COMPARING CONSTRUCTS FOR ASSESSING TEACHERS' PREPARATION TO TEACH MATHEMATICS WITH TECHNOLOGY

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This study examines the preparation of prospective teachers (PSTs) in teaching secondary mathematics with technology. It compares the assessment of PSTs' preparedness using two constructs: Vision of High-Quality Mathematics Instruction with Technology (VHQMIw/T) and Technological Pedagogical Content Knowledge (TPACK). To unpack this, we explore the journey of Avery, a prospective secondary teacher, within the context of Teaching Secondary Mathematics with Technology course. The study finds that while TPACK focuses on technological integration, VHQMIw/T may offer a more comprehensive understanding of PSTs' preparedness, especially in envisioning instructional practices with technology. The authors recommend using both constructs to assess PSTs' preparedness effectively.

Keywords: Instructional Vision, Preservice Teacher Education, Technology

The field of mathematics education has long agreed on the importance of secondary mathematics teachers being prepared to support students' learning of mathematics using technology that supports students' mathematical reasoning and sense making (ISTE, 2000, 2017; AMTE, 2017, 2022; NCTM, 2014, 2023). However, assessing the development of prospective mathematics teachers (PSTs) toward this goal is difficult (e.g., Abbitt, 2011). Researchers have called for the use of PSTs' instructional vision (Hammerness, 2001) to assess the development of their pedagogical practices during teacher preparation programs (Fieman-Nemser, 2001; Arbaugh et al., 2021). Munter (2014) described instructional vision as "ways of seeing the world that encompass horizons not yet reached" (p. 587). While instructional vision has been shown to be a helpful construct to assess PSTs' preparedness to teach mathematics (e.g., Arbaugh et al., 2021; Walkowiak, et al., 2015), to date there is scant research on the use of instructional vision to assess PSTs preparedness to teach mathematics with technology. Rather, the most common way of assessing PSTs' preparedness to teach with technology is through attending to the development of their specialized knowledge for teaching mathematics with technology referred to as technological pedagogical content knowledge (TPACK; Mishra & Koehler, 2006). The purpose of this paper is to compare and contrast what we can learn about PSTs' preparation to teach secondary mathematics with technology through attending to these two constructs, vision of high-quality mathematics instruction with technology (VHQMIw/T) and TPACK.

Theoretical Frameworks

Our approach to preparing PSTs to teach mathematics with math action technologies (MAT: Dick & Hollebrands, 2011) suggests two different ways of framing, and assessing, their development, 1) attending to the development of their VHQMIw/T or 2) attending to the development of their TPACK. We describe both theoretical approaches in the sections that follow.

VHQMIw/Tech

Instructional vision is a discourse that teachers (including PSTs) "employ to characterize the kind of ideal classroom practice to which they aspire but have not yet necessarily mastered" (Munter & Wilhelm, 2021, p. 343). As such, one's instructional vision is an expression of their appropriation of the principles, frameworks, and ideas about teaching and learning that they have encountered through their personal and professional learning experiences (Munter & Wilhelm, 2021). Munter (2014) described a specific vision of mathematics instruction deemed "high-quality" that is aligned with the literature on effective mathematics instruction, guiding frameworks in mathematics education (e.g., NCTM, 2014) and data collected from the Middle School Mathematics and the Institutional Setting of Teaching project (Cobb & Smith, 2008). As our work is in the context of using MATs, we refer to discourse about high-quality mathematics instruction that incorporates MATs in ways that are aligned with the literature on effective teaching with technology one's VHQMIw/T.

Like Munter's VHQMI, researchers have long characterized successful technology integration with a specific vision toward "constructivist, student-centered technology use" that includes "active and collaborative learning through authentic problem solving and knowledge construction" (Kopcha et al., 2020 p. 730). In fact, Kopcha et al. (2020) point out that many of the frameworks used to describe technology integration characterize student-centered approaches as high-quality. Thus, when considering a VHQMIw/T in the context of using MATs, the only real difference from Munter's VHQMI should be in the type of task used during instruction.

To operationalize how closely one's vision is aligned with the specific VHQMI described in the literature, Munter (2014) created three interrelated rubrics: *role of teacher, classroom discourse,* and *mathematical tasks*. Each dimension has its own 5-point rubric indicating a trajectory of VHQMI with 4 as the highest and 0 as the lowest. We extensively adapted the *mathematics tasks* rubric since the curriculum materials used in our work focused specifically on the use of tasks that include MATs. On the adapted VHQMIw/T *technology-enhanced mathematics task* rubric the descriptions are parallel to Munter's. A score of 0 or 1 indicates that the PST does not envision using a MAT and either "does not view tasks as a manipulatable features of classroom instruction" (0) or "emphasizes tech tasks that provide students with an opportunity to practice a procedure before applying it conceptually to a problem" (1). To score a 2 or higher it must be clear that a MAT is being used in the task. A score of 2 emphasizes "'reform'-oriented aspects of MAT tasks [e.g., "explore," "higher-order"] without elaborating on their function in terms of learning mathematics—often more about motivation/engagement", 3 emphasizes "MAT tasks with multiple solution paths, potential for complex thinking/problem-solving, but no emphasis on generalization, connections btw strategies/representations, etc.", and a score of 4 is characterized by an emphasis on MAT use in ways "that have the potential to engage students in 'doing mathematics' Munter (2014, p. 633)".

TPACK

Researchers in teacher education have built upon Shulman's (1986) notions of teachers' pedagogical content knowledge (PCK). Grossman (1989, 1990), Even (1990), and Hill et al. (2008) drew upon ideas of PCK and further delineated specific constructs for mathematics within PCK. However, none of this work considered the knowledge that comes with teaching mathematics with technology. In 2005, Niess adapted Grossman's (1989, 1990) components of PCK to take technology into consideration and referred to technology-enhanced PCK. In 2006, Mishra and Koehler identified such knowledge as TPACK. The TPACK framework describes the type of knowledge teachers need to understand how to use technology effectively to teach specific subject matter. In 2013, Neiss identified four components of TPACK with detailed descriptors and claimed they provide insight for "developing a transformed knowledge [of] (TPACK)" (p. 196). These four components: "(1) an overarching conception of what it means to teach a particular subject integrating technology in the learning; (2) knowledge of instructional strategies and representations for teaching particular topics with technology; (3) knowledge of students' understandings, thinking, and learning with technology in a particular subject; and (4) knowledge of curriculum and curriculum materials that integrate technology with learning in the subject area" (Neiss, 2005, p. 511) capture the skills teachers need to develop TPACK.

Many strategies have been used to assess TPACK including teacher interviews, team planning and classroom observations, self-reported surveys, open-ended questionnaires, and performance-assessment instruments (Mouza et al., 2014). Studies that use validated instruments to assess performance-assessments or artifacts of teaching have proven quite useful (e.g., Harris et al., 2010; Hofer et al., 2011; Lyublinskaya & Tournaki, 2012) in providing a glimpse into teachers' TPACK applied in their classrooms. Lyublinskaya and colleagues used the four levels as a guide to develop a validated TPACK Levels Rubric (Lyublinskaya & Tournak, 2012; Lyublinskaya & Kaplon-Schilis, 2022) used to code teachers' technology-enhanced teaching artifacts (e.g., lesson plans) for evidence of their TPACK.

Lyublinskaya and Tournak's (2012) rubrics include five levels of TPACK development (1-Recognizing, 2-Accepting, 3-Adapting, 4-Exploring and 5-Advancing) which are applied to each of Neiss' (2009, 2013) four components of TPACK. At the lowest level, 1-Recognizing, teachers use technology as a motivational tool, not to support students' mathematical thinking and are focused on rote practice. At level 2-Accepting, the use of technology is instructor led and focused on teacher delivery of information often mirroring traditional textbook material. For the middle level 3-Adapting, teachers begin to use technology as a source of student inquiry to support students' mathematical thinking under direct teacher guidance and without opportunity for student reflection; math action technologies may or may not be used at this level. At level 4-Exploring, students become the primary driver of explorative technology making full use of math action tools within the technology, however the teacher "still guides the students to see the meaningful consequences of those actions" (Lyublinskaya & Tournak, 2012). At the highest level, 5-Advancing, students are provided opportunities to explore, make conjectures, reflect and develop their own conceptual understanding of mathematical concepts.

Methods

This is an instrumental case study (Stake, 1995) of a single PST, Avery, who participated in a course titled *Teaching Secondary Mathematics with Technology*. Avery was a mathematics major and secondary mathematics education minor. This was the first mathematics specific methods course. The course was designed using principles of practice-based teacher education (Grossman et al., 2009), used curriculum materials from the Preparing to Teach Mathematics with Technology – Examining Student Pracices project [PTMT-ESP], and the course text was *Exploring Math with Technology: Practices for Secondary Teachers* (McCulloch & Lovett, 2024). Throughout the course Avery had many opportunities to engage in high-quality technology-enhanced math tasks as a learner, to analyze the design of these tasks, to analyze video of students and teachers engaged with these tasks, to create technology-enhanced math tasks, and to plan lessons that incorporate technology-enhanced math tasks. Through these experiences, Avery was developing both their VHQMIw/T and their TPACK. Avery was selected as the case for this study because they scored high on both VHQMIw/T and TPACK at the end of the semester, we hoped this similarity would provide insight to what we can learn about a PSTs' preparedness from these two different perspectives. As such, we aim to answer the following research question: What are the similarities and differences in what we can learn about PSTs' preparation to teach mathematics with technology-enhanced mathematics tasks by attending to their VHQMIw/T and their TPACK?

For the purposes of this study, we are focusing on artifacts from the end of the semester to understand Avery's preparation to teach secondary mathematics with technology at that point in time. This includes his description of his VHQMIw/T and a technology-enhanced math task that he created along with its accompanying lesson plan. The data sources and our analysis of them are described in the sections that follow.

TPACK: Data Source and Data Analysis

All PSTs enrolled in the *Teaching Secondary Mathematics with Technology* course along with Avery, were asked to design a technology task and an accompanying lesson plan. In a prior lesson PSTs had engaged in a sequence of approximations of practice related to a Desmos Activity designed to introduce amplitude, midline, and period of the sine function (tinyurl.com/IntroSine). This included anticipating student thinking, noticing student thinking, scripting whole class discussion, and analyzing video of the classroom teacher as she monitored small groups and facilitated a whole class discussion. At the end of this sequence the PSTs were assigned to design a follow up lesson. They had the option to create a task and lesson that either a) provides an opportunity for the students *to further develop* their understanding of amplitude, midline, and period related to sine functions and their graphs, or b) provides an opportunity for her students *to apply* their knowledge of amplitude, midline, and period to a real context through modeling, or c) an investigative task intended *to introduce* phase shift to go along with amplitude, midline, and period. For this assignment, Avery chose option c.

To analyze the tasks and accompanying lesson plans, we used the TPACK Levels Rubric (Lyublinskaya & Tournak, 2012; Lyublinskaya & Kaplon-Schilis, 2022) to capture the PSTs' TPACK levels across the four dimensions: overarching conception, knowledge of student understanding, knowledge of curriculum, instructional strategies. Each dimension has its own 5-point rubric indicating a growth trajectory of TPACK with 5 as the highest and 1 as the lowest. All tasks and accompanying implementation plans were coded by four researchers and then discrepancies were discussed until consensus was reached across the four coders. A composite score was computed (i.e., the sum of the four dimensions).

VHQMIw/T: Data Sources and Data Analysis

Similarly, all PSTs enrolled in the course with Avery responded to a vision prompt adapted from Munter (2014): If you were asked to observe a technology-using math teacher's classroom for one or more lessons, what would you look for to decide whether the mathematics instruction (including the use of technology) was high quality? In your response make sure you describe what you would expect to see/hear from the teacher, students, and mathematical tasks during your observations.

To analyze the PSTs' vision statements, we used our adapted version of Munter's VHQMI rubrics. These rubrics include 3 interrelated dimensions: role of teacher, discourse, and technology-enhanced math tasks. Each rubric indicates alignment with a research informed VHQMIw/T with 4 being the highest, 0 the lowest, and N/A indicating the dimension was not included in the PSTs' vision statement. Like the TPACK analysis, all vision statements were coded by four researchers and then discrepancies were discussed until consensus was reached across the four coders.

Findings

Avery's scores for both VHQMIw/T and TPACK are found in Table 1. Based on the analysis of their vision statement and lesson plan artifacts, Avery would be described as having a VHQMIw/T that is aligned with the literature on effective teaching and learning with technology and TPACK that is aligned with the advancing level of technology integration. In what follows we unpack what we learn from these rubric scores about Avery's preparedness to teach mathematics with technology-enhanced tasks. We begin with VHQMIw/T, then TPACK, and finally we compare and contrast the two.

VHQMIw/T Rubric Scores (max: 4)		TPACK Rubric Scores (max: 5)	
Role of Teacher	4	Overarching conception	5
Discourse	4	Knowledge of student understanding	5
Tech-enhanced math task	4	Instructional strategies	4
		Knowledge of curriculum	5
Composite Score	4	Composite Score	4.75

Table 1: Avery's TPACK and VHQMIw/T Rubric Scores

Avery's VHQMIw/T

When responding to the VHQMIw/T prompt, Avery began by clearly stating their overarching VHQMIw/T and then went on to describe how to achieve that vision. Avery articulated their VHQMIw/T as,

> A high quality, equitable math instruction with technology would include the following: students exploring the mathematics with dynamic math technology, each student/group working with the technology equally, and a lesson designed to incorporate the student's understandings and work into a discussion that furthers the whole-class understanding around the mathematics topic.

In terms of the *role of the teacher*, Avery expanded on this statement by describing that when students are working on a task, a teacher's role is to "ask students assessing questions that help the teacher see students' understanding then ask advancing questions that lead students to a more developed understanding." Avery envisions that an instructor "guides and facilitates, occasionally may need to step in to reach the necessary daily goals, but should almost never directly explain a topic in full without student input." Avery scored a 4 on the *role of the teacher* rubric since they described the teacher's role as more than just a facilitator. By describing that the teacher "should almost never directly explain a topic in full without student input" it is clear that Avery's vision is the teacher as a more knowledgeable other.

With respect to *discourse* Avery also wrote that their VHQMIw/T includes a "full-class discussion around understanding the mathematics." Avery expanded on this stating that "an instructor should facilitate a discussion on the topics using the students' work. This is done through carefully cultivating the responses and ordering them for discussion." Avery went on to explain that a teacher "should lead the discussion by having open-ended questions posed to them that allow them to identify, compare, contrast, and critique the responses." Avery provides concrete images of students learning from each other. Therefore, Avery scored a 4 on the *classroom discourse* rubric.

Finally, with respect to t*echnology-enhanced tasks*, Avery noted that students should explore mathematics and expanded this idea noting that a high-quality technology-enhanced math task "must be dynamic, it must allow students to explore and notice facts and relationships about the topics being presented." Here Avery focused on connections between the mathematical ideas and described a task that would align with "doing mathematics" (Smith & Stein, 1998), therefore Avery scored a 4 on this rubric.

Avery's TPACK

For three of the four components of TPACK – *overarching conception, knowledge of student understanding,* and *instructional strategies* – Avery scored at the advancing level (5). With regard to their *overarching conception*, Avery's lesson plan included a high-cognitive demand (Smith & Stein, 1998), technology-enhanced task built in Desmos Activity Builder. The task focused on developing students' understanding of phase shift and provided opportunities for inquiry and reflection. For example, Avery's task asked students to explore the relationship between the parameters of the sine function when a new parameter was added. Students were then asked to write down what they notice when examining the slider for *h*. This example demonstrates the connection to conceptual understanding through both inquiry and reflection that was seen throughout the task.

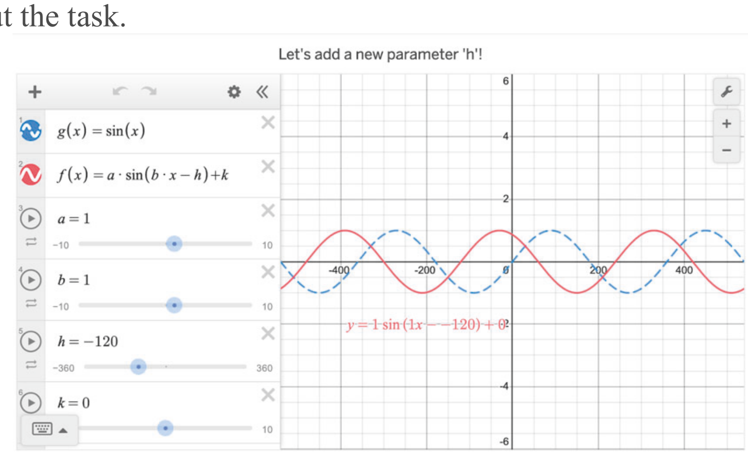

Figure 1: Students could explore the phase shift of the function using sliders

Within *knowledge of student understanding,* Avery's advanced rating of 5 was evident in Avery's task that relied on students taking mathematical actions (using sliders in Desmos) to examine the impact of *h* and *b*, see the consequences of their mathematical actions, and draw conclusions about phase shift (i.e., phase shift = *h/b*) based on these consequences. The lesson plan included how Avery planned to use the resulting student thinking to facilitate discussions and make connections across multiple representations, allowing student thinking to drive the direction of the lesson. For example, Avery included,

> After students have completed the activity, I plan to facilitate a whole class discussion on phase shift. I will ask students to share their noticing looking for…
>
> - An informal description of how the h slider alters the graph (horizontally shifts)
> - A description of the term phase shift
> - An informal description of the relationship between the slider and phase shift
> - A precisely described mathematical relationship between h, b, and phase shift (direct with h-h/b)

I want to make sure students connect the horizontal shift with phase shift and understand their slight differences…

Avery's advanced rating (i.e., score of 5) for *instructional strategies* was due to the use of sliders within the mathematical task that provided students with an inductive strategy that effectively supported students' exploration of phase shift and included prompts to promote reflection and sense making/reasoning.

With respect to the remaining component, *knowledge of curriculum*, Avery demonstrated an exploring level related to TPACK (i.e., score of 4). This was evidenced in how effectively the task was aligned to the learning and performance goals included in the lesson plan, and how the task provided students an alternative way to explore the mathematical topic (i.e, alternative to using paper and pencil methods) and expand on the mathematical ideas they build with respect to amplitude, midline, and period in the prior lesson through their exploration of the function. Avery's lesson did not score a 5 on the *knowledge of curriculum* rubric because the task did not make connections outside of the curriculum or challenge the traditional curriculum to have students learn different topics.

Comparing and Contrasting Avery's VHQMIw/T and TPACK

Both the VHQMIw/T and TPACK rubric scores suggest that Avery is well-prepared to teach mathematics with technology. What we are curious about is what we learn about Avery's preparation from each of these measures. We begin with the more commonly used construct, TPACK.

Since Avery was in a class focused on teaching mathematics with technology and the lesson planning assignment at the end of the semester required the inclusion of a technology-enhanced task that included a MAT, as long as they created a lesson that met the requirements of the assignment his TPACK rubric scores were going to all be 3, 4 or 5, leaving little room for variability. The TPACK rubrics highlight that Avery is well-prepared to design a high-cognitive demand technology-enhanced task, aligned with learning goals, that provides ways for students to interact with the mathematical objects and prompts them to both explore and reflect. However, we know less about how Avery plans for students and teachers to interact with each other when engaging with the task. The *knowledge of student understanding* and *instructional strategies* rubrics do indicate that the teacher plans to facilitate students' use of the technology in ways that lead to deep understanding of mathematics, and that they will use both deductive and inductive strategies to do so. So there is a sense from the rubric scores that Avery will carefully facilitate the implementation of the technology-enhanced task and resulting whole-class discussion, but this does not provide insight to how Avery's hypothetical students will interact with each other's ideas.

In contrast, the VHQMIw/T rubrics provide insight to not only what Avery envisions a high-quality technology-enhanced math task to be, but also provides further details for how Avery envisions the teacher and students' interacting around such a task. The *technology-enhanced mathematics task* rubric indicates that Avery designed a task that uses a MAT and is of high-cognitive demand, to score a 4 on this rubric they also had to explain how such a task would support student learning (i.e., function view). This additional explanation is not captured in the TPACK rubric. The *role of the teacher* rubric score indicates that Avery envisions the teachers' role as a more knowledgeable other who is proactively supporting students' learning through anticipating student thinking as part of the lesson planning process and then during the lesson, using student work to drive whole class discussions around the important mathematical concepts and connections. Thus, Avery is not only planning to use deductive and inductive strategies as indicated in the TPACK rubrics, but is going to leverage the students' thinking to drive the deepening of their understanding. Finally, the *discourse* rubric indicates that Avery envisions students' learning from each other, with the mathematical discourse often being student initiated and students talking to each other, not solely through the teacher. None of the TPACK rubrics capture the nature of the planned discourse.

Discussion and Conclusion

Comparing and contrasting VHQMIw/T and TPACK using Avery's work at the end of a course focused on preparing PSTs to teach secondary mathematics with technology does reveal some differences in what we can learn about PSTs' preparedness using these two constructs. The most striking finding is that the TPACK rubrics do not capture the nature of planned discourse, including how one envisions the role of the teacher during small group and whole class instruction. The VHQMIw/T *role of teacher* and *discourse* rubrics do capture these important aspects of mathematics instruction. In their review of technology-enhanced pedagogy in teacher learning, Zinger et al. (2017) called for less attention to PSTs' use of technological tools and more attention on the role of the teacher in using those tools to address problems of practice. Our findings suggest that VHQMIw/T might be a helpful framing for researchers taking on that work.

PSTs often do not have field experiences in courses in which they are learning to teach mathematics with technology (McCulloch et al., 2021), making assessing TPACK based on their practice difficult. With this in mind, rather than attending to their enacted instruction, researchers have called for attending to PSTs' instructional vision as an indication of their developmental progress during teacher preparation programs (e.g., Feiman-Nemser, 2001), noting that changes in instructional vision often occur before changes in practice (e.g., Munter, 2014). Our findings suggest that the VHQMIw/T rubrics do provide insight into how PSTs envision the design of technology-enhanced mathematics tasks – the main focus of the TPACK rubrics – while also providing insight to how they hope to one day facilitate students' working on such a task.

Based on the findings in this study, we would ultimately recommend using both the VHQMIw/T and TPACK constructs to understand PSTs' preparedness to teach mathematics with technology. However, assessing TPACK as a pre-test is difficult to do when PSTs have yet to be introduced to teaching mathematics with technology (i.e., it is unfair to ask them to design a technology-enhanced mathematics task and accompanying lesson plan when they have not yet been taught how to do so), yet VHQMIw/T can provide insight to what they aspire. It is not uncommon for researchers to use self-reported TPACK measures like self-efficacy or beliefs as pre/post measures alongside TPACK as post measures (e.g., Akapame et al., 2019). We recommend attending to instructional vision over other self-reported measures as the latter "suggest a relatively static set of decontextualized ontological commitments" and "vision is intended to communicate a more dynamic view of the future" (p. 587). To further compare these two constructs, future work following PSTs into the field to study how their VHQMIw/T informs their practice and whether or not their practice aligns with their VHQMIw/T and enacted TPACK would be useful.

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