# STUDENTS' CHANGING METARULES DURING AND AFTER WATCHING DIALOGIC INSTRUCTIONAL VIDEOS

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*Dialogic instructional videos feature authentic conversations of students as they engage in complex mathematical problems. Because these videos show students engaging in rich mathematical interactions students might use them as models for how they should engage in such interactions. In this study, we investigated how watching a dialogic video that showed two students creating pictures to illustrate mathematical relationships shaped what two pairs of students thought was necessary to include in their own pictures. We found that while the video the students watched did indeed shape what they thought was necessary to include in their pictures, the degree to which they felt they needed to mirror the pictures in the video varied considerably.*

## INTRODUCTION AND PERSPECTIVE

Instructional videos are appealing because they can flexibly offer additional instruction. Students can view them on their own time at their own pace or instructors can integrate them directly into classroom instruction. While the convenience and access to additional instruction that videos provide is compelling, educators should critically reflect on the quality of that instruction. Many instructional videos feature an expert explaining a concept or procedure (Bowers et al., 2012), essentially providing a lecture experience. However, a meta-analysis of classroom studies comparing lecture to alternatives suggests that the alternatives can be more productive (Freeman et al., 2014).

One way video creators have begun to go beyond recreating lecture on video is to create dialogic videos, those that feature the authentic dialogue of students as they engage with complex mathematical problems (e.g., Lobato et al., 2019). These videos have great potential because they allow students to indirectly participate in negotiating mathematical meanings, evaluating and critiquing the reasoning of others, and comparing peers' ways of reasoning to their own (Lobato et al., 2023). These practices mirror the types of rich interactions researchers and educators advocate for in classroom settings (National Council of Teachers of Mathematics, 2014). To investigate how viewing these videos shaped students' ways of interacting, we adopted a commognitive perspective (Sfard, 2008).

The commognitive perspective asserts that thinking is "an individualized version of interpersonal communicat[ion]" (Sfard, 2008, p. 81). Thus, instead of conceiving of learning as the acquisition of concepts, skills, and procedures, learning is defined as being able to participate in an expanding set of discourses. This requires learning the rules of these discourses.



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Sfard suggests that students learn two types of rules: object-level rules and meta-level rules. In general, "object-level rules are narratives about regularities in the behaviour of objects of the discourse, whereas meta-level narratives or meta rules define patterns in the activity of the learners trying to produce and substantiate object-level narratives." (Sfard, 2008, p. 204). For example, 2+3 = 5 is an object-level rule in arithmetic because it is a narrative about the relationship between the objects 2, 3, and 5. However, the rule "You can add the addends in either order (e.g., 2+3 or 3+2)" is meta-level because it governs how to produce object-level rules. Meta-level rules help us substantiate our claims. As such, the development of meta rules is important because they can help us to "become aware of new possibilities and arrive at a new vision of things" (Sfard, 2007, p.577).

While meta rules can seem firm because they govern how to endorse object-level narratives, they can change over time. This is because they are a result of patterned activity among a community's interlocutors. In this way, they are a product of, often tacit, social negotiation. This means the rules themselves are often tacit. However, this is not always the case. At times, participants in the community will make explicit the rules for arriving at object-level narratives. For this reason, Sfard distinguishes between *enacted* meta rules, the rules that seem to be governing interlocutors' actual behaviour, and *endorsed* meta rules, those that are explicitly stated as rules and agreed upon by the community members.

Our research seeks to provide insights into how viewing dialogic videos might shape students' development of meta rules. This led to the following research question, "As secondary students solve tasks and view dialogic videos of students solving similar tasks, what meta rules were developed and what changes occurred after watching the videos?"

## METHODS

Data were collected through four one-hour semi-structured interviews with four pairs of secondary students (grades 11 and 12). Conducting interviews in pairs allowed for meaningful interactions between students, not just student and interviewer. During the sessions, the pairs were tasked with solving a problem, followed by questions about their thought processes. They then watched a segment of a dialogic video featuring two students, Josh and Arobindo, solving a similar problem, and were given the option to revise their work.

The video the students watched showed Josh and Arobindo in the bottom right corner of the screen, with their work in the upper left. Viewers could choose to show or hide captions (see Figure 1). While the teacher's voice can be heard as he assigned tasks and prompted explanations from Josh and Arobindo about their solution paths, mathematical reasoning, and how they showed mathematical relationships, his presence was not visible. The video unit was on exponential functions, totalling 34 videos across 7 lessons. Throughout the unit Josh and Arobindo explored the exponential growth of magical beanstalks.

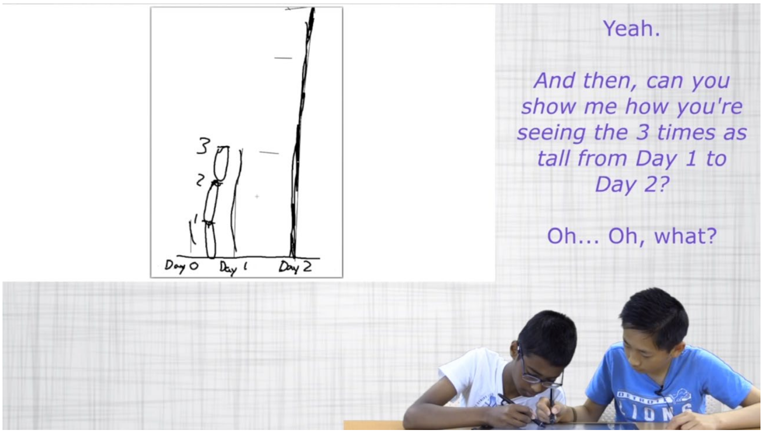


Figure 1. A screenshot of the video the interviewees watched during their interview.

Data analysis proceeded with us first creating descriptive accounts of the videotaped interviews. After reviewing across the accounts, we chose to focus our analysis on two of the pairs' responses to the first task we posed in the interview (Figure 2). This task was selected because the participants seemed to actively negotiate the meta rules for the interview session. Our focus on these two pairs is driven by the observation that the pairs reflect ends on a spectrum of how these videos can influence meta rule development.

Task 1: Consider a beanstalk whose height on day 0 is 1 cm and whose height triples each day. Draw a picture of the height of the beanstalk on Day 0, Day 1, and Day 2 that shows the tripling from one day to the next. Can you mark in any math relationships you see?

Figure 2. The interview task.

We began analysis by perusing the descriptive accounts to generate hypotheses about the students' meta rules. We focused on what seemed to count for the students as showing mathematical relationships, including showing the tripling in their pictures. The initial hypotheses were further refined by re-watching the videos and generating transcripts. Once we felt confident that we had inferred meta rules consistent with the students' actions and dialogue, we looked for changes in their meta rules before and after viewing the instructional video. Finally, we examined how those changes were related to the actions of dialogue of the students in video, Josh and Arobindo.

## FINDINGS: METARULES BEFORE AND AFTER

Our findings suggest that the video shaped the development of meta rules for both pairs of students, but in different ways. For Celina and Olympe the video seemed to make them more confident in their initial idea that drawing a graph or writing an equation counted as "showing a mathematical relationship." This is because Celina felt that her initial drawing showed the same relationships as Josh and Arobindo's and was thus sufficient. On the other hand, Olympe recognized that her drawing was quite different from Josh and Arobindo’s, but she felt that it was still sufficient because it made sense to her and she understood what Josh and Arobindo were saying. In contrast, Daniel and Peter wanted to change their picture after watching the video. They originally focused on using their picture to find the height of the beanstalk on Day 2. However, after watching the video, they revised their picture to be more similar to Josh and Arobindo’s. We provide more detail about our analysis that supports these claims below.

**Olympe and Celina**

Alicia, the interviewer for Olympe and Celina, asked them to engage in the task by saying, "I’d like for you to talk to each other…yeah… let me know when you’re done and I’ll ask you questions about your work." Olympe began by inquiring of Celina, "What do you want to do?" Celina responded, "So, should we…” as she drew a long vertical line on the paper. Olympe responded with, “Wait, can we do, erm…, can we…?” and drew the axes to a graph. Alicia replied, "You can do whatever you want." Celina then decided to continue with her line, while Olympe decided to create a graph. After she finished her graph, Olympe asked Alicia, "Does that work?" Alicia replied, “Yeah, yeah, yeah, it’s up to you, like I said, no right or wrong answers; just wanted to see how you’re thinking about it.”

In the above exchange we see evidence for the metarule (MR) that Olympe and Celina first appeared to operate with, *MR 1: The objective of this task is to show our thinking, but we’re not sure what counts and we seek approval.* Olympe and Celina grappled with uncertainty about the task, negotiating their drawings and settled on different pictures (Figure 3). Olympe's seeking approval from Alicia suggested a perceived need for approval of the solution path, yet the specific requirements remained unclear. This reinforced the idea of seeking approval for acceptability. Olympe continued with her drawing, seeking Alicia's approval once more, with Alicia reiterating her freedom of representation.

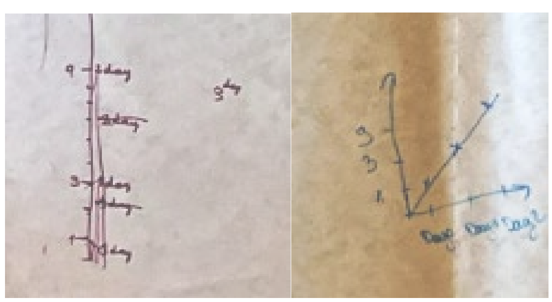

Figure 3. Celina's picture on left, Olympe's on right

After she had finished her drawing, Celina re-read the task statement, which asked them to mark in the math relationships they saw. Alicia then asked them to explain those relationships, but backtracked as she realized they were still grappling with the question. Celina said to Olympe, “Okay, we’re not done, mark in math relationships that you see. You just drew something. That’s not a math relationship.” Olympe responded by trying to find an equation. She said, "Well I put $x$ at the third or whatever, and that wouldn't work because one, one exponential three, would be one, so that doesn't work, oh wait, no…" Celina then joined her in trying to find an equation. Eventually, Alicia said "You don't have to put it into an equation, we want to see sort of what your pictures sort of look like. But it's fine if you want to." Celina stopped, and said "Well, yeah, that's my picture then, I guess (see Figure 3)."

During this exchange we believe Olympe was initially operating under the meta rule, *MR 2: Drawing a graph is acceptable as a solution.* However, it seems Celina was operating under a different meta rule, *MR 3: Drawing a graph is does not count as marking in mathematical relationships.* Together, they seemed to develop a new meta rule, *MR 4: An equation counts as a math relationship*. Olympe had stopped writing before Celina's comment, "Okay we're not done, mark in math relationships that you see. You just drew something. That's not a math relationship." This suggests that Olympe thought her graph was sufficient (MR 2), while Celina did not (MR 3). Olympe then started to create an equation (MR 4) as she said, "Well I put $x$ at the third whatever, and that wouldn't work because one, one exponential three, would be one, so that doesn't work, oh wait, no…" This meta rule may have been stunted as Alicia again suggested that she does not have to put it into an equation and a drawing is sufficient.

Olympe and Celina then watched a clip from a dialogic instructional video that showed Josh and Arobindo drawing a picture that showed the height of the beanstalk on Day 0, Day 1, and Day 2 and illustrated mathematical relationships. They represented the height of the beanstalk with vertical lines and showed mathematical relationships by drawing ovals next to those lines. Specifically, they showed that the height increased by a factor of three from Day 0 to Day 1 by drawing three ovals that were each the same height as the vertical line representing Day 0 next to the line representing Day 1 (see the screenshot in Figure 1). Similarly, they drew three large ovals, each with three smaller ovals inside them, next to the line representing Day 2. This showed that the Day 2 height was equivalent to 3 groups of 3 copies of the Day 0 height. Notably, these drawings were the result of some negotiation with the instructor (John) around what counted as "showing a mathematical relationship."

After Olympe and Celina summarized what happened in the video in their own words, Alicia asked, "How does your picture compare to theirs?" Celina responded, "I think that mine is pretty similar." In contrast, Olympe said, "I think mine is pretty far away." Alicia then asked if they thought their pictures showed the same relationships, and they both responded "Yeah." Alicia then asked if they wanted to change their picture, but neither did. Olympe said, "Well my drawing makes sense to me, but I probably couldn't explain it to someone. So, if I had to teach it to someone else I would probably use that [Josh and Arobindo's picture] because it's very clear. But in my head, it's very clear." While Celina responded, "Yeah I think I would probably use that one [Josh and Arobindo's], but I think mine is understandable."

After watching the video of Josh and Arobindo, Celina and Olympe's *MR1* seemed to change to *MR 5: The purpose of the task is to explain Josh and Arobindo's reasoning and compare our drawing to theirs. As such, we don't need approval anymore.* Furthermore, Celina seemed to develop a new metarule, *MR 6: My quantitative explanation was sufficient and showed the same relationships as Josh and Arobindo's* as did Olympe, with *MR 7: My drawing is a useful tool for my thinking, but for explaining to someone else, Josh and Arobindo's explanation is clearer.* Evidence for MR 6 includes Celina's statement "mine is pretty similar" and evidence for MR 7 includes Olympe choosing not to revise her picture and stating her thinking was "very clear."

**Daniel and Peter**

John interviewed Daniel and Peter. After he posed Task 1, Daniel and Peter worked together to draw the picture shown in see Figure 4. Peter, after asking Daniel if they "should also label the height," then asked, "wouldn't it be nine?" With some reassurance from John of "you're doing great, you're doing great," they continued with Peter asking Daniel "would you cube it to triple it?"

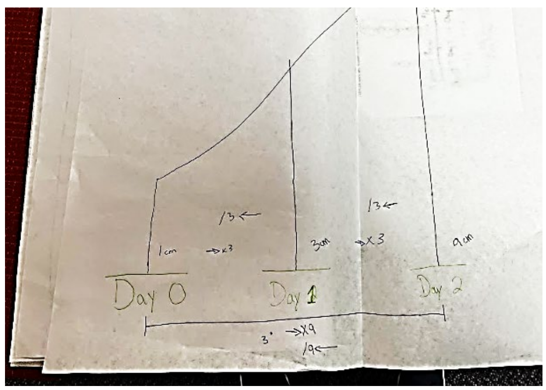


Figure 4. Daniel and Peter's initial picture

The first rule Daniel and Peter seemed to operate under was *MR 1: The purpose of engaging in this task is to solve the task accurately.* Because Daniel asked Peter "wouldn't it be nine?" and Daniel responded hesitantly with "yeah?," we inferred that the two interviewees were taking seriously the task of finding mathematical relationships, which supports our interpretation for MR 1.

After creating a line graph showing the beanstalk's height Days 0, 1, and 2, they began expressing and illustrating math relationships they saw by writing "x3" between the days. Daniel explained, "From each day, there's a multiple of three. After one day, after another day passes, it's a multiple three, so it increases by that much." John then asked for other math relationships, telling them the first one they found was "a great one." Daniel noticed that one could also say that, going in the opposite direction, the height was by divided by three over each day and wrote "/3" between the lines representing the beanstalk's heights. Peter then questioned if exponential growth qualifies as a relationship, showing uncertainty about the criteria.

In this episode, Daniel and Peter provided evidence they were operating under *MR 2: We think that a graph annotated with multiplication and division symbols counts as showing mathematical relationships, but we're not sure.* At this point the students had drawn a graph and marked in the factors by which the height changed. After some exchange, Daniel and Peter appeared to feel satisfied with their work, with Daniel indicating they were finished by saying, "Okay."

Daniel and Peter then watched the same clip that Olympe and Celina watched of Josh and Arobindo illustrating mathematical relationships by drawing sets of ovals. John then asked Daniel and Peter to explain what Josh and Arobindo's drawing showed. They pointed out the Josh and Arobindo showed the tripling from one day to the next with the ovals that they drew. They explained that the ovals showed, as Daniel put it, "how each segment of the previous day is built within the next height." Peter then elaborated, explaining that the ovals were showing the tripling from one day to the next. Daniel and Peter were then asked to redo the task and compare the picture they drew with the picture of Josh and Arobindo's. They redrew the three vertical lines representing the heights on Days 0, 1, and 2, similar to what they had drawn before, but this time they annotated the Day 1 picture with three segments to the side and the Day 2 picture with nine segments to the side. These segments seemed to serve the same purpose as Josh and Arobindo's ovals as Peter explained how they showed the tripling from one day to the next. In fact, when asked to compare, Peter made the connection explicit saying they were "like the ovals."

From their response we inferred they had developed two new meta rules, *MR 3: Josh and Arobindo's drawing, particularly the subdivision of the heights on each day, is an acceptable way to show the tripling relationship* and *MR 4: Our drawing should be more similar to Josh and Arobindo's.* We infer these meta rules from the fact that they revised their picture to show the same relationships that Josh and Arobindo showed.

## DISCUSSION

The dialogic instructional video the interviewees watched featured the authentic dialogue of two students as they worked together to draw a picture illustrating mathematical relationships. We hypothesized that having students watch videos that showed an example of creating a picture that showed mathematical relationships would shape the development of their own meta rules regarding how to communicate mathematical relationships. Our findings illustrate that our hypothesis was correct, though the meta rules the two pairs developed were quite different. Daniel and Peter developed meta rules that suggested they draw pictures that mirrored Josh and Arobindo's, ones that showed similar relationships in similar ways. In contrast, the video seemed to give Olympe and Celina confidence that their original pictures were sufficient. Since their original thinking was broadly consistent with Josh and Arobindo, they did not feel the need to revise their pictures to look like Josh and Arobindo's. This may be related to what they saw as the meta rules related to the purpose of the task, as

Olympe articulated that if she had needed to explain to another student, she might use Josh and Arobindo's representation.

These results suggest that dialogic instructional videos shape the development of students' meta rules. Both pairs of students seemed to attend to the videos and use Josh and Arobindo's work as a cue for what type of picture and explanation satisfied the task requirements. However, if teachers want to use dialogic videos to develop particular meta rules or establish particular expectations for drawings or explanations, they should be aware that they will need to go beyond simply showing the videos. This could include being explicit about what they found productive about the pictures and explanations featured in the videos. Similarly, they may want to consider intentionally eliciting and responding to students' developing meta rules.

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