9,10,11} However, these methods are usually applicable for the initialization stage and do not work for computational qubits during operations.^{9,12} Additionally, the number of ancilla qubits required to achieve sub-100 mK temperatures increases rapidly, leading to greater size and cost for quantum technology applications.^{9,13} Therefore, there remains a gap in effectively cooling interacting multi-qubit systems, particularly those based on spin qubits in quantum dots. This work aims to address this gap by exploring a DQD system as an alternative approach for quantum refrigeration. This new method could provide a more efficient and practical solution for cooling at higher operational temperatures using a single DQD, thereby facilitating the integration of control electronics and enhancing the scalability of quantum computing systems.

Microwave-Induced Cooling Mechanism for Double Quantum Dots

We propose leveraging the interplay among state transitions of a double quantum dot, microwave absorption at resonance, and detuning of the potential of the QDs to design a cooling cycle capable of reducing the system temperature to the millikelvin range using hole or electron spins. This cycle exploits the rapid response of hole spin qubits via spin-orbit coupling (SOC) to implement a dual cooling mechanism involving (i) microwave-induced state depopulation and (ii) phonon filtering (Figure 1). By adjusting gate potentials, the system is manipulated to drive the thermal population back to the ground state, surpassing natural thermal transition rates. Additionally, this method uses the energy imbalance induced by microwave absorption to extract energy from specific phonon modes, significantly lowering the system's energy. Repeating this process cyclically can achieve temperatures well below the surrounding environment.

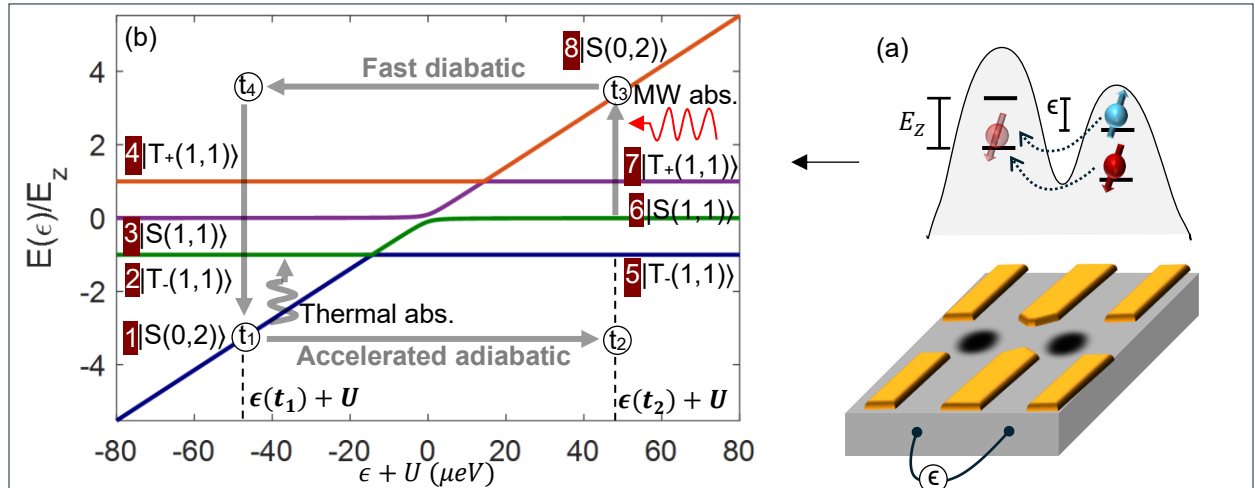


Figure 1: (a) A gate-defined DQD with two HH spins with Zeeman splitting E_Z and level detuning $\epsilon = \epsilon_R - \epsilon_L$. Both the spin-conserving and spin-flipping tunneling are allowed through DQD in the presence of SOC (see the section of theoretical formalism). The level detuning energy is controlled by the gate potential. (b) The lowest energy levels of the Hamiltonian H in the plotted energy range. Here, $\epsilon(t_1)$ and $\epsilon(t_2)$ are the difference between the on-site energies at times t_1 and t_2 , respectively. Initially, thermal excitations transfer population fractions from the spin ground state $|S(0,2)\rangle$ to the excited states $|T-(1,1)\rangle$, $|S(1,1)\rangle$, and $|T+(1,1)\rangle$ according to $p_j = \exp(-E_j/k_B T_0) / \sum_i \exp(-E_i/k_B T_0)$, where p_j is the population fraction at energy level E_j , and T_0 is the bath temperature. To cool below T_0 , energy is cycled between two adjacent QDs by detuning from $\epsilon(t_1)$ to $\epsilon(t_2)$ using an accelerated adiabatic passage, followed by microwave absorption and then quickly returning to $\epsilon(t_1)$ via a fast diabatic path. This process transfers the excited population to the ground state, cooling the DQD. The microwave signal applied to the DQD at time t_2 induces a transition from $|S(1,1)\rangle$ to $|S(0,2)\rangle$. Upon quick returning to $\epsilon(t_1)$, the system remains at $|S(0,2)\rangle$ but at a lower energy, effectively driving the thermal population back to the ground state. This transition must occur faster than the tunneling rate to prevent charge transfer to $|T_{\pm}(1,1)\rangle$ or $|S(1,1)\rangle$.

Initially, the system is at a detuning energy $\epsilon(t)$. The ground state at this point is $|S(0,2)\rangle$, with the first excited state $|T-(1,1)\rangle$, the second excited state $|S(1,1)\rangle$, and the third excited state at $|T+(1,1)\rangle$ (labeled as 1, 2, 3, and 4, respectively, in Figure 1). The system transitions at time t_1 from $\epsilon(t_1)$ to $\epsilon(t_2)$ by tuning the differential potential across the gates. This transition is executed through accelerated adiabatic transitions

over time $t_{12}=t_2-t_1$. Consequently, populations p_1, p_2, p_3 , and p_4 transition to p_5, p_6, p_7 , and p_8 , respectively. At time t_2 , a microwave pulse resonant with the energy difference between states $|6\rangle$ and $|8\rangle$ transitions the p_6 population to p_8 . The pulse duration is adjusted so that the entire p_6 (and p_5 and p_7) population is transferred to p_8 at time t_3 .

At t_3 , the potential across the gates is quickly returned to the starting voltage, setting $\epsilon(t_4) = \epsilon(t_1)$. This transition must be faster than the tunneling rate to ensure the charge remain in the $|S(0,2)\rangle$ configuration. Without charge transfer, there is no population transfer to p_2, p_3 , and p_4 . Consequently, the entire population is transferred to the ground state 1. This process moves the population in the excited states 2, 3, and 4 to the ground state 1, leading to cooling of the DQD system.

The second aspect, microwave-induced phonon filtering, happens at the end of the first cycle. At this time, the DQD is at a lower local temperature compared to the lattice, causing it to absorb phonons to reach equilibrium with the environment. By cycling the DQD back and forth between the two detuning points, more phonons are absorbed, effectively filtering the phonons locally where the DQD is situated. Phonons resonant with the energy difference of states 1 and 2 are expected to be filtered more efficiently. This energy can be adjusted to match the Zeeman energy splitting of a nearby computational spin qubit. Therefore, phonons in resonance with the Zeeman energy splitting of the computational qubit can be filtered around the qubit. Consequently, the cold DQD acts as a filter for specific phonon modes, particularly those resonant with the energy levels of the spin qubit. By selectively interacting with these local phonon modes, the cold DQD reduces the effective phonon noise experienced by the spin qubit, enhancing its dephasing time.

The following sections describe the theory and computational results of the proposed DQD cooler.

Theoretical Formalism

We examine a planar DQD system containing two heavy holes (HHs), as depicted in Figure 1(a). The overall Hamiltonian is expressed as $H = H_{DQD} + H_B + H_{SOC}$ where the first term is defined as:

$$H_{DQD} = \sum_{i=L,R} \sum_{\sigma=\uparrow,\downarrow} \epsilon_i c_{i\sigma}^\dagger c_{i\sigma} + U \sum_{i=L,R} n_{i\uparrow} n_{i\downarrow} - t_N \sum_{\sigma=\uparrow,\downarrow} (c_{L\sigma}^\dagger c_{R\sigma} + Hc)$$

Here, ϵ_i (for $i=L,R$) represents the energy levels of the left and right dots, U is the intradot Coulomb energy (on-site Hubbard interaction), and t_N is the spin-conserving tunneling amplitude between adjacent dots. The operators $c_{i\sigma}$ ($c_{i\sigma}^\dagger$) annihilate (create) a hole with spin σ in dot i , and $n_{i\sigma}$ is holes number operator. A magnetic field B , perpendicular to the plane of the dots, induces a Zeeman splitting. The interaction of the magnetic field with the HHs in the DQD is given by the following Hamiltonian:

$$H_B = \frac{1}{2} g^* \mu_B B \sum_{i=L,R} (n_{i\uparrow} - n_{i\downarrow})$$

where g^* is the effective Landé factor and μ_B is the Bohr magneton. We assume a homogeneous magnetic field and g factor. The spin-orbit coupling (SOC) is described by:

$$H_{SOC} = i\alpha E_\perp (\sigma_+ p_3^- - \sigma_- p_3^+) - \beta (\sigma_+ p_- p_+ p_- + \sigma_- p_+ p_- p_+)$$

The ladder operators are defined as $\sigma_\pm = (\sigma_x \pm i\sigma_y)/\sqrt{2}$, and the momentum operator $p_\pm = p_x \pm ip_y$ is given by $\mathbf{p} = -i\hbar\nabla + \frac{e^*}{c} \mathbf{A}$, where e^* is the effective charge of the hole, c is the speed of light, and $\mathbf{A}$ is the magnetic vector potential. This equation represents the Rashba SOC (α) due to structure inversion asymmetry, controlled by the effective electric field $E_\perp$ from the accumulation gate, and Dresselhaus SOC (β) due to bulk inversion asymmetry.

Since we consider a closed system, it is convenient to write the Hamiltonian matrix in the molecular basis of singlets: $|S(1,1)\rangle \equiv (|\uparrow, \downarrow\rangle - |\downarrow, \uparrow\rangle)/\sqrt{2}$, $|S(0,2)\rangle \equiv |0, \uparrow\downarrow\rangle$ and $|S(2,0)\rangle \equiv |\uparrow\downarrow, 0\rangle$, and triplets $|T_0(1,1)\rangle \equiv (|\uparrow, \downarrow\rangle + |\downarrow, \uparrow\rangle)/\sqrt{2}$, $|T_-(1,1)\rangle \equiv |\downarrow, \downarrow\rangle$, and $|T_+(1,1)\rangle \equiv |\uparrow, \uparrow\rangle$. The nonvanishing spin-flip tunneling matrix elements are between the polarized triplet states and the double- and single-occupied singlet states as:

$$\langle T_\pm(1,1) | H_{SOC} | S(0,2) \rangle = \lambda_2$$

$$\langle T_{\pm}(1,1) | H_{SOC} | S(2,0) \rangle = \lambda_2$$

$$\langle T_{\pm}(1,1) | H_{SOC} | S(1,1) \rangle = \lambda_1$$

where $\lambda_1 = \sqrt{2}S\lambda_2$. The overlap between the wave functions in each dot is defined as $S = \langle L | R \rangle = \exp(-d^2/2l^2)$, where d is the distance between dots and l is the extent of the wave function centered at each dot. The matrix element λ_2 also depends on these parameters.

Under a constant magnetic field B , the unpolarized triplet state $|T_0(1,1)\rangle$ does not interact with any other states. The anticrossing between $|T_+(1,1)\rangle$ and the singlet states occurs at a detuning near $\epsilon + U \approx -2t_N^2/E_Z + E_Z$, where ϵ is the detuning energy controlled by the gate potential. Additionally, if the detuning is sufficiently large ($\epsilon > -U$), the double-occupied singlet state $|S(2,0)\rangle$ is energetically distant and does not play a role.

To obtain the eigenenergies and eigenvectors of the total Hamiltonian H , we construct its matrix in the basis $|T_-(1,1)\rangle, |S(0,2)\rangle, |S(1,1)\rangle, |S(2,0)\rangle, |T_+(1,1)\rangle, |T_0(1,1)\rangle$. Considering that $\epsilon = \epsilon_R - \epsilon_L$ and $\epsilon_R + \epsilon_L = 0$, the action of $H = H_{DQD} + H_B + H_{SOC}$ on these states are given by:

$$\begin{aligned} H|T_0(1,1)\rangle &= H_{SOC}|T_0(1,1)\rangle = 0, \\ H|T_+(1,1)\rangle &= E_Z|T_+(1,1)\rangle + \lambda_2^*(|S(0,2)\rangle + |S(2,0)\rangle) + \lambda_1^*|S(1,1)\rangle, \\ H|T_-(1,1)\rangle &= -E_Z|T_-(1,1)\rangle + \lambda_2^*(|S(0,2)\rangle + |S(2,0)\rangle) + \lambda_1^*|S(1,1)\rangle, \\ H|S(0,2)\rangle &= (\epsilon + U)|S(0,2)\rangle - \sqrt{2}t_N|S(1,1)\rangle + \lambda_2(|T_+(1,1)\rangle + |T_-(1,1)\rangle), \\ H|S(2,0)\rangle &= (-\epsilon + U)|S(2,0)\rangle - \sqrt{2}t_N|S(1,1)\rangle + \lambda_2(|T_+(1,1)\rangle + |T_-(1,1)\rangle), \\ H|S(1,1)\rangle &= -\sqrt{2}t_N(|S(0,2)\rangle + |S(2,0)\rangle) + \lambda_1(|T_+(1,1)\rangle + |T_-(1,1)\rangle), \end{aligned} \quad (1)$$

where we have used:

$$\begin{aligned} \hat{H}_{soc}|T_+(1,1)\rangle &= \lambda_2^*(|S(0,2)\rangle + |S(2,0)\rangle) + \lambda_1^*|S(1,1)\rangle, \\ \hat{H}_{soc}|T_-(1,1)\rangle &= \lambda_2^*(|S(0,2)\rangle + |S(2,0)\rangle) + \lambda_1^*|S(1,1)\rangle, \\ \hat{H}_{soc}|S(0,2)\rangle &= \lambda_2(|T_+(1,1)\rangle + |T_-(1,1)\rangle), \\ \hat{H}_{soc}|S(2,0)\rangle &= \lambda_2(|T_+(1,1)\rangle + |T_-(1,1)\rangle), \\ \hat{H}_{soc}|S(1,1)\rangle &= \lambda_1(|T_+(1,1)\rangle + |T_-(1,1)\rangle), \end{aligned}$$

The vanishing act of the Hamiltonian on the unpolarized state indicates that this state does not interact with the other states. Thus, the total Hamiltonian H is a 5×5 matrix, and is expressed in the basis $(|T_-(1,1)\rangle, |S(0,2)\rangle, |S(1,1)\rangle, |S(2,0)\rangle, |T_+(1,1)\rangle)$ as:

$$H = \begin{bmatrix} -E_Z & \lambda_2 & \lambda_1 & \lambda_2 & 0 \\ \lambda_2^* & \epsilon + U & -\sqrt{2}t_N & 0 & \lambda_2^* \\ \lambda_1^* & -\sqrt{2}t_N & 0 & -\sqrt{2}t_N & \lambda_1^* \\ \lambda_2^* & 0 & -\sqrt{2}t_N & -\epsilon + U & \lambda_2^* \\ 0 & \lambda_2 & \lambda_1 & \lambda_2 & E_Z \end{bmatrix}$$

Here, the Zeeman splitting in each quantum dot is given by $E_Z = g^*\mu_B B$, where g^* is the effective g-factor, μ_B is the Bohr magneton, and B is the magnetic field. The terms λ_1^* and λ_2^* are the complex conjugates of λ_1 and λ_2 , respectively. The spin-orbit coupling terms represented by λ_1 and λ_2 capture the spin-flip tunneling processes between the states. The diagonal terms account for the energy contributions from the Zeeman effect and the detuning energies, while the off-diagonal elements represent tunneling and SOC.

For sufficiently small values of SOC λ_1 and λ_2 , as indicated by the relations in equation (1), the states $|T_{\pm}(1,1)\rangle$ are the eigenstates of the Hamiltonian H , with the eigenenergies $\pm E_Z$. These eigenenergies are depicted in Figure 1 with the horizontal orange-purple and green-blue lines at $E(\epsilon)/E_Z = \pm 1$. Moreover, for very small values of spin-conserved tunneling energy, the singlet states are also eigenstates of the system: the doubly occupied singlet states $|S(0,2)\rangle$ and $|S(2,0)\rangle$ with eigenenergies $\epsilon + U$ and $-\epsilon + U$, respectively,

and the state $|S(1,1)\rangle$ with a vanishing eigenenergy. In Figure 1, the eigenstate $|S(0,2)\rangle$ is represented by the off-diagonal blue-green-purple-orange line with $E(\epsilon)/E_z = (\epsilon + U)/E_z$, and the eigenstate $|S(1,1)\rangle$ is represented by the purple-green horizontal line at $E_z = 0$. The eigenenergy $-\epsilon + U$ is a diagonal line that is distant from the other eigenenergies and does not appear in our working energy window.

By turning the SOCs and spin-conserved tunneling energy, states repulsions make deformations in the energy spectrum. These deformations are seen as three avoided level crossing points, one with λ_2 -dependent energy gap appeared at $\epsilon + U = 0$, and two with t_N -dependent energy gaps appeared around the $\epsilon + U = 0$ point, symmetrically.

In the absence of the SOCs, the total Hamiltonian is block diagonal and the diagonalization of the following Hamiltonian

$$H_{3 \times 3} = \begin{bmatrix} \epsilon + U & -\sqrt{2}t_N & 0 \\ -\sqrt{2}t_N & 0 & -\sqrt{2}t_N \\ 0 & -\sqrt{2}t_N & -\epsilon + U \end{bmatrix},$$

gives the eigenstates of the system. The eigenenergies are obtained as:

$$E_0 = \frac{2U}{3} - \frac{U^2 + 3\epsilon^2 + 12t_N^2}{3\left(x_1 + \frac{\sqrt{x_2}}{2}\right)^{\frac{1}{3}}} - \frac{1}{3}\left(x_1 + \frac{\sqrt{x_2}}{2}\right)^{\frac{1}{3}},$$

$$E_{\pm} = \frac{2U}{3} - \frac{U^2 + 3\epsilon^2 + 12t_N^2}{\frac{3}{2}(-1 \pm \sqrt{3}i)\left(x_1 + \frac{\sqrt{x_2}}{2}\right)^{\frac{1}{3}}} - \frac{-1 \pm \sqrt{3}i}{6}\left(x_1 + \frac{\sqrt{x_2}}{2}\right)^{\frac{1}{3}},$$

with $E_0 < E_- < E_+$. The corresponding eignestates are given in terms of the singlet states as:

$$|\Phi_0\rangle = \frac{1}{\sqrt{\left(-1 + \left(\frac{E_0}{\sqrt{2}t_N}\right)^2 + \frac{E_0(\epsilon - U)}{2t_N^2}\right)^2 + \left(\frac{E_0 + \epsilon - U}{\sqrt{2}t_N}\right)^2 + 1}} \left(\left(-1 + \left(\frac{E_0}{\sqrt{2}t_N}\right)^2 + \frac{E_0(\epsilon - U)}{2t_N^2}\right) |S(0,2)\rangle - \left(\frac{E_0 + \epsilon - U}{\sqrt{2}t_N}\right) |S(1,1)\rangle + |S(2,0)\rangle \right),$$

$$|\Phi_{\pm}\rangle = \frac{1}{\sqrt{\left(-1 + \left(\frac{E_{\pm}}{\sqrt{2}t_N}\right)^2 + \frac{E_{\pm}(\epsilon - U)}{2t_N^2}\right)^2 + \left(\frac{E_{\pm} + \epsilon - U}{\sqrt{2}t_N}\right)^2 + 1}} \left(\left(-1 + \left(\frac{E_{\pm}}{\sqrt{2}t_N}\right)^2 + \frac{E_{\pm}(\epsilon - U)}{2t_N^2}\right) |S(0,2)\rangle + \left(\frac{-E_{\pm} - \epsilon + U}{\sqrt{2}t_N}\right) |S(1,1)\rangle + |S(2,0)\rangle \right),$$

Here, $x_1 = U(U^2 + 18t_N^2 - 9\epsilon^2)$, and $x_2 = -4(U^2 + 3\epsilon^2 + 12t_N^2)^3 + 4x_1^2$.

To achieve an adiabatic transfer from the triplet state $|T-(1,1)\rangle$ to the singlet state $|S(1,1)\rangle$ in a DQD system, the procedure involves designing a detuning pulse that allows the system to evolve along the instantaneous eigenstate. This process is detailed in reference [14]. The goal is to transition from $|T-(1,1)\rangle$ to $|S(1,1)\rangle$ by manipulating the detuning $\epsilon(t)$ between the quantum dots. The accelerated quasi-adiabatic protocol has been proposed to achieve high-fidelity state transfer [14]. The protocol adjusts the detuning $\epsilon(t)$ dynamically to maintain adiabatic conditions, minimizing diabatic transitions. In brief, the adiabatic condition for an N -level system is:

$$C = \hbar \sum_{k \neq i}^N \left| \frac{\langle \phi_i(t) | \partial_t H(t) | \phi_k(t) \rangle}{[E_i(t) - E_k(t)]^2} \right|$$

where $|\phi_k(t)\rangle$ are the instantaneous eigenstates and $E_k(t)$ the corresponding eigenenergies, and C is a dimensionless constant. By solving the differential equation:

$$\dot{\epsilon} = \frac{C}{\hbar} \sum_{k \neq i}^N \left| \frac{[E_i(\epsilon) - E_k(\epsilon)]^2}{\langle \phi_i(\epsilon) | \partial_{\epsilon} H(t) | \phi_k(\epsilon) \rangle} \right|$$

the time-dependent driving parameter $\epsilon(t)$ is obtained, ensuring a constant value of C during the transfer. The fidelity of the protocol is defined as $\mathcal{F} \equiv |\langle S(1,1) | \Psi(t_f) \rangle|^2$, where $|\Psi(t_f)\rangle$ is the final state achieved subsequent of the accelerated adiabatic transfer. The protocol shows an oscillatory behavior of fidelity, tending asymptotically to unity as the total time t_f increases. The boundary conditions for the detuning $\epsilon(t_1)$ and $\epsilon(t_2)$ significantly affect the transfer fidelity. Higher fidelities are achieved by increasing the detuning range $\Delta\epsilon = \epsilon(t_2) - \epsilon(t_1)$ [14].

Thermal and Microwave Excitations

The four states at $\epsilon(t_1) + U$ are in constant interaction with phonons, leading to thermodynamic equilibrium at the bath temperature T_0 . The population fraction rate equations in matrix form can be expressed as:

$$\frac{d}{dt} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix} = \begin{bmatrix} -W_{12} - W_{13} - W_{14} & W_{21} & W_{31} & W_{41} \\ W_{12} & -W_{21} - W_{23} - W_{24} & W_{32} & W_{42} \\ W_{13} & W_{23} & -W_{31} - W_{32} - W_{34} & W_{43} \\ W_{14} & W_{24} & W_{34} & -W_{41} - W_{42} - W_{43} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}$$

where W_{ij} and W_{ji} are the spontaneous absorption and emission, respectively, between states i and j at temperature T . W_{ij} can be estimated using thermodynamic relations $W_{ij} = W_0 n(E_{ij})$ and $W_{ji} = W_0 (n(E_{ij}) + 1)$. $n(E_{ij})$ represents the occupation number of phonons with energy $E_{ij} = E_j - E_i$ and follows the Bose-Einstein distribution at temperature T . The negative signs behind the diagonal matrix elements are an indication of population reduction due to transition. The above set of rate equations is formulated and solved at times t_1 and t_4 among states $|1\rangle$, $|2\rangle$, $|3\rangle$, and $|4\rangle$.

During the interval from t_3 to t_4 , between states $|6\rangle$ and $|8\rangle$, there is coherent interaction with the microwave, causing Rabi oscillations. The Rabi oscillations are characterized by: $p_6(t) = \cos^2(\Omega t/2)$ and $p_8(t) = \sin^2(\Omega t/2)$, where Ω is the Rabi frequency. Their derivatives are: $dp_6(t)/dt = -\Omega/2 \sin(\Omega t)$ and $dp_8(t)/dt = \Omega/2 \sin(\Omega t)$. There are also spontaneous transitions among all these states, characterized by W_{ij} . Therefore, the population fraction rate equations in matrix form can be written as:

$$\frac{d}{dt} \begin{bmatrix} p_5 \\ p_6 \\ p_7 \\ p_8 \end{bmatrix} = \begin{bmatrix} -W_{56} - W_{57} - W_{58} & W_{65} & W_{75} & W_{85} \\ W_{56} & -W_{65} - W_{67} - W_{68} & W_{76} & W_{86} \\ W_{57} & W_{67} & -W_{75} - W_{76} - W_{78} & W_{87} \\ W_{58} & W_{68} & W_{78} & -W_{85} - W_{86} - W_{87} \end{bmatrix} \begin{bmatrix} p_5 \\ p_6 \\ p_7 \\ p_8 \end{bmatrix} + \begin{bmatrix} 0 \\ -\Omega/2 \sin(\Omega t) \\ 0 \\ \Omega/2 \sin(\Omega t) \end{bmatrix}$$

The negative sign in the second term indicates a trade-off between the population of the states 6 and 8 due to Rabi oscillations, i.e., following an increase in the population of 8, a reduction in the population of the state 6 occurs.

Numerical Results

For the numerical calculations, we assume the following parameters: Rabi frequency $\Omega=2$ MHz, $g^*=0.3$ (the effective g -factor of HHs in Ge), accelerated adiabatic transition time $t_{12}=1$ μ s, microwave radiation time $t_{23}=5.25$ μ s, quantum dot separation $d=50$ nm, the extent of the wave function $l=25$ nm, effective mass $m^*=0.36m_0$, $\mu_B = 2.58 \times 10^{-23} J/T$ (corresponding to the density of states hole effective mass in Ge), detuning energy $\epsilon(t_2) = 55.8$ μ eV $- U$, $U = 2$ meV, and spin-conserving tunneling amplitude $t_N=1$ μ eV. The spin-flip tunneling matrix element $\lambda_2=0.1$ μ eV, and the corresponding value between single-occupied states $|T_{\pm}(1,1)\rangle$ and $|S(1,1)\rangle$, λ_1 , is 0.02 μ eV, calculated as $\lambda_1 = \sqrt{2} \exp(-d^2/2l^2) \lambda_2$. W_0 is chosen as 50 kHz to fit the empirical value of the hole spin relaxation time T_1 . $W_0 = 5$ kHz results in a hole spin relaxation time of 200 μ s at 20 mK between the excited state $|T-(1,1)\rangle$ and ground state $|S(0,2)\rangle$ at $\epsilon(t_1) + U = -65$ μ eV. At this detuning energy, the energy splitting is 50 μ eV corresponding to typical spin qubits. For consistency, W_0 is kept constant for all W_{ij} , and they only differ from each other through $n(E_{ij})$ term.

Figure 1b is plotted assuming these parameter values and $B=0.3$ T. However, in the plots presented in this section, wherever the energy levels change with variables such as the magnetic field and tunneling matrix elements, the energy levels are calculated accordingly for consistency. The equivalent temperature T at time $t=t_4$ is calculated by solving this equation for T :

$$\frac{1}{Z(t_4)} \sum_{i=1,2,3,4} E_i(t_4) \exp(-E_i(t_4)/k_B T) = \sum_{i=1,2,3,4} p_i(t_4) E_i(t_4)$$

where $Z(t_4) = \sum_{j=1,2,3,4} \exp(-E_j(t_4)/k_B T)$.

Figure 2 illustrates the transient population dynamics of states $p_5(t)$, $p_6(t)$, $p_7(t)$, and $p_8(t)$ starting at time t_2 with an initial temperature of 1 K during the first cycle of microwave radiation. The Rabi frequency is assumed to be 2 MHz. At the outset, the population in state $p_8(t_2)$ is much lower than the other states due to its higher energy. Microwave radiation, resonant with the energy difference between states $|6\rangle$ and $|8\rangle$, induces Rabi oscillations, causing the population to shift from $|6\rangle$ to $|8\rangle$. Concurrently, population transfer occurs from $|5\rangle$ to $|6\rangle$ via spontaneous absorption and from $|7\rangle$ to $|6\rangle$ via spontaneous emission. These interactions continuously pump the population from $|5\rangle$ and $|7\rangle$ into $|6\rangle$ and from $|6\rangle$ to $|8\rangle$. As Rabi oscillations proceed, states $|5\rangle$ and $|7\rangle$ become completely depopulated, and the population oscillates between $|6\rangle$ and $|8\rangle$ in resonance with the microwave radiation. This dynamic effectively drives the system toward the desired cooling state by manipulating the populations of the energy levels through controlled microwave interactions.

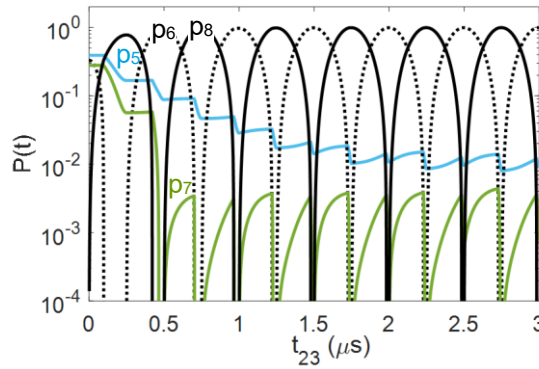


Figure 2: Transient population dynamics of states p_5 (solid blue), p_6 (dotted black), p_7 (solid green), and p_8 (solid black) starting at time t_2 with an initial temperature of 1 K during the first cycle of microwave radiation. The Rabi frequency is assumed to be 2 MHz. At the outset, the population in state $p_8(t_2)$ is much lower than in the other states due to its higher energy. Microwave radiation, resonant with the energy difference between states $|6\rangle$ and $|8\rangle$, induces Rabi oscillations, causing the population to shift from $|6\rangle$ to $|8\rangle$. Concurrently, population transfer occurs from $|5\rangle$ to $|6\rangle$ via spontaneous absorption and from $|7\rangle$ to $|6\rangle$ via spontaneous emission. These interactions continuously pump the population from $|5\rangle$ and $|7\rangle$ into $|6\rangle$ and from $|6\rangle$ to $|8\rangle$. As the Rabi oscillations proceed, states $|5\rangle$ and $|7\rangle$ become completely depopulated, and the population oscillates between $|6\rangle$ and $|8\rangle$ in resonance with the microwave radiation.

Figure 3 provides an analysis of the system temperature and the normalized populations of the ground state (p_1) and first excited state (p_2) as functions of the detuning energy $\epsilon(t_1) + U$. Panel (a) illustrates the system temperature at $B=0.3$ T for three different bath temperatures: $T_0=1$ K (solid black line), $T_0=4$ K (solid blue line), and $T_0=10$ K (solid red line). At $\epsilon(t_1) + U \approx 20$ μeV and $t_{34}=10$ ns, the system temperature reaches 10 mK for $T_0=1$ K. As the detuning potential is further reduced, the temperature approaches 2 mK. When t_{34} increases from 10 ns to 1 μs , the system temperature rises to 18 mK. For bath temperatures of $T_0=4$ K and $T_0=10$ K with $t_{34}=10$ ns, the system temperature increases to 15 mK and 26 mK, respectively. Panel (b) shows the normalized populations of the ground state (p_1 in black) and the first excited state (p_2 in blue) relative to their equilibrium values at steady state. As the system temperature decreases, the population of the ground state (p_1) increases, while the population of the excited state (p_2) decreases. This analysis emphasizes the sensitivity of the system temperature to the detuning energy and the diabatic return time, t_{34} , under varying bath temperatures.

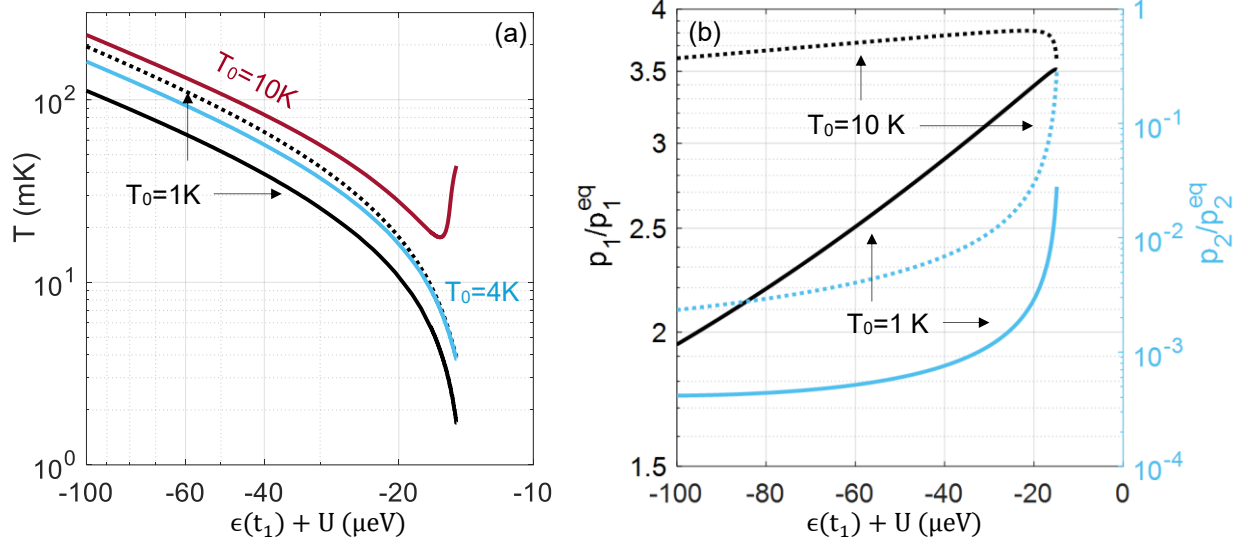


Figure 3: (a) System temperature and (b) normalized population of the ground state p_1 (blue) and the first excited state p_2 (black) at 1 K (solid line) and 10 K (dotted line) with respect to their equilibrium values at steady state versus detuning energy $\epsilon(t_1) + U$. In panel (a), the results are plotted for $B=0.3\text{ T}$ at three bath temperatures: $T_0=1\text{ K}$ (solid black), $T_0=4\text{ K}$ (solid blue), and $T_0=10\text{ K}$ (solid red). The system temperature at $\epsilon(t_1) + U \approx -20\text{ }\mu\text{eV}$ and $t_{34}=10\text{ ns}$ reaches 10 mK for $T_0=1\text{ K}$ and approaches 2 mK as the detuning potential is further reduced. This temperature increases to 18 mK when t_{34} rises from 10 ns to $1\text{ }\mu\text{s}$ (dotted black line). For $T_0=4\text{ K}$ and 10 K , with $t_{34}=10\text{ ns}$, the system temperature increases to 15 mK and 26 mK, respectively. Panel (b) shows that as the temperature decreases, the ground state population increases while the excited state population decreases.

Figure 4 provides a detailed examination of the system temperature's dependency on the fast diabatic return time (t_{34}) and the dwelling time (t_{dwell}), which is the duration which the system interacts with phonons before initiating the next cooling cycle. These results highlight the critical role of the fast diabatic return time and the dwelling time in achieving and maintaining effective cooling in DQD systems. Both panels (a) and (b) assume a bath temperature (T_0) of 1 K and a detuning energy ($\epsilon(t_1) + U$) of approximately $-20\text{ }\mu\text{eV}$. Panel (a) illustrates the system temperature as a function of the fast diabatic return time (t_{34}) for two different dwelling times: $t_{\text{dwell}}=10\text{ ns}$ (solid line) and $t_{\text{dwell}}=1\text{ }\mu\text{s}$ (dotted line). When $t_{\text{dwell}}=10\text{ ns}$, the system temperature remains between 10 mK and 20 mK over a wide range of t_{34} from 1 ns to $20\text{ }\mu\text{s}$. However, for $t_{\text{dwell}}=1\text{ }\mu\text{s}$, the system temperature increases to a range of 40 mK to 50 mK over a similar range of t_{34} . This indicates that longer dwelling times can lead to higher system temperatures, highlighting the importance of optimizing t_{dwell} for effective cooling. Panel (b) depicts the system temperature as a function of the dwelling time (t_{dwell}) for two different fast diabatic return times: $t_{34}=10\text{ ns}$ (solid line) and $t_{34}=1\text{ }\mu\text{s}$ (dotted line). The temperature increases with t_{dwell} , as expected, and saturates at approximately 1 K (the bath temperature) when t_{dwell} approaches $100\text{ }\mu\text{s}$. In practice, the dwelling time should be optimized to be short enough to maintain a low system temperature, yet long enough to allow the spins to interact and effectively filter phonons through absorption.

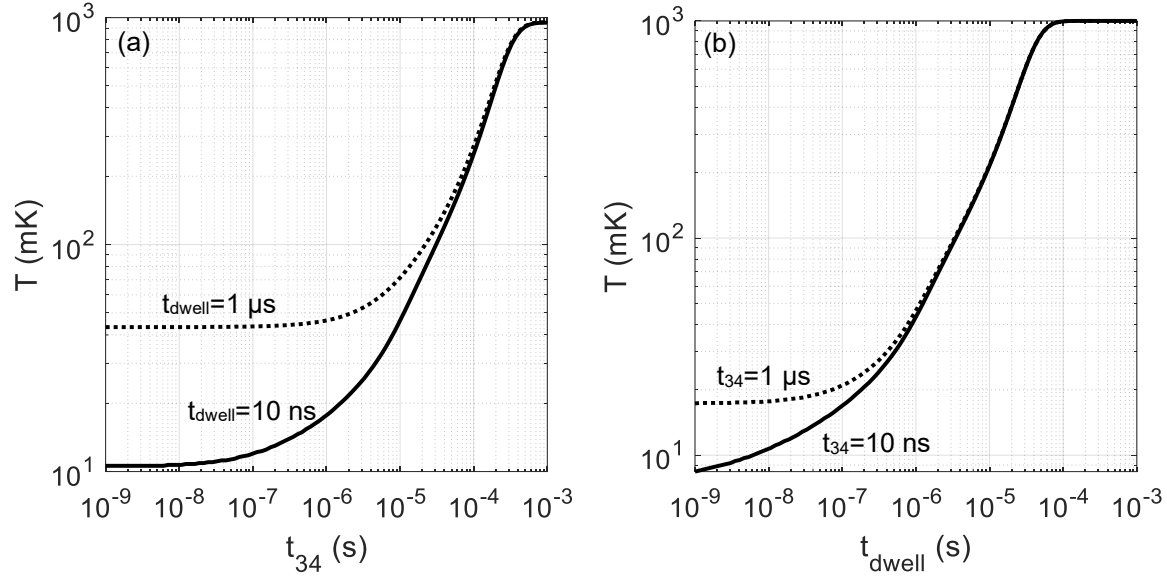


Figure 4: (a) System temperature versus fast diabatic return time t_{34} and (b) system temperature versus dwelling time t_{dwell} , which is the duration when the system interacts with phonons before starting the next cooling cycle. (a) shows the results for $t_{\text{dwell}} = 10$ ns (solid line) and $t_{\text{dwell}} = 1$ μs (dotted line). The system temperature for $t_{\text{dwell}} = 10$ ns ranges between 10 mK and 20 mK over a wide range of fast diabatic return times from 1 ns to 20 μs. For $t_{\text{dwell}} = 1$ μs, the system temperature increases to between 40 mK and 50 mK over a similar range of t_{34} . (b) shows the results for $t_{34} = 10$ ns (solid line) and $t_{34} = 1$ μs (dotted line). The temperature increases with t_{dwell} as expected, saturating at approximately 1 K when $t_{\text{dwell}} \approx 100$ μs. Both panels (a) and (b) assume $T_0 = 1$ K and $\epsilon(t_1) + U \approx -20$ μeV.

Clarifying Energy Conservation During Gate Voltage Transitions

The key issue with energy conservation in this system is understanding where the energy goes when the gate voltage transitions. It might initially be thought that the transition releases energy in the form of photons or phonons. However, during the transition from $|S(0,2)\rangle$ at $\epsilon(t_2)$ to $|S(0,2)\rangle$ at $\epsilon(t_1)$, there is no charge transfer. Given the lack of charge transfer between the quantum dots, these hypotheses are incorrect.

The correct explanation is that the energy is conserved by being returned to the power supply. The gate and the QD can be modeled as a capacitor. The power supply provides energy to increase the electrostatic energy of the holes when the gate voltage transitions the system from $\epsilon(t_1)$ to $\epsilon(t_2)$. This increase is more pronounced for the $|S(0,2)\rangle$ state, as shown in Figure 1b. The energy is returned to the power supply when the gate voltage switches back to negative.

Additionally, microwave absorption during time t_{23} adds energy to the system by increasing the population fraction in the $|S(0,2)\rangle$ state. Consequently, when the voltage switches back to negative, more energy is returned to the power supply than was initially provided, resulting in a lower total energy of the system. This work, done by the power supply and microwave radiation, is equal to $\sum_{i=1,2,6} [p_i(t_1) - p_i(t_4)]$.

Furthermore, one might consider that changes in the wave function distribution in the QD due to voltage changes could lead to photon radiation. However, this contribution depends on the rate at which the gate voltage changes. Since the energy difference between the two states is fixed, the released energy does not depend on the rate of gate voltage change and therefore cannot account for the observed energy conservation.

Practical Considerations

For practical implementation of the accelerated adiabatic transition, the ideal pulse shape derived from the protocol can be discretized into a series of linear ramps to mimic experimentally feasible pulses.

The second consideration is for microwave excitation at time t_2 . At a detuning of $\epsilon(t_2) + U = 55.8$ μeV and $B = 0.3$ T, the energy difference between $|S(1,1)\rangle$ and $|S(0,2)\rangle$ is 41 μeV, requiring

a microwave excitation frequency of 10 GHz for the $|S(1,1)\rangle$ to $|S(0,2)\rangle$ transition. Increasing the detuning energy at time t_2 results in a higher energy difference. Therefore, the system can be designed by adjusting $\epsilon(t_2)$ to match the available microwave sources in the laboratory. Other parameters, such as the magnetic field and geometrical dimensions of the dots, would also affect the required microwave frequency. These adjustments are crucial for ensuring the efficient operation of the cooling loop within the experimental setup.

Another practical consideration is the sensitivity of the energy levels to the DQD parameters. Key parameters include λ_1 , λ_2 , t_N , g^* , and μ_B . The energy levels are significantly more sensitive to the values g^* , t_N , and μ_B than λ_1 and λ_2 . The dots' separation d is usually a multiple of the extent of the wavefunction l . Considering $d = 2l$, the overlap parameter is $S \sim 0.14$, and $\lambda_1/\lambda_2 \sim 0.2$. This indicates that the transition between different charge configurations, $|T_{\pm}(1,1)\rangle$ and $|S(0,2)\rangle$, due to SOC is more probable compared to the transition between $|T_{\pm}(1,1)\rangle$ and $|S(1,1)\rangle$. For materials with strong SOC, like GaAs, this ratio is about 10^{-2} , and without loss of generality, one can ignore the effects of the spin-flip tunneling matrix element λ_1 .

As an illustration, Figure 5 demonstrates how the system temperature varies with λ_2 for two different values of λ_1 : $0.015\lambda_2$ and $0.2\lambda_2$. The system temperature remains relatively stable across a wide range of λ_2 . However, the temperature increases significantly when λ_2 exceeds the spin-conserved tunneling energy t_N , which can occur under very large SOC.

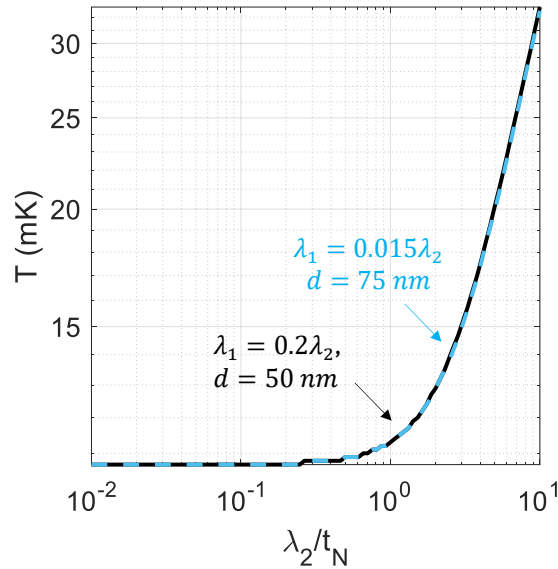


Figure 5: System temperature as a function of λ_2 for two different values of $\lambda_1 = 0.2\lambda_2$ and $0.015\lambda_2$. Assuming the extension of the wavefunction l is 25 nm, the corresponding distances between the QDs d are 50 nm and 75 nm. The system temperature remains relatively stable over a wide range of λ_2 . However, at very high values of λ_2 , where spin-flip tunneling exceeds spin-conserved tunneling, the system temperature increases with λ_2 . The system temperature shows little sensitivity to changes in λ_1 .

Another practical consideration is the importance of the accelerated adiabatic transition in achieving the predicted low temperatures. Given that the energy states can be very close to each other depending on the detuning energy, achieving complete adiabatic transitions might be challenging in practice. Figure 6 illustrates the system temperature as a function of detuning energy at t_1 , considering the population transfer from states $|1\rangle$, $|2\rangle$, $|3\rangle$, and $|4\rangle$ to states $|5\rangle$, $|6\rangle$, $|7\rangle$, and $|8\rangle$ via two different paths: accelerated adiabatic transfer and complete population transfer to $|7\rangle$ ($|T_+(1,1)\rangle$). The path where the population is transferred to $|7\rangle$ results in a higher system temperature; however, this increase is not significant. Consequently, the accelerated adiabatic transition is not strictly necessary for the cooling loop to function and can be relaxed without a substantial loss in the achievable low temperature.

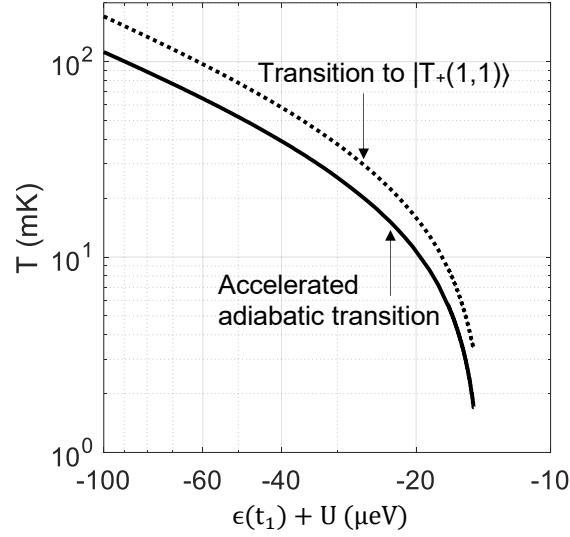


Figure 6: System temperature versus detuning energy at t_1 assuming the population is transferred from states $|1\rangle$, $|2\rangle$, $|3\rangle$, and $|4\rangle$ to $|5\rangle$, $|6\rangle$, $|7\rangle$, and $|8\rangle$ via two different paths: accelerated adiabatic transfer, and all population transfer to $|7\rangle$. The second path, where all the population is transferred to $|7\rangle$ ($|T_+(1,1)\rangle$), results in a higher system temperature; however, the difference is not significant. Therefore, the accelerated adiabatic transition is not strictly necessary for the cooling loop to function and can be relaxed without a substantial loss in the achievable low temperature.

This behavior occurs because, at time t_3 , regardless of the initial population distribution among states $|5\rangle$, $|6\rangle$, $|7\rangle$, and $|8\rangle$, microwave radiation resonant with the energy difference between states $|6\rangle$ and $|8\rangle$ induces Rabi oscillations, causing the population to shift from $|6\rangle$ to $|8\rangle$. Concurrently, population transfer occurs from $|5\rangle$ to $|6\rangle$ via spontaneous absorption and from $|7\rangle$ to $|6\rangle$ via spontaneous emission. These interactions continuously pump the population from $|5\rangle$ and $|7\rangle$ into $|6\rangle$ and from $|6\rangle$ to $|8\rangle$. As the Rabi oscillations proceed, states $|5\rangle$ and $|7\rangle$ become completely depopulated, and the population oscillates between $|6\rangle$ and $|8\rangle$ in resonance with the microwave radiation. This dynamic effectively drives the population to $|8\rangle$ regardless of the initial population distribution, explaining why the different paths result in similar final system temperatures.

The lowest achievable temperature may vary with different values of magnetic field (B), as the cooling effect arises from the interplay of relative energy level spacings that influence the thermal excitation rate. Figure 7 presents a contour map of the system temperature as a function of the magnetic field and detuning energy at time t_1 . A minimum temperature of 2 mK is achievable under optimal parameters. Each temperature corresponds to a specific range of magnetic field and detuning energy ($\epsilon(t_1) + U$). For instance, to achieve a temperature of 2 mK, the detuning energy can range from near zero to $-50 \mu\text{eV}$, with the magnetic field between near zero and 1 T. As the temperature increases, these constraints become more relaxed, providing greater flexibility for experimental design.

It is important to note that some points in the contour map may not correspond to the pure states shown in Figure 1 at time t_1 . Therefore, the optimal detuning energy and magnetic field must be determined based on the specific parameters of the DQD system. The lowest achievable temperature is ultimately determined by the competition between state population changes due to thermal excitation and microwave-induced state depopulation. Consequently, in practice, the DQD parameters must be carefully considered, and the energy levels must be measured or accurately calculated when designing the cooling loop.

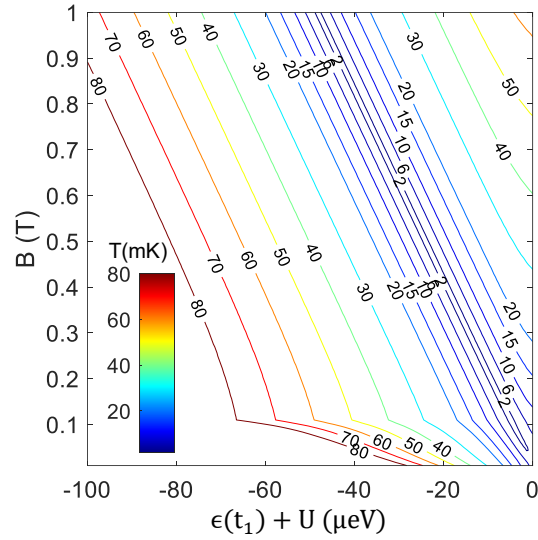


Figure 7: Contour map of the system temperature as a function of the magnetic field and detuning energy at time t_1 . A minimum temperature of 2 mK is achievable under optimal parameters. Each temperature corresponds to a specific range of magnetic field and detuning energy ($\epsilon(t_1) + U$). For instance, to achieve a temperature of 2 mK, the detuning energy can range from near zero to -50 μeV , with the magnetic field between near zero and 1 T.

Another important consideration is the hole spin phase shift caused by interactions with phonons, particularly during the thermal transition from states $|6\rangle$ to $|8\rangle$. In this transition, a hole tunnels between the left and right dots, and interacts with a phonon field via the following Hamiltonian:

$$H_{h-ph} = \sum_{\vec{q}} [(\alpha_{\vec{q}} |S(1,1)\rangle \langle S(1,1)| + \beta_{\vec{q}} |S(0,2)\rangle \langle S(0,2)|)(a_{-\vec{q}} + a_{\vec{q}}^\dagger)]$$

where $a_{\vec{q}}^\dagger$ is creation operator for a phonon of mode $\vec{q}$, and $\alpha_{\vec{q}}$ and $\beta_{\vec{q}}$ are hole-phonon matrix elements. In general, the hole-phonon coupling can be asymmetric ($\alpha_{\vec{q}} \neq \beta_{\vec{q}}$). The hole-phonon interaction locally changes the energy of the hole, depending on whether the hole is in the left dot or in the right. During tunneling, hole's wave function experiences an additional phase shift due to this coupling. This phase shift is zero if the coupling is identical in both dots ($\alpha_{\vec{q}} = \beta_{\vec{q}}$). In the case of non-identical coupling, the phase shift depends on the state of the phonon field ($e^{i\vec{q} \cdot \vec{d}}$ with $|\vec{d}| = d$, the separation of QDs). In our model calculations this effect can be practically compensated by adjusting the microwave pulse width to ensure p6 is minimized and p8 is maximized at the end of the microwave pulse.

Additionally, the dwelling time between cycles is a crucial parameter. This time must be optimized so that the DQD has sufficient time to absorb specific phonons, yet remains short enough to prevent the local environment around the DQD from becoming isothermal with the rest of the lattice. The appropriate duration depends on factors such as the material properties, operating temperature, and the area intended for local cooling to effectively manage the nearby computational spin qubits.

The interaction of phonons with the charge and spin in the DQD is critical for understanding the DQD's effectiveness in filtering local phonons. Phonon interactions with the DQD impact the energy transfer efficiency and overall cooling performance of the system. In-depth studies are needed to elucidate the precise mechanisms by which the DQD selectively absorbs and filters phonons, thereby reducing thermal noise around the computational qubits. Understanding these interactions is essential for refining the cooling technique and ensuring its practical application in large-scale quantum computing systems.

In cooling systems, quantum correlations and entanglement in the eigenstates of the Hamiltonian can negatively impact the achievement of desired temperatures. For instance, in spin-star cooling systems, the Heisenberg interaction between a central spin and surrounding ancilla qubits includes a transverse component, leading to entanglement between the central and ancilla qubits. This entanglement generation

is detrimental to the cooling process.⁹ Conversely, when qubit interactions follow the Ising model, the ground state and all excited states of the spin-star system are factorized, resulting in negligible entanglement. Consequently, the minimum achievable temperature in the Ising model is lower than that in the Heisenberg model.

Therefore, it is pertinent to examine entanglement behavior in the context of our DQD cooling system. Quantum Fisher Information (QFI), a measure of multipartite entanglement,¹⁵ has recently been studied for QDs with a universal Hamiltonian.¹⁶ As demonstrated in [16], exchange interactions and confinement effects can induce unexpected entanglement within the system, which may influence the cooling mechanism. Understanding these entanglement dynamics will be valuable for refining our cooling approach and enhancing its effectiveness.

Conclusion

This paper presents a novel cooling mechanism for spin qubits in gate-defined quantum dots using microwave-induced state depopulation and phonon filtering to achieve local temperatures as low as 2 mK at a bath temperature of 1 K. The proposed system leverages fast cyclic detuning of quantum dot potentials and Rabi oscillations to efficiently transfer thermal populations to the ground state, surpassing natural thermal transition rates. By employing a combination of adiabatic and diabatic transitions, along with resonant microwave pulses, the cooling cycle effectively reduces the system's temperature through continuous depopulation of excited states.

Numerical calculations have demonstrated the feasibility of achieving these low local temperatures, highlighting the significant influence of detuning energy, magnetic field strength, and diabatic return time on the cooling performance. This approach addresses the critical gap in effectively cooling interacting multi-qubit systems, particularly those based on spin qubits in quantum dots. The enhanced cooling capability at higher operational temperatures facilitates the integration of control electronics and paves the way for more practical and scalable quantum computing systems.

Acknowledgement

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