# Performance Evaluation of Two-Machine Bernoulli Lines with Conveyor Buffers

Yishu Bai, Tianyu Zhu, Liang Zhang

***Abstract*—Serial production lines with conveyor buffers are commonly seen in production practice, where the work-in-process parts are initially positioned on the conveyor belt and then transported to the downstream machine to be processed. The time taken for this transfer, referred to as transportation time, is a significant factor in the overall efficiency of the production line and cannot be overlooked. In this paper, an analytical approach is developed, providing exact solutions to transient and steady-state performance metrics evaluation in two-machine Bernoulli lines with conveyor buffers. Numerical experiments demonstrate the precision and efficiency of the proposed approach. A sensitivity test highlights the significant impact of conveyor speeds on productivity optimization.**

## I. Introduction

Performance evaluation of production lines stands as a critical area of study in the domain of production systems engineering and operations research and has attracted significant attention from researchers (see [1]–[4]). Among various mathematical models for production line analysis, Bernoulli serial lines represent a significant archetype due to their convenient application in the manufacturing process such as automotive assembly and electronics manufacturing discussed in [5]. As the simplest layout for a mass production system, the serial production line serves as the fundamental building block for both practical implementation and theoretical analysis of complex production systems. A serial production line is a highly structured manufacturing process in which work-in-process items are produced sequentially, moving from one machine (or workstation) to the next in a fixed, linear order. An intermediate buffer is typically present between two consecutive machines, serving as a temporary storage area to accommodate work-in-process jobs. These buffers play a crucial role in maintaining the smooth operation of the production line by mitigating the effects of variability in processing times and machine downtimes (see [6], [7]). For the sake of simplicity, most research models an intermediate buffer as a static storage area with a fixed capacity. This capacity is defined as the maximum number of parts that the buffer can accommodate at any given time. The buffer inventory level, which represents the current number of parts within the buffer, is used to denote the buffer state. Performance evaluation for such systems in both transient and steady-state have been extensively studied [8]–[11]. In addition, the latest research explores several advanced topics including identifying real-time bottleneck operations to achieve productivity optimization [12], developing control policies to minimize energy consumption [13], [14], investigating optimal buffer storage strategies such as echelon buffer policy [15], leveraging machine learning techniques to perform preventive maintenance [16], etc.

Despite this exciting progress, the studies mentioned above predominantly focused on Bernoulli lines with static buffers (i.e., with no transportation time in the buffer between consecutive operations), while those with conveyor buffers receive limited attention in the literature. One of the distinctive features of conveyor buffers, as opposed to static buffers, is the introduction of non-negligible delivery time for work-in-process parts. This distinction becomes increasingly relevant with the rise in manufacturing complexity, where static buffer models fall short in capturing the dynamics of real-world production systems, as conveyor buffer systems more accurately reflect the continuous movement of work-in-process items through production lines. However, the dynamic nature of conveyors requires a more sophisticated approach to modeling and analysis by considering factors such as conveyor speed, buffer capacity, and the position of each work-in-process part. Two studies presented in [5] and [17] describe an approximation method that converts a conveyor buffer into a static buffer with a calculated capacity based on conveyor length and travel speed. This approximation technique allows for a simplified steady-state analysis with fair accuracy in most cases. However, it tends to yield large approximation errors particularly when the two consecutive machines are unbalanced or the transportation time is large (see Section IV for more details). Also, the approximation error may accumulate and propagate when extending to longer production lines.

To the best of our knowledge, no exact analytical model is available to accurately model conveyor buffers and then carry out performance metrics calculation in production systems with conveyor buffers. Addressing the complexities associated with transportation times and conveyor dynamics will provide a more comprehensive understanding of production systems and enhance the applicability of research findings to real-world manufacturing scenarios. Thus, this study is devoted to achieving this for the case of Bernoulli serial lines with two machines and one conveyor buffer. Extension to longer lines will be carried out in future work. In addition, it is important to distinguish between the conveyor loop system

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with loading and unloading stations in Flexible Manufacturing Systems (FMS), as introduced in [18], and the conveyor buffer concept discussed in this study. In an FMS, multiple loading and unloading stations are connected as a closed loop through a conveyor belt. Each loading station independently inputs jobs under a certain rate onto the conveyor loop, where jobs travel until reaching the next available unloading station and exit the loop, thereby, the loading and unloading stations operate without a predetermined sequence. In this paper, the conveyor buffer serves as the immediate storage area between two predetermined consecutive machines in a serial production line.

The remainder of this paper is structured as follows: Section II describes the model assumptions and performance metrics of interest. Section III derives the mathematical model and formulas for transient and steady-state performance evaluation. Section IV presents the experiment setup and the results of our analysis. Finally, Section V concludes the paper with a summary of our findings and suggestions for future research directions.

## II. System Model Assumptions and Performance Metrics

### A. Model assumptions

The system considered in this paper, as illustrated in Fig. 1, is defined by the following assumptions:

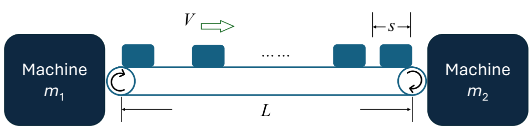


Fig. 1. System diagram

- (i) The system consists of two machines, $m_1$ and $m_2$, connected by a conveyor buffer $b$.
- (ii) The cycle times of the machines are identical and equal to $\tau$ seconds.
- (iii) The time axis is discrete with each time slot having a duration equal to $\tau$ seconds.
- (iv) The machines follow the Bernoulli reliability model, i.e., during each time slot, machine $m_i$ is up with probability $p_i$ and down with probability $1-p_i$, $i = 1, 2$. Parameter $p_i$ is referred to as the efficiency of machine $m_i$. The status of the machines (up or down) is determined at the beginning of each time slot.
- (v) Each part occupies a linear space of $s$ ft when it is on the conveyor (including the dimension of the part itself and necessary spacing before and after the part).
- (vi) The conveyor has a total length of $L$ ft which is a multiple of $s$. In other words, the conveyor can accommodate a total of $K = L/s$ parts at maximum. $K$ is also referred to as the capacity of the conveyor in subsequent discussions.
- (vii) The conveyor does not experience breakdowns and moves continuously at a constant speed of $V$ ft/second. Taking the part space $s$ and the cycle time $\tau$ into account, we further define the *normalized travel speed* of the conveyor as $v = \frac{V\tau}{s}$ part-space per cycle time.
- (viii) When a part reaches the end of the conveyor, it remains there until being picked up by $m_2$, while other parts continue moving towards the end, maintaining the space requirement $s$ for each part.
- (ix) If machine $m_2$ is up during a time slot and a part is present at the end of the conveyor, the machine picks up the part, processes it during the time slot, and releases it from the production system at the end of this time slot. Otherwise, no production activity on $m_2$ takes place during this time slot.
- (x) If machine $m_1$ is up during a time slot and a space of at least $s$ ft is available at the input of the conveyor, then the machine picks up one raw part, processes it during the time slot, and places it on to the conveyor at the end of the time slot. Otherwise, no production activity on $m_1$ takes place during this time slot.

**Remark 1:** Without loss of generality, assume that the moving velocity of the conveyor is at least $\frac{s}{\tau}$ ft/sec. In other words, if not full, the conveyor can create a space to accommodate at least one new part in one cycle time and, thus, does not slow down production.

**Remark 2:** It follows from assumptions (vi) and (vii), it takes at least $\frac{(K-1)s}{V}$ seconds for a part released by $m_1$ to reach the other end of the conveyor. However, since the machines start operation at discrete time instants with interval $\tau$ seconds, it would take at least $T = \lceil \frac{K-1}{v} \rceil$ time slots (cycle times) for a part to reach $m_2$.

### B. Performance metrics

Under the assumptions defined in Subsection II-A, the performance metrics of the production system in transient and steady states can be defined as follows:

*Production Rate*, $PR(t)$ = the expected number of finished parts produced by $m_2$ during the time slot $t$;

*Work-In-Process*, $WIP(t)$ = the expected number of parts on the conveyor buffer $b$ by the beginning of time slot $t$;

*Machine Starvation*, $ST(t)$ = the probability that $m_2$ is starved by buffer $b$ at the beginning of time slot $t$;

*Machine Blockage*, $BL(t)$ = the probability that $m_1$ is blocked by buffer $b$ at the beginning of time slot $t$;

The steady-state performance measures represent the long-term behavior of production lines and, thus, the transient performance measures will converge to steady-state measures when $t \to \infty$.

$$\begin{aligned} &PR_{ss} = PR(t), WIP_{ss} = WIP(t), \\ &ST_{ss} = ST(t), BL_{ss} = BL(t), t \to \infty \end{aligned} \quad (1)$$

In this paper, we will develop analytical methods to calculate the system state evolution and the transient and steady-state performance metrics.

## III. Mathematical Model

Under assumptions discussed in Section II-A, a Bernoulli serial line with a conveyor buffer is characterized by an ergodic Markov chain with the states defined by both the number of parts in the buffer and the position of each part on the conveyor. In this section, we formulate the mathematical representation of the system states, derive an algorithm to identify all feasible states, develop a procedure to calculate state transition probabilities and provide formulas for performance metrics calculation.

### A. System state representation

For the system considered in the paper, let $h$ denote the number of parts in the buffer (i.e., on the conveyor) at the beginning of a time slot. Apparently, $h \in \{0, 1, 2, .., K\}$. For all $h$ parts on the conveyor, assign index $i$, $i \in \{0, 1, \dots, h\}$ to each one from the input end of the conveyor to the output end of the conveyor. Then, let $n_i$ denote the position of part $i$ on the conveyor, measured from the input end of the conveyor to the farthest side of the part in the unit of part-space (see Fig. 2 for illustration).

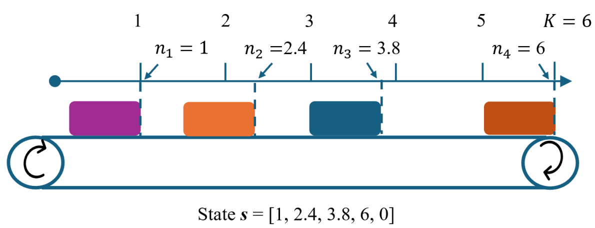


Fig. 2. Illustration of system state

Under this formulation, a part that is just finished by $m_1$ and placed on the conveyor shall have index $i = 1$ and position $n_1 = 1$; similarly, the part at the output end of the conveyor waiting to be processed by $m_2$ shall have index $h$ and position $n_h = K$. Clearly, $n_i$ is a real number within the range $[1, K]$ for all $i \in \{1, 2, \dots, h\}$. Based on this formulation, we define the state of a system defined by assumptions (i)-(x) as a 1-by-$K$ row vector $\boldsymbol{s}$ with the first $h$ element being $n_1, n_2, \dots, n_h$, and the rest are 0's; for the case of $h = 0$, i.e., when the conveyor is empty, all elements of the vector is 0; for the case of $h = K$, i.e., when the conveyor is full, the parts are position one next to the other and the corresponding state vector is $\boldsymbol{s} = [1,\ 2,\ \dots\ K]$. This leads to

$$\boldsymbol{s} = \begin{cases} [0,\ 0,\ \dots, 0] & \text{for } h = 0 \\ [n_1,\ n_2,\ \dots\ n_h,\ 0,\ \dots,\ 0], & \text{for } h \in \{1, \dots, K-1\} \\ [1,\ 2,\ \dots, K] & \text{for } h = K \end{cases} \tag{2}$$

It should be noted when the conveyor is neither empty nor full, i.e., when $h \in \{1, \dots, K-1\}$, different positioning of the parts on the conveyor may be possible, thus leading to different state vectors corresponding to the same $h$. Moreover, as parts travel downstream on the conveyor, accumulation may occur at the output end when $m_2$ is down. As a result, the accumulated parts at the end of the conveyor may be still or move at a speed slower than $v$. Based on the above formulation and taking into account the accumulation phenomenon, the part positions $n_i$'s must satisfy the following constraints:

$$\min\{n_i - n_{i-1}\} = 1, \quad i = 2, \dots, h, \tag{3}$$

$$n_i \in \{x | x = 1 + vt, t = 0, 1, \dots, T\} \cup \{K - (h - i)\}, \quad i = 1, 2, \dots, h. \tag{4}$$

We refer to the state vector $\boldsymbol{s}$ that satisfies the above constraints as *feasible* system states as only those can appear in the system considered in this paper. Let $\boldsymbol{S}^h$ denote the set of all feasible system states under buffer occupancy $h$ and let $N_h$ denote the number of state vectors in $\boldsymbol{S}^h$. Clearly, $\boldsymbol{S}^0 = [0, 0, ..., 0]$ and $\boldsymbol{S}^K = [1, 2, \dots, K]$ when $h = 0$ and $K$, respectively, and $N_0 = N_K = 1$. An example of all feasible system states is shown in Table I for a system with $v = 1.2$ part-space per cycle time and $K = 5$ to illustrate the possible states of such a system.

TABLE I
EXAMPLE OF SYSTEM STATES IN A TWO-MACHINE BERNOULLI LINE WITH A CONVEYOR BUFFER

| $h$ | $\boldsymbol{S}^h$ | | |
|---|---|---|---|
| 0 | | [0, 0, 0, 0, 0] | |
| 1 | [1, 0, 0, 0, 0] | [2.2, 0, 0, 0, 0] | [3.4, 0, 0, 0, 0] |
| | [4.6, 0, 0, 0, 0] | [5, 0, 0, 0, 0] | |
| 2 | [1, 2.2, 0, 0, 0] | [1, 3.4, 0, 0, 0] | [1, 4.6, 0, 0, 0] |
| | [1, 5, 0, 0, 0] | [2.2, 3.4, 0, 0, 0] | [2.2, 4.6, 0, 0, 0] |
| | [2.2, 5, 0, 0, 0] | [3.4, 4.6, 0, 0, 0] | [3.4, 5, 0, 0, 0] |
| | [4, 5, 0, 0, 0] | | |
| 3 | [1, 2.2, 3.4, 0, 0] | [1, 2.2, 4.6, 0, 0] | [1, 2.2, 5, 0, 0] |
| | [1, 3.4, 4.6, 0, 0] | [1, 3.4, 5, 0, 0] | [1, 4, 5, 0, 0] |
| | [2.2, 3.4, 4.6, 0, 0] | [2.2, 3.4, 5, 0, 0] | [2.2, 4, 5, 0, 0] |
| | [3, 4, 5, 0, 0] | | |
| 4 | [1, 2.2, 3.4, 4.6, 0] | [1, 2.2, 3.4, 5, 0] | [1, 2.2, 4, 5, 0] |
| | [1, 3, 4, 5, 0] | [2, 3, 4, 5, 0] | |
| 5 | | [1, 2, 3, 4, 5] | |

To identify all feasible system states for any given $v$ and $K$, an algorithm is developed based on definition (2) and constraints (3) and (4). The pseudo-code of the algorithm is presented in Algorithm 1. The algorithm first generates all feasible state vectors with $h = 1$, i.e., the possible positions of the part when there is only one part on the conveyor, and stores them into $\boldsymbol{S}^1$. Then, for each state vector obtained, the algorithm proceeds to explore all possible ways to insert an additional part between the current part, specified in the state vector with $h = 1$, and the output end of the conveyor. This will generate all feasible states with $h = 2$. Next, this process is repeated to generate the feasible states for $h = 3, 4, \dots, K-1$, iteratively. Lastly, the state vectors for $h = 0$ and $h = K$, i.e., $\boldsymbol{S}^0$ and $\boldsymbol{S}^K$, are added to the set of the state vectors obtained. When Algorithm 1 completes all its steps, it will output a matrix $\boldsymbol{States_{all}}$ with $K$ columns and with each row representing one feasible state vector of the system.

### B. State transition probability

To derive the transition probability of this Markov chain, we first arrange the system states sequentially. In this paper, this is accomplished using the same order that each state vector is stored in matrix $\boldsymbol{States_{all}}$ after the completion of Algorithm 1. For the same example in Table I (i.e., for $v = 1.2$ and $K = 5$), the corresponding system state number assignment is shown in Table II.

Let $N$ denote the total number of feasible system states. Then, for each state $i = 1, 2, \dots, N$, we can develop an algorithm to deduce its next possible states and the corresponding probabilities. It is noted that the number of next possible states of each current state can be different. For

**Algorithm 1:** Buffer State Derivation

**Initialization**: Set $\boldsymbol{S}$ and $\boldsymbol{States_{all}}$ as two matrices with $K$ columns and an undecided number of rows, and $\boldsymbol{s} = [0, ..., 0]$.
**for** $t = 1 : T + 1$ **do**
  $n_1 = \min(1 + v(t-1), K)$;
  $\boldsymbol{s}(1) = n_1$;
  $\boldsymbol{S}(t, :) = \boldsymbol{s}$;
  Reset $\boldsymbol{s} = [0, ..., 0]$.
**end**
$\boldsymbol{S}^1 \leftarrow \boldsymbol{S}$;
Reset $\boldsymbol{S}$ and append $\boldsymbol{S}^1$ into $\boldsymbol{States_{all}}$;
**for** $h = 2 : K - 1$ **do**
  $N_{h-1} \leftarrow$ Number of rows of $\boldsymbol{S}^{h-1}$ ;
  **for** $j = 1 : N_{h-1}$ **do**
    $t = 1$;
    $\boldsymbol{s} = \boldsymbol{S}^{h-1}(j, :)$;
    $n_h \leftarrow \min(\boldsymbol{s}(h-1) + vt,\ K)$;
    **while** $n_h < K$ **do**
      $\boldsymbol{s}(h) = n_h$;
      Append $\boldsymbol{s}$ to $\boldsymbol{S}$;
      $t = t + 1$;
      $n_h \leftarrow \min(\boldsymbol{s}(h-1) + vt,\ K)$;
    **end**
    $\boldsymbol{s}(h) = n_h$;
    **for** $k = h : -1 : 2$ **do**
      **if** $\boldsymbol{s}(k) - \boldsymbol{s}(k-1) < 1$ **then**
        $\boldsymbol{s}(k-1) = \boldsymbol{s}(k) - 1$;
      **else**
        break;
      **end**
    **end**
    Append $\boldsymbol{s}$ to $\boldsymbol{S}$;
  **end**
  Eliminate repeated states in $\boldsymbol{S}$;
  $\boldsymbol{S}^h \leftarrow \boldsymbol{S}$;
  Reset $\boldsymbol{S}$ and append $\boldsymbol{S}^h$ to $\boldsymbol{States_{all}}$;
**end**
Set the first row of $\boldsymbol{States_{all}} = \boldsymbol{S}^0 = [0, 0, \ldots, 0]$;
Set the last row of $\boldsymbol{States_{all}} = \boldsymbol{S}^K = [1, 2, \ldots, K]$.

TABLE II
SYSTEM STATES RANK

| State no. $j$ | $s_j$ | | |
|---|---|---|---|
| 1 | | $[0, 0, 0, 0, 0]$ | |
| $2, 3, 4$ | $[1, 0, 0, 0, 0]$ | $[2.2, 0, 0, 0, 0]$ | $[3.4, 0, 0, 0, 0]$ |
| $5, 6$ | $[4.6, 0, 0, 0, 0]$ | $[5, 0, 0, 0, 0]$ | |
| $7, 8, 9$ | $[1, 2.2, 0, 0, 0]$ | $[1, 3.4, 0, 0, 0]$ | $[1, 4.6, 0, 0, 0]$ |
| $10, 11, 12$ | $[1, 5, 0, 0, 0]$ | $[2.2, 3.4, 0, 0, 0]$ | $[2.2, 4.6, 0, 0, 0]$ |
| $13, 14, 15$ | $[2.2, 5, 0, 0, 0]$ | $[3.4, 4.6, 0, 0, 0]$ | $[3.4, 5, 0, 0, 0]$ |
| 16 | $[4, 5, 0, 0, 0]$ | | |
| $17, 18, 19$ | $[1, 2.2, 3.4, 0, 0]$ | $[1, 2.2, 4.6, 0, 0]$ | $[1, 2.2, 5, 0, 0]$ |
| $20, 21, 22$ | $[1, 3.4, 4.6, 0, 0]$ | $[1, 3.4, 5, 0, 0]$ | $[1, 4, 5, 0, 0]$ |
| $23, 24, 25$ | $[2.2, 3.4, 4.6, 0, 0]$ | $[2.2, 3.4, 5, 0, 0]$ | $[2.2, 4, 5, 0, 0]$ |
| 26 | $[3, 4, 5, 0, 0]$ | | |
| $27, 28, 29$ | $[1, 2.2, 3.4, 4.6, 0]$ | $[1, 2.2, 3.4, 5, 0]$ | $[1, 2.2, 4, 5, 0]$ |
| $30, 31$ | $[1, 3, 4, 5, 0]$ | $[2, 3, 4, 5, 0]$ | |
| 32 | | $[1, 2, 3, 4, 5]$ | |

example, consider state $\boldsymbol{s}_1$ shown in Table II being the current state. From this state, there are only two potential next states: state $\boldsymbol{s}_1$ with a probability of $1 - p_1$, and state $\boldsymbol{s}_2$ with a probability of $p_1$. This is due to the potential starvation of $m_2$ by buffer $b$ causing $m_2$ to be incapable of processing a part regardless of its up or down, while state $\boldsymbol{s}_{15}$ will have four potential next states as illustrated in Figure 3. By performing this process for each state, we can derive the system state transition probability matrix $\boldsymbol{P}$ to represent the system dynamics.

Introduce $x_i(t), i = 1, 2, \ldots, N$ as the probability of the Markov chain is in state $i$ at the beginning of time slot $t$,

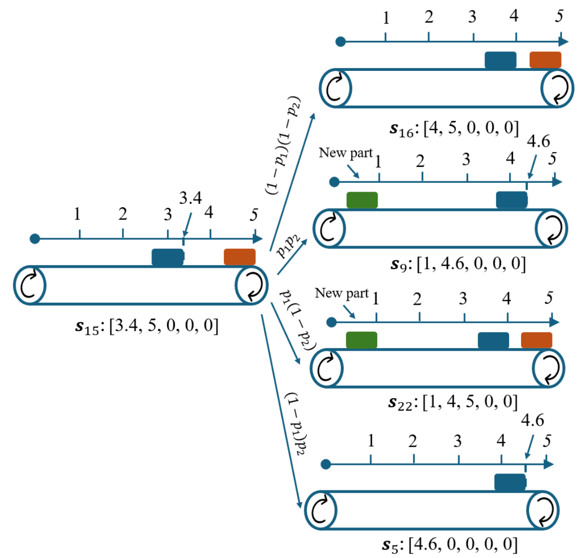


Fig. 3. An example of transition between states

and the state vector $\boldsymbol{x}(t) = [x_1(t), x_2(t), \ldots, x_N(t)]^T$ as the state vector of the system. Then, the evolution of the state vector $\boldsymbol{x}(t)$ can be described as,

$$\boldsymbol{x}(t+1) = \boldsymbol{P}\boldsymbol{x}(t), t = 1, 2, \ldots, \infty \tag{5}$$

It should be noted that $\sum_{i=1}^{i=N} x_i(t) = 1$ for all values of $t$.

### C. Performance metrics calculation

To calculate the performance metrics of a production line with a total number of $N$ states, we first introduce a set denoted as $\boldsymbol{Z_1}$, comprising all state indices $j$ for which the final element of the corresponding state vector $\boldsymbol{s}_j$ is equal to $K$, i.e., a part is at the output end of the conveyor. Then, the performance metrics can be calculated as

$$\begin{aligned} PR(t) &= p_2\boldsymbol{c_1}\boldsymbol{x}(t) \\ WIP(t) &= \boldsymbol{H}\boldsymbol{x}(t) \\ ST(t) &= p_2\boldsymbol{c_2}\boldsymbol{x}(t) \\ BL(t) &= p_1(1-p_2)\boldsymbol{c_3}\boldsymbol{x}(t) \end{aligned} \tag{6}$$

where $\boldsymbol{c_1}$, $\boldsymbol{c_2}$, and $\boldsymbol{c_3}$ are three binary row vectors with

$$\boldsymbol{c}_1(j) = \begin{cases} 1, & \text{if } j \in \boldsymbol{Z_1}, \\ 0, & \text{otherwise}, \end{cases} \tag{7}$$

$$\boldsymbol{c}_2(j) = \begin{cases} 0, & \text{if } j \in \boldsymbol{Z_1}, \\ 1, & \text{otherwise}, \end{cases} \tag{8}$$

$$\boldsymbol{c}_3 = [0\ 0\ \ldots\ 0\ 1], \tag{9}$$

and $\boldsymbol{H}$ vector is defined as

$$\boldsymbol{H} = [0\ \ \mathbf{1}_{N_1}\ \ 2 \cdot \mathbf{1}_{N_2}\ \ldots\ (K-1) \cdot \mathbf{1}_{N_{K-1}}\ \ K] \tag{10}$$

with $\mathbf{1}_D$ indicates a $D$-dimensional row vector of 1's

Similarly, when the steady-state probability distribution $\vec{\boldsymbol{x}}$ of the Markov chain is obtained (e.g., from the probability transition matrix $\boldsymbol{P}$), and the steady-state performance metrics are evaluated as below:

$$\begin{aligned} PR_{ss} &= p_2\boldsymbol{c_1}\vec{\boldsymbol{x}}, \\ WIP_{ss} &= \boldsymbol{H}\vec{\boldsymbol{x}}, \\ ST_{ss} &= p_2\boldsymbol{c_2}\vec{\boldsymbol{x}}, \\ BL_{ss} &= p_1(1-p_2)\boldsymbol{c_3}\vec{\boldsymbol{x}}. \end{aligned} \tag{11}$$

## IV. Numerical Experiments

### A. Transient and steady-state performance metrics

Using the mathematical model derived in Section III, we can use equations (6) and (11) to conduct an exact analysis of performance metrics for the production lines considered in this paper in both transient and steady-state operations. This precise analytical approach eliminates the need to rely on simulation for validating the proposed method. Alternatively, as mentioned in Section I, there exists an approach, presented in [5] and [17], is available to approximate the system performance metrics by converting the conveyor buffer to a static buffer with capacity $N_0 = K - T$. Then the performance metrics for such equivalent systems can be evaluated through the existing methods (such as [10] and [5] for transient and steady-state performance of Bernoulli serial lines). To illustrate this, an example Bernoulli line with parameters $p_1 = 0.7667, p_2 = 0.8611, v = 1.34$, and $K = 5$ is considered. Its transient and steady-state performance measures under both approaches are presented in Figure 4, with an initially empty buffer. As one can see from the figure, it is evident that the approximation method fails to capture the transportation delay time of the conveyor buffer. Notably, the approximation system starts to produce at $t = 2$, in contrast to the original system initiates at $t = 5$ due to a transportation time $T = 3$. Correspondingly, the approximation system experiences blockage earlier and has a larger probability of being blocked because of its less buffer capacity. In addition, as demonstrated in the figure, both systems reach steady-state operations quickly and the approximation method shows good accuracy in steady-state system performance estimation.

To further understand the approximation accuracy of the approximation approach in [5] and [17], 1,000 two-machine Bernoulli lines with randomly selected parameters from

$$p_i \in [0.75, 0.95],\ v \in [1, 4],\ K \in [3, 10], i = 1, 2 \qquad (12)$$

are analyzed. The approximation errors can be evaluated based on

$$\begin{aligned}
\epsilon_{PR_{ss}} &= \frac{|PR^a_{ss} - PR^{approx}_{ss}|}{PR^a_{ss}} \times 100\% \\
\epsilon_{WIP_{ss}} &= \frac{|WIP^a_{ss} - WIP^{approx}_{ss}|}{K} \times 100\% \\
\epsilon_{ST_{ss}} &= |ST^a_{ss} - ST^{approx}_{ss}| \\
\epsilon_{BL_{ss}} &= |BL^a_{ss} - BL^{approx}_{ss}| \qquad (13)
\end{aligned}$$

where the subscripts $a$ and $approx$ indicate the analytical and approximation methods, respectively. The results are documented in Table III and Figure 5. As one can see, the approximation method has higher accurate approximating $PR$ on average, but $\epsilon_{WIP}$ has fewer and lower outliers compared to $\epsilon_{PR}$. After a detailed investigation, it is observed that the approximation approach tends to have larger $\epsilon_{PR}$ if the original system has longer transportation time $T$, while the $\epsilon_{WIP}$ is larger when the efficiency of $m_1$ is significantly greater than $m_2$. It should be noted that, although the approximation approach may get good results under certain conditions, it is not robust and the error may accumulate and propagate when extending to longer lines as reported in [17].

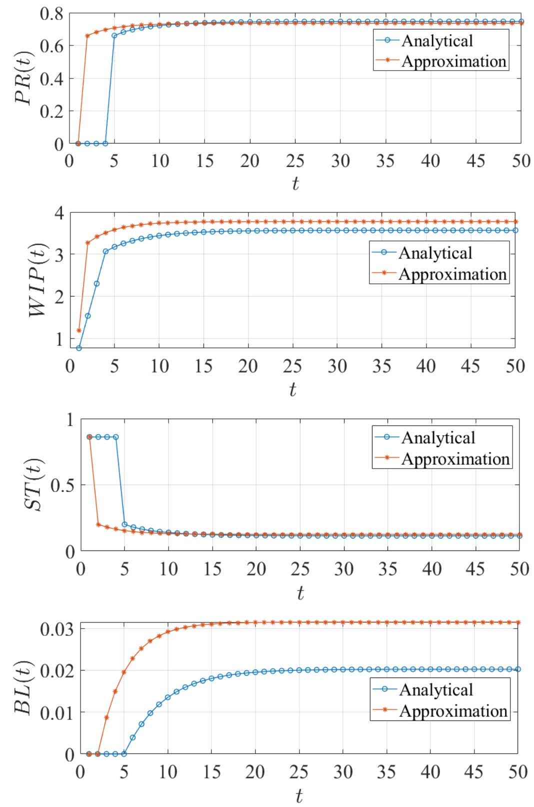


Fig. 4. An example of transient and steady-state performance measures of a two-machine Bernoulli line under both approaches

TABLE III
AVERAGE ACCURACY OF THE APPROXIMATION METHOD IN [5] AND [17]

| $\epsilon_{PR}$ | $\epsilon_{WIP}$ | $\epsilon_{ST}$ | $\epsilon_{BL}$ |
|---|---|---|---|
| 0.83% | 1.79% | 0.0064 | 0.0064 |

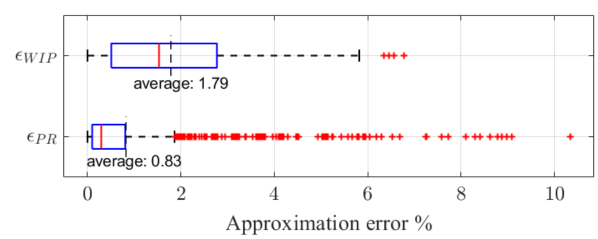


Fig. 5. Boxplot of $\epsilon_{PR}$ and $\epsilon_{WIP}$ of the approximation approach

### B. Effects of conveyor speed on $PR$ and $WIP$

In this section, we investigate the relationship between the conveyor speed and system performance metrics $PR$ and $WIP$. Intuitively, a higher conveyor speed will lead to a smaller transportation time $T$, resulting in shorter delay, larger production rate, and lower $WIP$. To demonstrate this, we experiment on a Bernoulli line with identical machine efficiency $p_1 = p_2 = 0.9$, conveyor buffer capacity $K = 5$, and velocity $v \in \{1, 1.1, 1.2, \ldots, K\}$. The transportation time can be calculated within the range $T \in \{4, 3, 2, 1\}$. The results of $PR$ and $WIP$ as functions of $v$ are presented in Figure 6. As one can see, there exists a step-wise, increasing relationship between $PR$ and $v$, with the *jumps* coinciding with the jumps in $T$ as a function of $v$. Specifically, the initial rise of $PR$ occurs as the conveyor velocity $v$ increases from 1 to about 1.35, at which the $T$ reduces from 4 to 3, then $PR$ remains constant. Subsequently, the second increase is observed at $v = 2$, coinciding with a decrease in $T$ to 2, and once again, $PR$ remains constant until $v$ increases to 4. Beyond this point, further increments in $v$ up to 5 do not influence the $PR$ as $T = 1$ (i.e., no transportation delay for the work-in-process part). $WIP$ demonstrates a similar relationship with $v$ but in a decreasing manner.

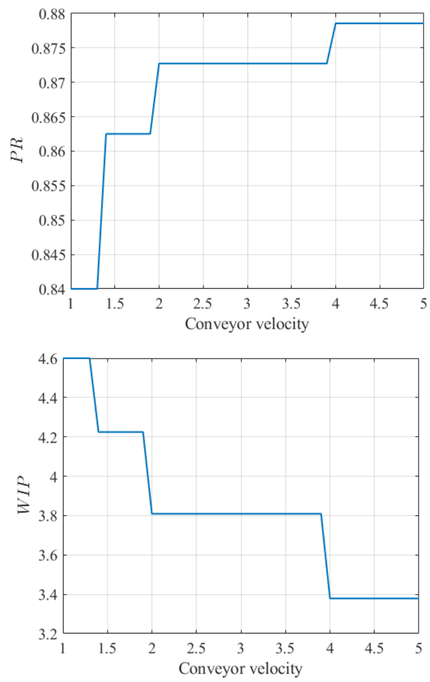


Fig. 6. The $PR$ and $WIP$ values under different conveyor speed $v$

From these observations, it is evident that changes in $PR$ are intrinsically correlated with discrete shifts in $T$, suggesting a threshold effect where $PR$ only increases when $T$ falls, particularly noticeable when there is an initial decrease in $T$ as $v$ is raised from a lower starting point. Between two jumps on $PR$, increases in $v$ do not appear to influence $PR$, implying $PR$ is optimized or constrained regardless of further enhancements in conveyor velocity. For instance, given that $v$ values of 2 and 3 yield an identical $PR$ and $WIP$, it implies that there is no advantage to operating the conveyor at the higher speed of 3, as doing so could potentially increase the risk of conveyor malfunction and lead to unnecessary energy consumption. We believe the proposed analytical approach holds the potential to significantly enhance the understanding and implementation of conveyor buffer systems in industrial settings. However, due to the page limitation, we are unable to add a case study that is dedicated to exploring the optimal conveyor speed and analyzing its impact on operational efficiency and productivity, and this will be included in one of our future research suggestions.

## V. Conclusion and Future Work

This paper presents an analytical approach to model two-machine Bernoulli serial production lines with conveyor buffers, deriving formulas for performance metrics in both transient and steady states. Numerical experiments highlight the limitations of the existing approximation method and reveal interesting behaviors of system performance metrics concerning conveyor speed. Future research includes extending the model to multiple-machine lines, investigating optimal conveyor speeds through case studies, and generalizing the approach to production systems with different machine reliability models.

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