Preservice Secondary Mathematics Teachers' Anticipations of Student Thinking on a Technology-Enhanced Task

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Abstract: The practice of anticipating student thinking is fundamental for planning to inform instructional decisions. However, it is not an intuitive practice for secondary preservice teachers (PSTs). When using technology-enhanced mathematical tasks, the complexity for anticipating student thinking increases as the teacher needs to consider both students' mathematical strategies and technology interactions. This study aims to examine the development of PSTs' anticipations of student thinking on a technology-enhanced mathematics task. PSTs were explicitly introduced to the practice of anticipating student thinking on technology-enhanced tasks and the connections between anticipating and noticing student thinking using a framework in a methods course. While PSTs either focused on their own thinking or did not anticipate mathematically important components on the pre-assessment, PSTs' focus shifted to student thinking on the post-assessment. Findings suggest that explicitly introducing a framework to support PSTs' anticipations of student thinking on technology-enhanced tasks and connecting anticipating to noticing student thinking has promise.

Introduction

Anticipating student thinking, an important component of planning student-centered instruction, is a teaching practice that needs to be developed (Fernandez & Yoshida 2004; Stein, Engle, Smith, & Hughes 2008). The process of anticipating requires teachers to consider the myriad of ways students might approach a task and sets the stage for what they will pay attention to as the students engage with that task. When technology is included as part of a mathematical task, this adds an additional layer of complexity for anticipating student thinking. Now, the teacher not only needs to consider the mathematical strategies, conceptions, and misconceptions students may exhibit, but also students' interactions with the technology, which inform these. Despite the importance of anticipating student thinking, there is little research to help us understand how PSTs engage in the practice of anticipating (Morrissey, Popovic & Kartal 2019), especially in the context of using technology-enhanced math tasks. In this study, we examine the development of PSTs' anticipations of student thinking on a mathematics task which incorporates technology with dynamically linked representations to explore the concept of vertical asymptotes for rational functions.

Background Literature

Anticipating student thinking (sometimes referred to as "informed predicting") is an important teaching practice because it prepares teachers for making instructional decisions informed by student thinking (e.g., Fernandez & Yoshida 2004; Stein et al. 2008). Anticipating involves

> developing considered expectations about how students might mathematically interpret a problem, the array of strategies—both correct and incorrect—they might use to tackle it, and how those strategies and interpretations might relate to the mathematical concepts, representations, procedures, and practices that the teacher would like his or her students to learn (Stein et al. 2008, p. 322).

In fact, anticipating student thinking is such an important practice that it is identified by name in the Standards for Preparing Teachers of Mathematics (Association of Mathematics Teacher Educators; AMTE 2017), which states that well-prepared beginning teachers of mathematics should be able to "anticipate and attend to students' thinking about mathematics content" (SPTM Indicator C.3.1, p. 18).

As mentioned in this indicator, there is a tight tie between the practice of anticipating student thinking and noticing student thinking (i.e., attending, interpreting, and deciding how to respond; Jacobs, Lamb, & Philipp 2010). In fact, Smith, Steele, and Sherin (2020) explain anticipating student thinking as "planning to notice". Specifically, they state that anticipating enables teachers to proactively recognize and make sense of student approaches and respond effectively to students. Research has shown when teachers anticipate student thinking on a mathematics task prior to engaging in noticing student thinking, their noticing improves (e.g., Güner & Akyüz 2020; Jacobs et al. 2010; Ulusoy & Çakıroğlu 2018).

Scholars strongly agreed on the importance of anticipating student thinking as one of the quality aspects of lesson planning for improving the quality of math teaching (e.g., Akyuz et al. 2013; Taylan 2018). However, research on the nature of teachers' anticipations is limited. The scant studies that exist have found that teachers and PSTs often focus their anticipations on their own problem-solving strategies (e.g., Şen Zeytun, Çetinkaya & Erbaş 2010) or solely on student errors (e.g., Janike 2019; Morrissey et al. 2019). This is consistent with the results of a systematic review of recent studies on teacher competence in mathematics lesson planning which showed the need to support teachers to improve their awareness of student thinking (Cevikbas et al. 2023). In addition, Morrissey et al. (2019) found when anticipating student errors, PSTs often focus on procedural errors without considering students' underlying conceptual understanding. In another study, senior PSTs struggled to anticipate students' correct thinking in their lesson plans while they were better at providing examples of incorrect student thinking (Taylan 2018).

The practice of anticipating student thinking can be developed by including opportunities to analyze student work (e.g., Didiş Kabar & Erbaş 2021; Nickerson & Masarik 2010) or to plan lessons (Hughes 2006). Didiş Kabar and Erbaş (2021) investigated PSTs' anticipation of student responses as PSTs engaged in examining and analyzing authentic student work and found that PSTs' anticipations shifted throughout the study from focusing on their own thinking to considering tasks from students' perspectives. Hughes (2006) noted that PSTs demonstrated significant growth in their anticipation of student thinking when planning a lesson after participating in a course which emphasized students' mathematical thinking as a key element of lesson planning. Similarly, Nickerson and Masarik (2010) found positive shifts in secondary teachers' anticipations after the teachers participated in a professional development program designed to improve teachers' interpretations of student thinking by analyzing student work. At the beginning of the study, teachers either focused on where students might have difficulties or did not anticipate student thinking at all. Later, they all articulated possible student responses, both correct and incorrect. Overall, these findings show that asking teachers and PSTs to anticipate student thinking during lesson planning and examining artifacts of student work can result in positive shifts in their practice.

In our review of the literature, we only found studies focused on anticipating student thinking on paper and pencil tasks. What is missing are studies on the development of anticipating student thinking in the context of using technology-enhanced math tasks. When technology involved, teachers need to consider both students' mathematical strategies and their interactions with the technology. Given the additional layer of complexity added by technology, we can infer that PSTs need support in the development of the practice of anticipating student thinking on technology-enhanced math tasks.

Conceptual Framework

As stated previously, Smith et al. (2020) describe anticipating as "planning to notice student thinking" (p. 50). Thomas and colleagues (Thomas, Fisher, Jong, Schack, Krause & Kasten 2015) directly connect the attend component of noticing student thinking (Jacobs et al. 2010) to anticipating student thinking, noting that one anticipates by "adopting a student's perspective and considering the strategies, questions, and difficult points that may arise as students complete the chosen task" (p. 240). In the context of our work, we are specifically interested in PSTs' anticipations on tasks that include math action technology (i.e., technology that responds to user actions in mathematically defined ways; Dick & Hollebrands 2011). We refer to these tasks as technology-enhanced math tasks. Thus, when anticipating student thinking on a technology-enhanced math task adopting a students' perspective includes "anticipating the ways you expect students to engage with the technology, where their eyes might be drawn, what representations they might favor, and how they will make sense of their actions with the technology" (Preparing to Teach Mathematics with Technology: Examining Students' Practices in Algebra and Function; PTMT-ESP 2023, p. 5).

We take a practice-based approach (e.g., Grossman, Compton, Igra, Ronfeldt, Shanhan, & Williamson 2009) to preparing PSTs to teach secondary mathematics with technology. From this perspective, PSTs need opportunities to enact core components of the work of teaching in simplified contexts. Anticipating student thinking is a core teaching practice as it prepares one for making instructional decisions informed by student thinking (e.g., Fernandez & Yoshida 2004; Stein et al. 2008). The practice of anticipating has been identified as a foundational component of multiple student-centered instructional frameworks, including Simon's (1995) Hypothetical Learning Trajectory, the structured problem-solving lessons used in Japanese Lesson Study (Fernandez, Cannon & Chokshi 2003), and the 5 Practices for Orchestrating Productive Mathematics Discussion (Stein et al. 2008). Given its relationship to other core practices, it is important to provide novice teachers (PSTs) with scaffolded opportunities to study and practice (Forzani 2014).

From a practice-based perspective, to support PSTs in developing their anticipating practice, PSTs must first build an understanding of the practice through analyzing representations of the practice—unpacking (or decomposing) them into their constituent parts (Grossman et al. 2009). Since the practice of anticipating student thinking is in service of planning to notice student thinking, Thomas et al. (2015) suggested that noticing frameworks could support PSTs in their decomposition of the practice of anticipating. To that end, to support PSTs in building an understanding of the practice of anticipating student thinking on technology-enhanced math tasks, we used a framework for noticing student thinking on technology-enhanced math tasks, the NITE framework (Dick, McCulloch, & Lovett 2021; Dick, Lovett, McCulloch, Cayton, Bailey, & Yalman Ozen 2022; McCulloch, Dick, & Lovett 2023). The NITE framework (Fig. 1) builds on Jacobs et al.'s (2010) conceptualization of noticing students' mathematical thinking and highlights that students' mathematical thinking can be expressed through their engagement with math action technologies. In our work with PSTs, we follow Grossman et al.'s (2009) framework by having PSTs first build an understanding of the practice of anticipating student thinking on technology-enhanced math tasks using the NITE framework to guide this work, then approximate the practice of anticipating student thinking on such a task by providing opportunities for deliberate practice in a safe environment.

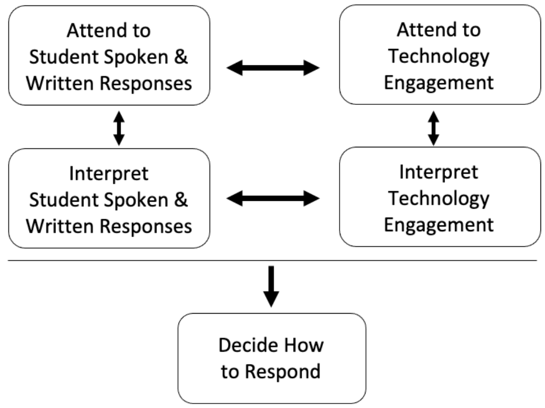

Figure 1. The Noticing in Technology-Mediated Environments (NITE) Framework

Methods

Participants and Context

This qualitative study examined the anticipations of PSTs at the beginning and end of a methods course in which PSTs learned about anticipating student thinking on technology-enhanced math tasks. The participants included 12 secondary PSTs enrolled in one of the three methods courses at three different universities. The PSTs were familiar with anticipating student thinking on paper and pencil tasks prior to this course (Smith et al. 2020). During the course, PSTs engaged with curricular materials designed to extend the practice of anticipating into technology-mediated tasks.

The curricular materials included eight modules designed to engage PSTs in carefully crafted approximations of practice focused on anticipating and noticing student thinking in technology-mediated environments (PTMT-ESP 2019; see website for a full description of the design of the materials used in the course). The first module explicitly introduced PSTs to the practice of anticipating student thinking on technology-enhanced math tasks and the connections between anticipating and noticing student thinking using the NITE framework (Dick et al. 2021, Dick et al. 2022, McCulloch et al. 2023). The seven remaining modules focused on specific algebraic/function topics (e.g., algebraic equivalence; comparing/contrasting linear, quadratic, and exponential rates of change; key features of quadratic functions; trigonometric functions; and characteristics of function families). Each module followed specific design principles that included engaging with a technology-enhanced task as a learner, anticipating student thinking on the task, and then noticing student thinking on the same task using carefully selected video clips of middle and high school students engaging with the task.

Throughout the course, PSTs had multiple opportunities (at least 5, not including the pre- and post-assessment) to engage in the practice of anticipating and noticing student thinking on technology-enhanced algebra and function tasks throughout the semester. The purpose of this study was to investigate the development of PSTs' anticipation of student thinking on technology-enhanced tasks when explicitly introduced to the practice of anticipating as a means of planning to noticing student thinking.

Data Collection and Analysis

The data consisted of screen recordings of PSTs' anticipations of student thinking on a technology-enhanced task designed to be an introduction to vertical asymptotes for secondary students. PSTs completed screen recordings once at the beginning of the course and once at the end (i.e., pre- and post-assessment). The instructions for the assessment stated:

> Part of planning to use a technology-enhanced task is to anticipate student thinking. Take some time to work through the task yourself. Screencast record yourself as you do so. As you work, make sure you talk aloud to share your thinking.

Because of the nature of a screencast assessment, we only had access to what PSTs shared while engaging in the task, and we could not probe their thinking as they screen recorded themselves.

The task was built in Desmos Classroom and included several pages with prompting questions and definitions. The early pages contained a warm-up and the definition of vertical asymptote (Fig. 2). Subsequently, a page featuring a Desmos Graphing Calculator presented dynamically linked representations (e.g., graphical, numerical, and symbolic) for the function, *f(x) = k / (ax+b)*. This dynamic page prompted the user to manipulate the three sliders for parameters *k*, *a*, and *b* (Fig. 3). For the purpose of this study, we are focusing only on this page of the task.

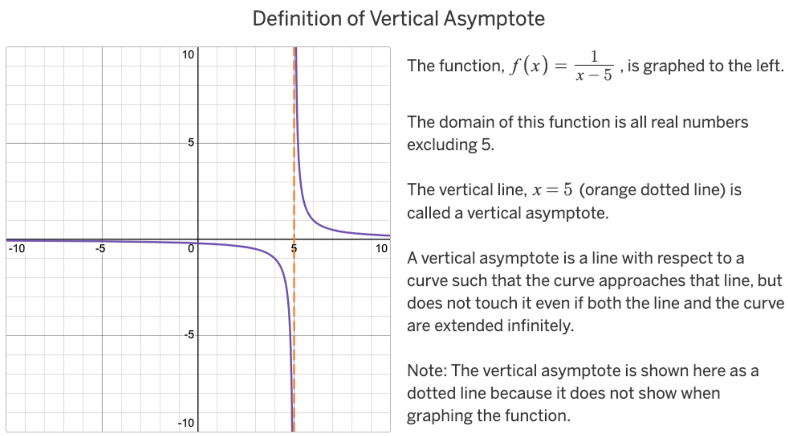

Figure 2. The Definition Page from the Introduction to Vertical Asymptotes Desmos Task

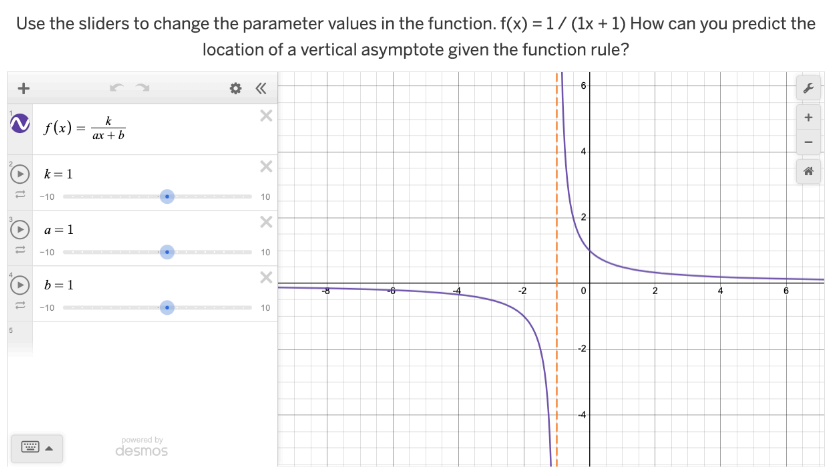

Figure 3. The Math Action Technology Page from the Introduction to Vertical Asymptotes Desmos Task

The screencast data was analyzed using a three-step process. First, we identified the mathematically significant details (Leatham, Peterson, Stockero, & van Zoest 2015) that would be important to consider when anticipating how students might interact with the dynamic sketch and what they would see as a result of those interactions (e.g., dragging all of the sliders, parameter changes in the hot text). Second, we created a rubric to capture the various strategies and representations that PSTs might include in their anticipation of student thinking on the task.

The coding rubric (Tab. 1) included anticipation of strategies using dynamic graphical representations (e.g., testing key values on sliders), numerical representations (e.g., using a cursor or slider to test specific values), and symbolic representations (e.g., sensemaking about the dynamic function rule). For each strategy, we also coded the focus of the PSTs' anticipation (i.e., no evidence, self, student, both). This is similar to Didiş Kabar and Erbaş's (2021) coding although they refer to this as an "approach". For example, exploring key values was considered as mathematically important. Thus, testing $a=0$ was included in the rubric. An example of each of the focus codes including a description of what the PST would say and/or do to receive a given code follows:

> *No Evidence*: There was no evidence in the video of the PST considering $a=0$ in their anticipation (e.g., either did not use the sliders at all or did not stop on $a=0$ as they did use them).
>
> Focus on *Self*: PST drags slider a, pauses on $a=0$ and sees a horizontal line on the graph and says "oh", then continues to drag the slider to the left.
>
> Focus on *Student*: As the PST drags the sliders, they say "Depending on how comfortable they are with sliders, they might set them all to 0 and see what is happening. They might note that when a is 0 it is a horizontal line. I think that is an important thing for students to notice. It's something I'd want students to notice so I can build on that later, about why that is."
>
> Focus on *Both*: PST is dragging all 3 sliders. When doing so, they stop at $a=0$ a few times, pausing while changing the other parameters. The PST then sets both a and b to 0 and says "everything goes away and they're going to go 'huh, that's interesting'". The PST then drags b to the right and adds "there needs to be some value down here for there to be any sort of graph at all and a needs to be something in order for there to be a vertical asymptote. If a is 0, then all you have is a vertical line that moves around."

Finally, we watched the screencasts and applied the rubric to code the PSTs' anticipations. Each video was coded by two members of the research team, and codes were recorded in a spreadsheet. Then, codes were compared, and any discrepancies were discussed and reconciled with the other research team members.

Results

The evidence and focus of PSTs' anticipations on the vertical asymptote task are summarized in Table 1. We identified six mathematically significant details that would be important to consider when anticipating, which resulted in 72 instances to analyze across 12 PSTs. On the pre-assessment, there were only three instances (4%) of the PSTs focusing on students' thinking in their anticipations. Instead, PSTs either focused on their own thinking (33 instances; 46%) or did not include important thinking in their anticipations at all (36 instances; 50%). On the post-assessment, PSTs' focus on students' thinking in their anticipations demonstrated a 10-fold increase (30 instances; 42%), with only 21% (15 instances) focusing on their own thinking and 37% (27 instances) not including important thinking in their anticipations. Findings with respect to thinking with the dynamic representations are included in the sections that follow.

Anticipations with Graphical Representations

When examining PSTs' anticipations within graphical representations, we considered if PSTs tested all sliders, including key values ($a=0$, $k=0$). With respect to anticipations related to the testing of all sliders, we saw positive changes from the pre- to post-assessment. On the pre-assessment, four out of 12 (33.3%) PSTs did not test any sliders, compared to only two (16.6%) on the post-assessment. Additionally, six PSTs (50%) tested sliders for themselves only, in contrast to two (16.6%) on the post-assessment. Most promising was that only two PSTs (16.6%) tested sliders from both self and student perspectives on the pre-assessment, but eight (66.6%) did so on the post-assessment. Thus, we saw both an improvement in the number of PSTs testing the sliders regardless of focus, as well as more PSTs focusing on students' responses rather than just themselves.

Table 1. Anticipations Related to Different Representations on the Desmos Introduction to Vertical Asymptotes Task

	Graphical Representations						Numerical Representations		Symbolic Representations			
	Test all sliders		Test key values (*a=0*)		Test key values (*k=0*)		Use cursor/slider to identify individual points to use to test values		Identify the dynamic rule at the top of the page		Evidence of use (or not) of the dynamic function rule at the top of the page	
PST	**PRE**	**POST**	**PRE**	**POST**	**PRE**	**POST**	**PRE**	**POST**	**PRE**	**POST**	**PRE**	**POST**
1	Self	Self	Self	N/E	Self	Self	Self	Self	N/E	Self	N/E	N/E
2	Self	Student	Self	Student	N/E	Student	Self	Student	N/E	Student	N/E	Student
3	N/E	N/E	N/E	Self	N/E	N/E	Self	Both	N/E	Self	N/E	Self
4	Both	Both	Both	Both	Self	N/E	Self	Both	N/E	Both	N/E	Student
5	Both	Self	Self	Self	Self	N/E	Self	Self	N/E	N/E	N/E	N/E
6	Self	Both	Self	Self	Self	N/E	Self	Both	N/E	Student	N/E	Student
7	Self	Both	Self	N/E	Self	Self	Self	Self	N/E	N/E	N/E	N/E
8	Self	Both	Self	N/E	Self	N/E	Self	Student	N/E	Both	N/E	Both
9	Self	Both	Self	Student	Self	Student	Self	Student	N/E	Student	N/E	Student
10	N/E	Both	N/E	N/E	N/E	N/E	N/E	Self	N/E	N/E	N/E	N/E
11	N/E	N/E	N/E	N/E	N/E	N/E	Self	Student	N/E	N/E	N/E	N/E
12	N/E	Both	Self	N/E	Self	N/E	Self	Self	N/E	N/E	N/E	N/E
N/E	4 33.3%	2 16.6%	3 25%	6 50%	4 33.3%	8 66.6%	1 8.3%	0 0%	12 100%	5 41.6%	12 100%	6 50%
Self	6 50%	2 16.6%	8 66.6%	3 25%	8 66.6%	2 16.6%	11 91.6%	5 41.6%	0 0%	2 16.6%	0 0%	1 8.3%
Both/ Student	2 16.6%	8 66.6%	1 8.3%	3 25%	0 0%	2 16.6%	0 0%	7 58.3%	0 0%	5 41.6%	0 0%	5 41.6%

Note. *N/E means no evidence.*

For testing key values (*a=0, k=0*), results were mixed. These key values were included because it is important to anticipate student thinking at values for which the preconstructed file "breaks" (i.e., the function is no longer rational, but a vertical line still shows in the plane). The number of PSTs that did not test key values for *a=0* increased from three (24%) to six (50%) from pre- to post-assessment. Those that focused on testing *a=0* from the perspective of self declined from eight (66.6%) to three (25%), and those that focused on both self and student increased from one (8.3%) to three (25%). A similar pattern was noticed for *k=0*. While it is promising that the focus on self and student increased in both cases, we are not sure why half of the PSTs chose not to test the key values for *a=0* and *k=0* on the post-assessment.

Anticipations with Numerical Representations

PSTs' anticipations of students' responses with numerical representations in the vertical asymptotes task resulted in a large shift from PSTs focusing on self to a focus that included student responses. Only one PST (8.3%) showed no evidence of anticipations with numerical representations (e.g., using the cursor to identify specific ordered pairs) on the pre-assessment, 11 PSTs (91.6%) did so with a focus on self, and none with a focus that included anticipations of student responses. However, on the post-assessment, all PSTs anticipated in some form with five (41.6%) focusing on self and seven (58.3%) focusing that included student responses.

Anticipations with Symbolic Representations

With respect to the symbolic representations, analysis attended to identifying the symbolic representation (i.e., the dynamic function rule at the top of the page) and using it. On the pre-assessment, none of the PSTs identified or provided evidence of the use of the dynamic function rule at the top of the task page (Fig. 3). On the post-assessment, five PSTs (41.6%) and six PSTs (50%) did not identify or provide evidence of using the dynamic function rule, respectively. However, two (16.6%) PSTs identified and one (8.3%) provided evidence for using the dynamic function rule with a focus on self on the post-assessment. The largest positive shift was seen in anticipations that included student responses with the symbolic representation on the post-assessment. Five (41.6%) PSTs both identified and provided evidence for using the dynamic function rule.

Discussion and Implications

PSTs' anticipations of student thinking is an important part of planning for student centered instruction. Anticipating is not an intuitive teaching practice, as PSTs often focus on their own responses rather than students' thinking but can be developed through coursework or professional development (Didiş Kabar & Erbaş 2021, Hughes 2006, Nickerson & Masarik 2010). What has been under-studied to date is PSTs' anticipating practices on technology-enhanced math tasks. In this study, we set out to understand how introducing PSTs to anticipating student thinking on technology-enhanced math tasks through connecting it to the practice of noticing student thinking might support their development of the practice of anticipating. What is clear and consistent across PSTs' anticipations for the various dynamic representations is that there was an increase from pre- to post-assessment for PSTs' focus to include student responses (i.e., focus codes of "student" and "both"). Though the sample was small, and insight into PSTs' anticipations was limited to what they explicitly expressed in their screen recordings, these findings suggest that this instructional approach may be a fruitful one to further develop PSTs' anticipation of student thinking.

When examining PSTs' anticipations on the dynamic elements in the Introduction to Vertical Asymptotes task, we looked at both the representations they included in their anticipations, as well as the focus of those anticipations. On the pre-assessment, most of the PSTs focused their anticipations on their own thinking; this finding is similar to prior studies on teachers' and PSTs' anticipations on paper and pencil tasks (Didiş Kabar & Erbaş 2021; Şen Zeytun et al. 2010). However, on the post-assessment PSTs' focus shifted greatly from their own thinking to student thinking. This shift is consistent with other studies that included opportunities to examine authentic student work as PSTs worked on their anticipating practices (e.g., Didiş Kabar & Erbaş 2021; Hughes 2006; Nickerson & Masarik 2010; Ulusoy & Çakıroğlu 2018).

When we look at the specific representations that PSTs included in their anticipations, the largest difference from the pre- to post-assessment was anticipations related to the numerical representation. There was evidence that all PSTs anticipated using the numerical representation on the post-assessment, with more than half including a focus on student thinking, whereas none of them showed evidence on the pre-assessment. The second largest shift from pre- to post-assessment was anticipations related to the graphical representations. Two-thirds of PSTs showed evidence of anticipation with a focus on student thinking related to testing sliders, including key values in the post-assessment. With respect to the symbolic representation, none of the PSTs showed evidence of anticipation with the function rule on the pre-assessment, while just under half did on the post-assessment. In the context of vertical asymptotes, the function rule itself is important to consider as it is key to anticipating conceptual understanding (i.e., the location of the vertical asymptote is related to the domain of the function). The absence of the PSTs anticipating student thinking with the dynamic function rule aligns with prior research indicating a tendency to prioritize procedures over underlying concepts (Morrissey et al. 2019). It is promising that about half of the PSTs' anticipations included the function rule on the post-assessment, suggesting a potential for recognizing students' thinking related to this important conceptual connection. Anticipating student thinking with respect to all these different dynamic representations is crucial, as it is part of planning for making connections among multiple representations, a practice that has been found to be essential for supporting student learning in the context of algebra and function (Goldin & Shteingold 2001; Knuth 2000; Lesh, Post, & Behr 1987).

Findings from this study suggest that explicitly introducing the NITE framework to support PSTs' anticipations of student thinking on technology-enhanced mathematics tasks and connecting anticipating to noticing student thinking has promise. Furthermore, research has shown that PSTs' noticing improved when introduced to the

NITE framework, though it is still not consistently robust (Bailey, Yalman Ozen, Lovett, McCulloch, Dick, & Cayton 2022, Dick et al. 2022). Based on these findings, additional research is needed to examine the tie between anticipating and noticing, specifically how supporting PSTs' anticipating influences their noticing of student thinking on technology-enhanced mathematics tasks. In addition, prior research on both anticipating and noticing has shown that PSTs often do not show evidence of anticipating or interpreting students' conceptual understandings (Morrissey et al. 2019). Insight into students' conceptual understandings requires anticipating more than just correct and incorrect responses to a task. Future research should consider how we might leverage noticing frameworks to support PSTs' anticipating, attending to and interpreting students' conceptual understandings—especially those they do not yet understand.

Conclusion

The findings from this study demonstrate that anticipating student thinking on technology-enhanced tasks can be learned. To this end, the use of materials that explicitly introduce the practice of anticipating student thinking on technology-enhanced tasks and the connections between anticipating and noticing based on the NITE Framework have the potential to improve PSTs' anticipating practice.

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