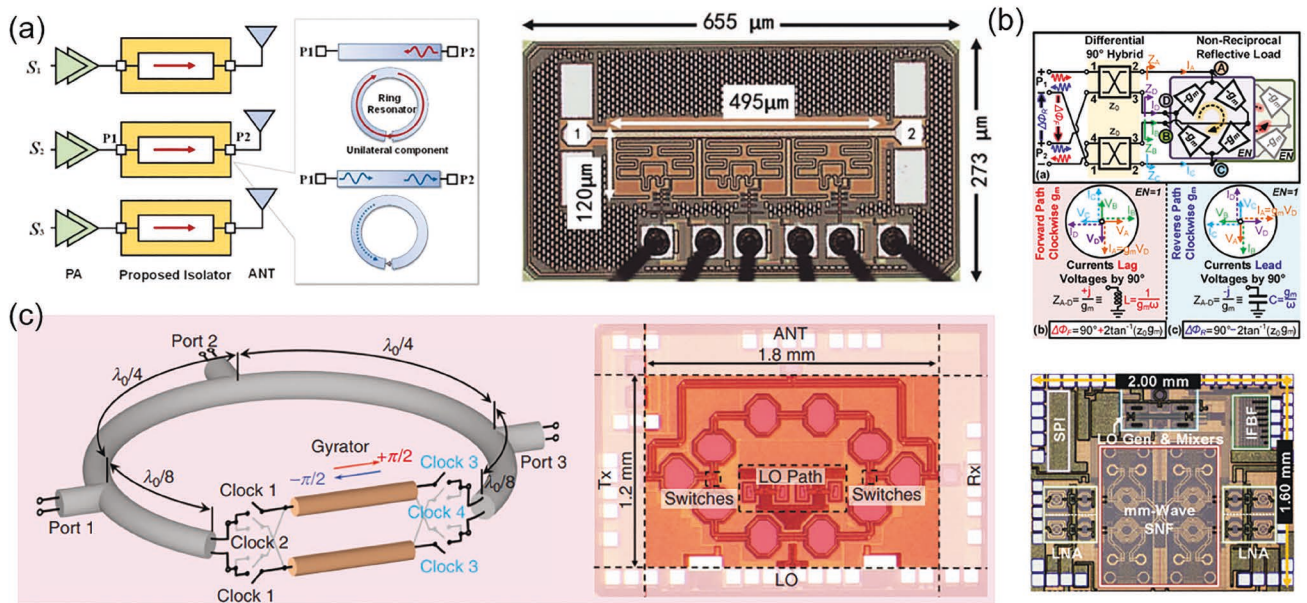


Asymmetric Transmittance and Nonreciprocity in Guided Wave Circuits

Fundamentals and IC topology



In a reciprocal system, all the waves travel in the same way backward as forward. When the exchange between the source and the detectors results in different transmit-

tance, non-Hermiticity is granted, but the nonreciprocity needs to be carefully evaluated. Although most of the integrated circuits (ICs) are reciprocal, an unexpected nonreciprocal response may emerge in the system, especially the tunable components that contain asymmetrically coupled resonators, travel-

ing wave electrodes, and hysteresis response. Nonreciprocity may result in unexpected signal redistribution, distortion, and errors in analog circuits of electrical and photonic networks. With proper engineering, nonreciprocity can be leveraged and optimized for suppressing the laser noise in photonic systems

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as isolators, reducing the circuits duplication as circulators. RF nonreciprocity can be used for protecting high power amplifiers from oscillation and damage. Asymmetric coupling can also be useful in simplifying circuit complexity and reducing crosstalk in the optical interconnect transceiver circuits.

Introduction

Nonreciprocity first emerges as an intriguing physical concept and has been found in wave-based systems out of equilibrium [1]. Nonreciprocity in high-speed electronic and photonic devices is triggered by similar mechanisms, such as rectification and hysteresis, involving magnetic-optic polarization rotations or spatially asymmetric time-varying responses. Absolute directional transport in traditional optical isolators is ensured by the underlying mechanism of nonreciprocity [2], [3], [4]. On semiconductor platforms, electro-optic modulation [5], [6], [7], [8], acoustic optics, and nonlinear optics have been explored for optical isolation effects. It is well understood among the physics community for nonreciprocities and non-Hermiticity, however, that presentations and discussions are still quite confusing for circuit engineers. In guided circuits with subwavelength width, the measurement of specific polarization or wave vector of supported traveling waves requires nonconventional instruments or metrology, while the approach to identify nonreciprocity with the most accessible tools, especially through a typical power transmission measurement, is not well discussed nor understood.

Nonreciprocity and Asymmetry

Identification of Nonreciprocity in Guided Wave Circuits

Sometimes nonreciprocity can be confused with asymmetric transmission due to the subtle difference

of its definitions. Nonreciprocity is stated as “a transmission channel works the same way if you swap the source and observation points.” The transmission channel connects one state or modes on each port. In guided optic systems, each mode is identified with unique polarization and wave vectors [Figure 1(a)] and cannot be easily characterized with optical power meters Figure 1(b). For example, in a simple two-port system, reciprocity applies to all pairs of modes on each port while the asymmetric transmittance counts only the power transmittance, which may involve many modes [9]. This

point is particularly important for the development of isolators and circulators as only the nonreciprocal response can be leveraged for suppressing all the back-reflection modes. Accidental reflected electromagnetic (EM) waves can disturb lasers, accidentally triggering oscillations in optical and RF amplifiers, or lead to unexpected multipath interference in integrated photonic or RF circuits.

A reciprocal system typically results in the same transmission ratio when the source and the detector are exchanged (most commonly for circuit designers) [10]. This

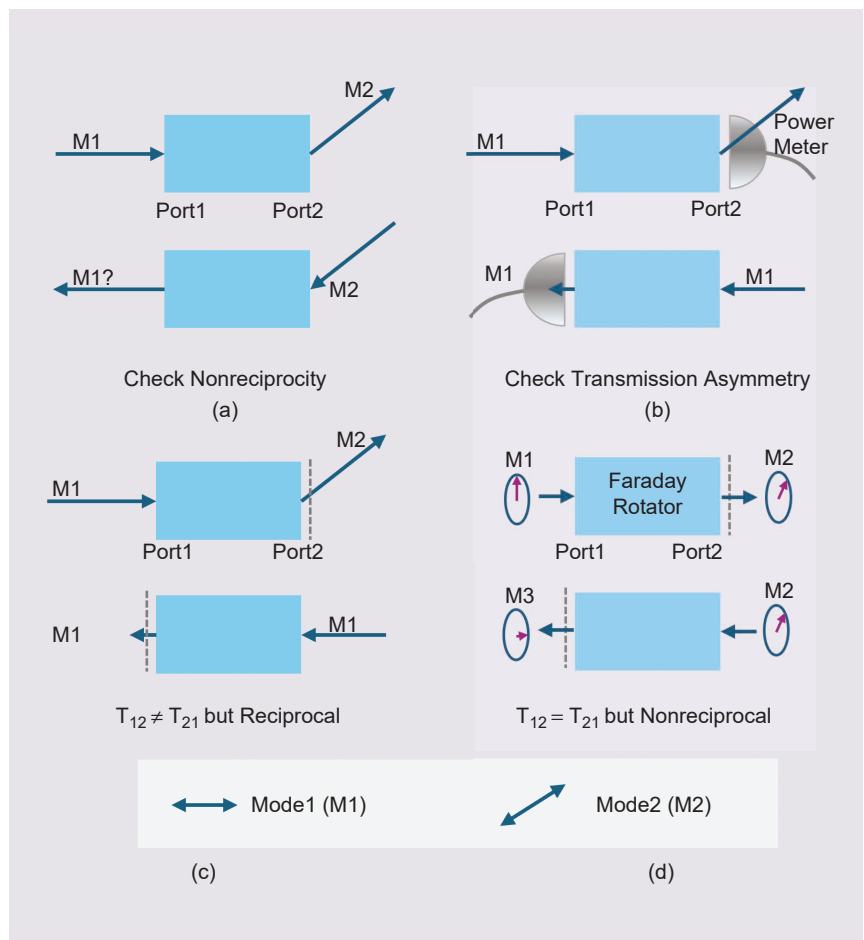


FIGURE 1: The difference between nonreciprocity and asymmetric transmission, exemplified in a two-port system (blue square). (a) An approach to check nonreciprocity, which requires identification of the high transmission mode (from M1 to M2) to excite the correct mode for backside excitation. (b) A typical approach to check transmission asymmetry. Regardless of the forward transmitted mode, the same mode for forward excitation is used for backward excitation. (c) An example of asymmetric transmission in a reciprocal system. (d) An example nonreciprocal system with symmetric transmission (a fiber waveguide with cylindrical symmetry). M1, M2, M3: mode 1, mode 2, mode 3; dashed gray line: position for placing the power meter (e.g., a photodetector).

Absolute directional transport in traditional optical isolators is ensured by the underlying mechanism of nonreciprocity.

principle implies that the propagation of waves occurs equally well in both directions. Reciprocity can thus be viewed as a form of more intricate symmetry in the behavior of wave propagation within these materials. However, if one observes asymmetric transmittance, it does not mean that the system must be nonreciprocal. One exemplary confusing case is given in Figure 1(c). The typical mode of excitation for guided wave circuits is the normally incident wave (mode 1). If high transmission is the result of coupling from mode 1 to mode 2, reverse excitation with mode 1 cannot check the reciprocity of the system [11]. Another example of a nonreciprocal system exhibiting symmetric transmittance is illustrated in Figure 1(d). Faraday rotation changes polarization of the wave, but transmittance remains the same given polarization-independent transmittance. Additional polarizers convert nonreciprocity to asymmetric transmittance. Polarization-sensitive transmittance can be incorporated into the waveguide design, which demonstrates a fully integrated guided-wave isolator [12].

Asymmetric Transmittance in Single and Multimode Waveguides

It is particularly confusing if a system exhibits asymmetric transmittance but remains reciprocal. There are many excellent works that discuss the difference between the two (for example [7], [10], and [13]). Typically, one can assert that a system is reciprocal by checking whether the system is not tunable (passive, nonmagnetic). However, this approach might be misleading as the coexistence of nonlinearity or electro-optic modulation does not necessarily result in nonreciprocity.

Among the conditions discussed in Figure 1, asymmetric transmittance cannot be equivalent to system nonreciprocity due to the existence of additional modes (guided or leaky) coupled to the excitation mode. If we limit the design of the input and output ports to be a single-mode waveguide (the same polarization and propagation constant, which is typical in silicon photonics) and the system is loss invariant, asymmetric transmittance can be equivalent to nonreciprocity [6], [15], [16], [17]. Note that in reciprocal but asymmetric transmission systems, there should be coexistence among

other physical forms of asymmetry between the input and output modes, such as different aperture size, polarization, or wave vectors.

Nonreciprocity and Non-Hermiticity

Scattering Matrix for Time-Reversal Symmetry

To identify the scattering matrix, the first step is locating the involved modes (include both guided and leaky/lossy modes). Figure 2(a) illustrates the scattering matrix of a simple two-port system. The inputs of the system (with complex amplitudes of A and D) are marked in red arrows, while the outputs (B and C) are in blue. The corresponding matrix S connecting the input and output vectors is marked beneath the structure.

In a lossless system, nonreciprocity is equivalent to the breaking of time-reversal symmetry [9]. A nonreciprocal system is characterized by a scattering matrix, whose transpose is not equal to the original matrix.

$$S^T \neq S \quad (1)$$

For example, the scattering matrix of a two-port nonreciprocal system means (Figure 2)

$$S_{12} \neq S_{21} \quad (2)$$

Note that each component in the scattering matrix (S) may need to be expressed as submatrix if more than one mode is involved. The dimension of the submatrix is equivalent to the number of modes involved (or group of modes), including both the input and output ports. The mode (or groups of the modes) for the input and output vectors needs to carefully select the covering for all the involved modes (guided or leaky/lossy) to check nonreciprocity.

Time-Reversal Symmetry in Hamiltonian

The same system can be written as Hamiltonian with redefined inputs and outputs [Figure 2(b)]. If the system is non-Hermitian, then it satisfies

$$H^T \neq H \quad (3)$$

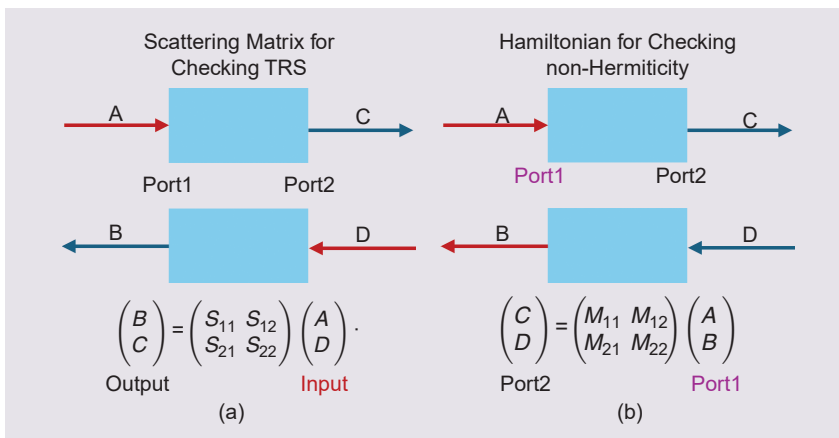


FIGURE 2: A comparison between (a) the scattering matrix and (b) Hamiltonian of the same system. The red arrows are the inputs.

where H is the Hamiltonian of the system [18]. It is noted that the non-Hermiticity of the Hamiltonian is not equivalent to the nonreciprocity of the system. This distinction arises because nonreciprocity is defined by the scattering matrix, not the Hamiltonian, which can often lead to misunderstandings.

However, it is still possible to check the reciprocity of the system through the Hamiltonian, which is investigated in the context of the transfer matrix model [19]. As the scattering matrix relates the outgoing waves to the incoming waves, as shown in Figure 2(a), while the transfer matrix (M matrix) relates two bidirectional waves [Figure 2(a)], thus corresponding nonreciprocal relationships can be identified in matrix format [19], [20]:

$$M_{11} \neq M_{22} \quad (4)$$

which is derived from the relationship between the scattering matrix and the Hamiltonian (Figure 2) as well as the relationships described in (2), which are established under the energy conservation law [21].

Nonreciprocity in an EM Wave Guided Wave Circuit

Resonance-enhanced wave-matter interactions lead to a low-threshold implementation of reciprocity, supporting applications in nonlinear optics, optomechanics, quantum optics, and information processing. Various integrated nonreciprocal devices have been successfully demonstrated using techniques such as spatiotemporal modulation, magnetic bias, Brillouin scattering, and optical nonlinearities.

Here, we introduce four types of nonreciprocal optical devices based on optical nonlinearity, electro-optics, magneto-optics (MOs), and acousto-optics for introducing the nonreciprocity into the guided wave circuits. The nonlinearity manifested subtle coupling (or loss) difference from the geometric asymmetry [Figure 3(a)–(c)]. Asymmetric electro-optic modulation can be introduced through both two lumped modulators (electrical or acoustic) with different phases,

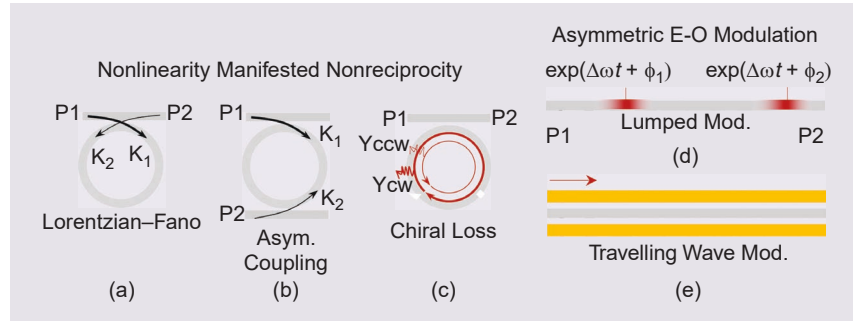


FIGURE 3: Nonreciprocities in monolithic integrated resonator circuits. Nonlinearity manifested nonreciprocity due to (a) an asymmetric coupling between the resonator and the waveguide for forward and backward propagating waves, (b) top and bottom ring-waveguide coupling, (c) a different resonator loss rate with clockwise (CW) and counter-clockwise (CCW) excitations [22]. (d) Asymmetric electro-optic (E-O) modulation achieved with two lumped modulators separated by correspondent RF wavelength spacing (a tandem phase modulator [5]), and (e) traveling wave electrodes with one-direction RF excitation. asym.: asymmetric; mod.: modulation.

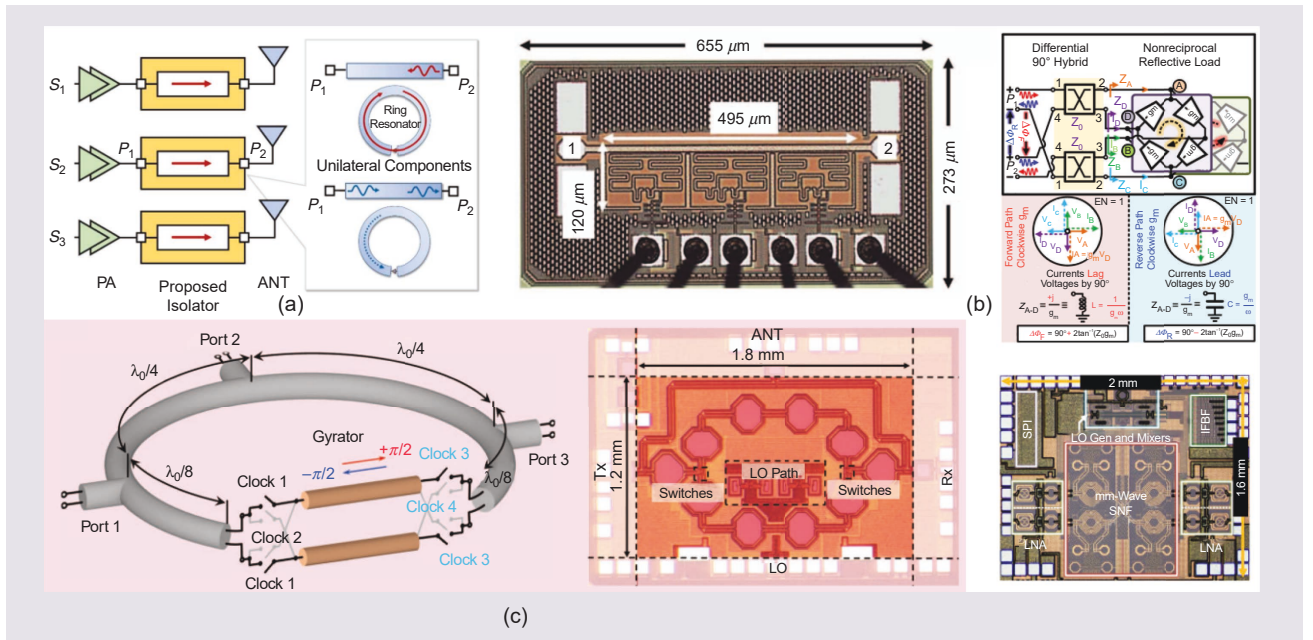


FIGURE 4: A nonreciprocal RFIC. (a) An MNM loop-resonator-based THz isolator [27]. (b) A millimeter-wave (mm-wave) spatial notch filter incorporated nonreciprocal phase-shifter architecture [28]. (c) An mm-wave circulator based on a switched transmission line [24].

In a lossless system, nonreciprocity is equivalent to the breaking of time-reversal symmetry.

or traveling wave electrodes with directional RF (acoustic) excitation directions [Figure 3(d) and (e)]. Here, we just used optical waveguides as examples, but the similar concept applies to tunable RFICs.

Nonreciprocal RFICs

Nonreciprocity has been employed in various RFICs, such as circulators and isolators [2], [3], [4], [23], [24], [25], [26]. Figure 4(a) demonstrates an integrated RF isolator using magnetless nonreciprocal metamaterials (MNMs) made of a loop resonator with a transmission line. A transistor-based nonreciprocal phase shifter (NRPS) defined as MNM was placed in a loop resonator. Depending on the direction of the incident wave applied to the transmission line, each incident wave is coupled to the loop resonator in a different loop direction. Consequently, this caused a unidirectional gain boost effect in the transistor. As a result, the proposed structure exhibits an isolation depth of 15.3 dB and an insertion loss (IL) of 3.6 dB in the 305–325-GHz range. Figure 4(b) shows another RF isolator example using the NRPS method. Traditional reciprocal phase shifters (RPSs) exhibit nonreciprocal characteristics through $\pm 180^\circ$ phase control. Consequently, conventional RPSs cannot eliminate blockers in all paths. To address this issue, the proposed study suggests using NRPSs to block blockers at all LNA outputs through $\pm 90^\circ$ phase control. Each NRPS can independently control its phase and can be turned off to reduce power consumption and noise when there are no spatial blockers. An integrated RF circulator can be realized by a switched transmission line [Figure 4(c)]. The generalized conductivity modulation concept using switched transmission

lines implements a differential transmission line delay. As a result, the waves traveling from left to right experience a transmission line delay with no sign flip over both halves of the clock period, while the waves traveling from right to left experience transmission line delay along with one sign flip. This setup generates a 180° nonreciprocal phase difference with infinite BW and without loss.

Nonlinearity-Induced Nonreciprocity in Electronic Resonators

Electric circuits are a versatile platform for exploring the basic concepts in nonreciprocal systems [29], [30], [31], [32], [33]. Nonreciprocity can be achieved within RF resonators that are made of capacitors and inductors. The circuit design allows replication of Lorentz and Fano cavities in low-frequency spectra. This simulation is instrumental in constructing nonreciprocal systems. For instance, by

utilizing asymmetric structures combined with third-order nonlinear effects, nonreciprocal systems can be created using electronic components. The construction of such a nonreciprocal system is effectively equivalent to connecting a Lorentz cavity and a Fano cavity in series. Significant isolation can be achieved at high-input power levels. When the input power reaches 21 dBm, the transmission curves S_{12} and S_{21} show substantial differences. The effective isolation is achieved through the series connection of the Lorentz and Fano cavities under high power conditions. These passive nonlinear isolators using nonlinear Lorentzian and Fano resonators achieve control over forward transmission and isolation [33].

Nonlinear Optical Resonator

One way to implement nonreciprocal devices is to introduce nonlinearity in asymmetric structures [15], [34], [35], [36], [37], [38]. In high-power regions, an asymmetric structure causes an excitation port-dependent energy built up inside the microresonator, which can be attributed to

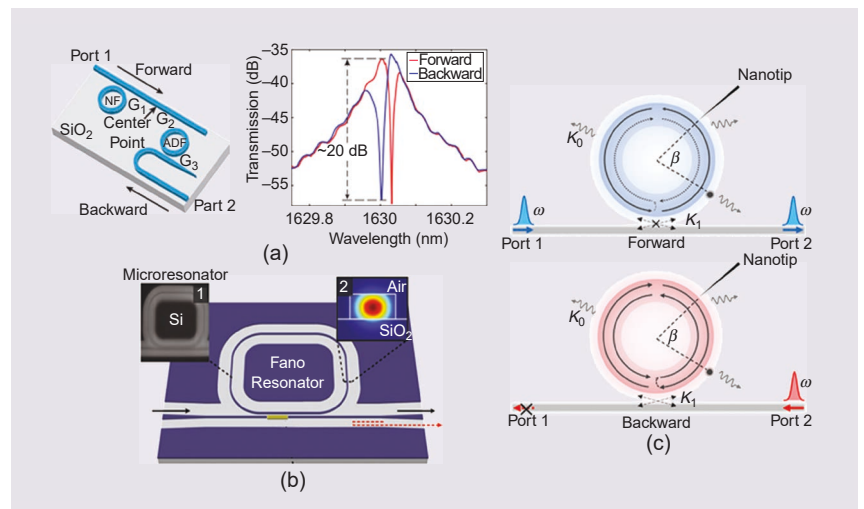


FIGURE 5: An experimental implementation of nonlinearity-induced nonreciprocity. (a) The optical diode consists of two resonance-matched filters. Forward and backward transmission spectra at input power level of 85 mW, showing strong optical nonreciprocity. (b) Schematic of the Fano nonreciprocal device. Inset 1: An SEM image of the resonator; inset 2: a simulated spatial mode profile. (c) Chirality-induced nonreciprocity. For forward excitation, the backscattering from the CCW mode to the CW mode vanishes. For backward excitation, both the CCW and CW modes heat the mode volume, leading to a stronger thermal broadening in the resonance. NF: notch filter; ADF: add-drop filter.

asymmetric coupling between add and drop waveguides, or a different loss rate between clockwise and counterclockwise modes.

As an example, here we discuss the optical diode consists of two resonance-matched filters (Figure 5), one notch filter and one add-drop filter. Transmit from port I(II) to port II(I) is defined as *forward (backward) propagation*. Due to a different coupling strength between the rings and the waveguide, the storing energy is different. As the figure shows, forward and backward transmission spectra at an input power level of 85 mW show strong optical nonreciprocity. With the input power increased to 85 mW, a nonreciprocal transmission ratio of 20 dB was observed at a wavelength of 1,630 nm. This optical diode performed similarly to electrical diodes with unidirectional transport. In the coupled Lorentz and Fano cavity, the key is to use a nonlinear response to break the system's time-reversal symmetry. Specifically, with high-power inputs, the frequencies shift differently for forward and backward signals through the chip. This difference

Each NRPS can independently control its phase and can be turned off to reduce power consumption and noise when there are no spatial blockers.

is mostly noticeable in the Fano cavity due to its sharp transition in transmission. By using these nonlinear effects, the photonic circuit achieves significant nonreciprocal capabilities. Interestingly, despite these differences, the circuit maintains high transmission efficiency. These features make it a promising platform for applications where nonreciprocal behavior is crucial, such as advanced photonics and optical communication systems. The combined effects of nonlinearity and chiral modes lead to a stronger thermal broadening in the resonance.

Asymmetric Time-Varying Excitation With Electro- and Acoustic EM Wave Modulation

Another way to implement nonreciprocal devices is through the use of electric-optic modulators [Figure 3(d) and (e)]. By incorporating electro-optic modulators,

it is possible to achieve an asymmetric scattering matrix. The primary method used to realize this involves integrating a time-dependent mechanism within the device, ensuring that the modulation varies with time to create the desired nonreciprocal behavior.

As an example, the tandem phase modulator scheme could achieve an optical isolator. Figure 5 shows a narrow-band optical isolator in InP using phase modulation and another optical isolator in III/V material, achieving a 30-dB isolation and an 8-dB excess IL with a single-sideband electro-optic modulator.

Some articles show advanced optical isolation techniques using acousto-optic interactions in integrated photonic circuits. They use phonon-mediated processes and spatiotemporal modulation to achieve one-way light propagation. These innovations aim to improve integrated photonic devices like isolators and modulators, which

TABLE 1. A SUMMARY OF DIFFERENT OPTICAL AND RFIC ISOLATION TECHNIQUES.

TECHNIQUE	STRUCTURE	BW	TC (DB)	IL (DB)	INPUT POWER	EXTERNAL POWER
MO material with the Faraday effect [46]	MZI	14 nm	30	14	N/A	3 mW
MO material with the Faraday effect [42]	Microring	2 nm	30	9	N/A	0.1 T
Asymmetric time-varying excitation [6]	Microring	0.16 nm	13	18	N/A	0.5 mW
Asymmetric time-varying excitation [16]	Microring	0.006 nm	10	0.1	−30–0 dBm	300 mW
Asymmetric time-varying excitation [40]	Microring	0.002 nm	13	1.13	N/A	794 mW
RF nonlinearity [33]	Lorentzian–Fano resonator	50 MHz	30	1	Min. 17 dBm	N/A
RF permittivity modulation [14]	LC bandstop filter	20 MHz	60	3.1	Max. 5 dBm	1.1 V
Optical nonlinearity [15]	Microring	0.3 nm	23	1.8–5.5	Min. 19 dBm	N/A
Optical nonlinearity [35]	Microring	Single	24	3.3	Min. 20 dBm	N/A

TC: transmission contrast; N/A: not applicable; min.: minimum; max.: maximum.

are crucial for high-quality information processing and quantum state control.

This principle has been used to design an optical isolator based on very large chiral asymmetry in a two-level photonic atom using phonon-mediated photonic Autler–Townes splitting. The measurement results from a 1,556-nm device for 3-GHz applied RF with 29-dBm power. Similar isolator measurement results from a 773-nm device for 5.05-GHz applied RF with 25 dBm of power [39].

Nonreciprocity Based on Magneto-EM Wave Interactions

The integration of magnetic materials has enabled the development of nonreciprocal guided wave devices [40], [41], [42], [43] for isolation and circulation in photonic and microwave ICs. When a magnetic optic (MO) waveguide is placed in a magnetic field, it creates a nonreciprocal phase shift that can be used in phase-sensitive interferometric photonic components [44], [45]. For the practical applications of those MO-EM wave isolators, on-chip generation of the magnetic field can be generated by the electric current in the metal microstrip [40]. Such integrated magneto-optical isolators up to 30 dB are achieved on silicon and silicon nitride waveguides for both transverse electric and transverse magnetic modes.

Conclusions

Here we discussed the approaches used to identify the nonreciprocal response in electronic and photonic guided wave circuits, and provided a few examples for control and engineering the nonreciprocity in both photonic and RFICs. By studying how nonreciprocal systems work and, breaking time-reversal symmetry, we can create devices that leverage nonreciprocal responses for advancing circuit functionalities and complications. Engineering nonreciprocity and non-Hermiticity may offer additional degrees of freedom for circuit engineering, with applications from controlling the cavity-qubit coupling to quantum network, reducing the circuit redundancy,

protecting RF power amplifiers to laser noise suppression. Table 1 summarizes conventional optical and RF isolation techniques. Typically, MO material-induced structures show relatively high transmission contrast (TC) and broad BW. However, they require external power to generate the Faraday effect and have higher IL. Asymmetric time-varying excitation techniques, such as electro-optic or acousto-optic modulation, can achieve intermediate TC and low IL, but they are limited by narrow BW and external power requirements. These techniques show trends that are similar to optical nonlinearity techniques, but they do not require external power because structural nonlinearity induces TC. Meanwhile, nonreciprocity is demonstrated in the RF Region with electrical circuits and lumped elements, such as varactor diodes. Isolation devices that use these various technologies provide the community with a wide range of solutions that are tailored to their purposes and are expected to lead to more practical isolation applications.

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