# Building Instructional Capacity Across Difference: Analyzing Transdisciplinary Discourse in a Faculty Learning Community focused on Geometry for Teachers Courses

Carolyn Hetrick
University of Michigan

Patricio Herbst
University of Michigan

Mike Ion
University of Michigan

Amanda Milewski
University of Michigan

*In response to enduring methodological tensions in analyzing collaborative discourse, we detail an assemblage of argumentation analytic methods that can support research about faculty learning communities interacting across substantive differences. Drawing on our research with a cross-institutional faculty online learning community, we use data to show how theories from discourse analysis, systemic functional linguistics, and argumentation modeling can be operationalized to support researchers in brooking methodological tensions, including framing argumentation as the topic of or a resource for investigation and considerations of collaborative discourse as both process and content. Our methodological findings illustrate an example of this operationalization, highlighting analysis of transdisciplinary, collaborative discourse in a community composed of instructors of college geometry courses required for pre-service teachers. We share possible uses for this methodological approach vis-a-vis research about the professional work of undergraduate mathematics education and pre-service teacher preparation.*

*Keywords:* Geometry, Argumentation, Professional Development

In explaining the exigencies of continued research about discourse in mathematics education, Sfard (2015) argued that researchers ought to investigate communication not merely as a means for learning, but as "the centerpiece of the story—the very object of learning" (p. 249). Such is our interest in faculty members' collaboration around instructional improvement, in their discourse is a product of their collaboration. However, as Sfard (2014) and others have detailed extensively, research about discourse in mathematics education is theoretically and methodologically diverse, and so does not rely on a common set of assumptions, conceptual models, or analytical tools (Niss, 2007; Ryve, 2011). In our own research with a faculty online learning community (FOLC) of college mathematics instructors, we encountered the need for methodological resources to support analysis of instructors' discourse—specifically, methods that would align with our goal of understanding the nature of the arguments among a diverse community of practitioners. This need is not unique to our research team. In an extensive review of research about instructors' collaborative discourse, Lefstein et al. (2020) found that "the field would benefit from greater coherence between theoretical perspectives and research designs" (p. 11). In mathematics education research, we see evidence of such coherence from scholars using argumentation theories and models to analyze classroom discourse (see, e.g., Conner et al., 2022) and teachers' knowledge and beliefs (see, e.g., Nardi et al., 2012). In turn, our methodological approach contributes careful consideration and practical examples of analyzing argumentation in undergraduate mathematics instructors' collegial discourse.

In this paper, we detail a qualitative methodological approach that supports the investigation of undergraduate mathematics instructors' cross-disciplinary knowledge resources—including synthetic geometry knowledge and mathematical knowledge for teaching geometry. We demonstrate this approach in the context of analyzing faculty members' collaborative discourse geared toward the development of a curricular resource for instructors of college geometry courses required for pre-service secondary mathematics teachers. With attention to relevant methodological disputes, we illustrate how an assemblage of social semiotic methods—guided by Toulmin's (2003) and Gilbert's (1997) argumentation models—can support analysis of instructors' collaborative discourse across disciplinary and institutional differences.

## Relation to the Literature: Argumentation Analysis in Mathematics Education

Discourse analytic research in mathematics education commonly adopts a fundamentally social constructivist stance towards human thought and knowledge (Sfard et al., 2001). That is, researchers assume that knowledge is socially and culturally, rather than individually, constructed; is dynamic rather than static; and is inherently contextual (Palincsar, 1998, p. 354). Unlike formal logic-based argumentation models, informal and quasi-logical argumentation models and theories involve inductive or diffuse disputes in which participants can "explore positions flexibly" (Nussbaum, 2008, p. 349) and in which different "ways of representing the world" can become "premises" supporting (a potential plurality of) conclusions (Fairclough & Fairclough, 2012, p. 86-87). In the data we share from our study with a FOLC, mathematics instructors' interactions are discursive activities situated in their development of a curricular resource. In many ways, the group's discourse represents their efforts to build a plane while they fly it (so to speak)—with the destination being instructional improvement. A review of research shows methodological challenges when conceptual fidelity requires researchers to consider this kind of collaborative discourse in which argumentation is more exploratory in nature.

Taylor (2001) named key tensions that researchers face in discourse analysis work: (1) investigating discourse as the topic of study versus analyzing discourse as a means of (or resource for) investigating something else and (2) fore-fronting the process or content of discourse. Conceptualizing discourse as process means focusing on the interactivity of discourse, including questioning the functions or effects of talk. In contrast, conceptualizing discourse as content typically focuses on the recurrence of themes, ideas, or other elements in the corpus of text. While these tensions exist across discourse analytic approaches, we identified how they are instantiated in argumentation analyses related to mathematics education research and research about educators' collaborative discourse.

Mathematics education research that analyzes argumentation often focuses on deliberate classroom interactions, such as patterns of teacher and student participation in formal (logical) argumentation. Such scholarship often frames argumentation as the topic of study. For instance, Forman et al. (1998) described their aims in analyzing transcriptions from a mathematics lesson as "understanding …the socialization of argumentation in her mathematics classroom…[and] to provide teachers and teacher educators with a detailed picture of argumentation in this classroom (p. 529). We also found several examples of researchers studying students' argumentation in the context of proof activities, illustrating further scholarly focus on argument as the main topic of investigation (see Harel & Sowder, 1998; Knipping, 2003; Rodd, 2000; Weber & Alcock, 2005). Still, many of these same researchers (and others) have related their findings to other phenomena, suggesting argumentation is a resource for engaging concepts relevant to social constructionist notions of learning. For instance, Krummheuer (2007) analyzed students' argumentation and theorized that their contributions reflected their learning autonomy.

Research about teachers' professional knowledge and learning communities has used argumentation as a resource to investigate other educational phenomena—most notably, teachers' beliefs, values, and knowledge (e.g., Conner & Singletary, 2021). Scholarship like this has demonstrated the need for researchers to analyze argumentation as both topic and resource. For instance, Nardi et al. (2012) showed that teachers' arguments could be understood in terms of mathematical accuracy and other professional concerns (including pedagogical, curricular, and personal considerations).

Mathematics education research in our review often elided distinctions between argumentation as content and process. We posit that the mathematical nature of logical argumentation and the cultural diffusion of mathematical thought into more popular and informal modes of argumentation (Keitel, 2006) mean that, in mathematics education research, the process and content of argumentation can both be objects of study vis-a-vis the same phenomenon (e.g., students' proving work in Hollebrands et al., 2010). Similarly, social constructionist frameworks can complicate distinctions between the content and process of argumentation. Researchers who have investigated the quality of collaborative discourse as a matter of process as much as content have faced this challenge (Lefstein et al., 2020). For instance, scholars noted that facilitators can help provide the *content* of expertise (Horn & Kane, 2015) and facilitate social *processes* (Kintz et al., 2015). We conclude that field-specific conceptions of mathematics and learning must inform these kinds of methodological distinctions.

## Conceptual Framework & Operating Theories

Theoretical underpinnings of our research guide our attention to how and why undergraduate mathematics instructors draw on diverse knowledge resources to engage in collaborative argumentation. We discuss our study, including the community, its members, and its formation, in this report's research methodology section. We focus on the theories and models of argumentation we have operationalized to facilitate analysis premised on treating discourse as both topic and resource and content and process. In particular, we discuss Toulmin's (2003) six-part model of argumentation and Gilbert's (1997) theory of coalescent argumentation.

Toulmin (2001) averred that analyzing argumentation should balance attention to the social situation of the argument and attention to its artifacts (i.e., texts). Gilbert (1997) wrote that "we need to shift the focus…from the *artifacts* that happen to be chosen for communicative purposes to the *situation* in which those artifacts function as a component" (p. 46). In both views, analyzing argumentation involves the particularities of the situation, including participants' motivations, goals, and positioned relationships. In bringing these two theories together, we do not want to reduce Toulmin's and Gilbert's robust bodies of work to narrow facets or conflate shared theoretical implications with common theoretical assumptions. Instead, we consider how their theories complement each other—allowing for dual-attention to argumentation as both (a) topic and resource and (b) content and process.

Toulmin's model includes six parts: claim (conclusion), data (evidence or grounds for the claim), warrants (justifications explaining the relationship between the data and the claims), backings (beliefs or evidence underlying warrants' logic), qualifier (claim's degree of certainty), and rebuttals (conditions that would make the claim untenable). Not all arguments fit this framework, as others have noted (Ellis, 2015; Schwarz, 2009). Still, as Inglis et al. (2007) wrote, Toulmin's framework is "less concerned with the logical validity of an argument, and more worried about the semantic content and structure in which it fits" (p. 4). That is, the Toulmin framework is useful for decomposing everyday reasoning, even if partially. On the other hand, coalescent argumentation captures how participants may join their justifications to achieve collaborative aims. Of note, Gilbert's theory directs attention to multi-modal, goal-oriented, and position-based argumentation (Gilbert, 1994; 1997; Godden, 2011)—a conceptualization compatible with Toulmin's larger theory of argumentation. Extending Toulmin-esque theories of informal argumentation, Gilbert suggested that models of argumentation could include warrants of varying sources. Warrants from other sources (what Gilbert calls "multi-modal" participation), include the emotional, the visceral (e.g., "That's a sensitive subject for me"), and the kisceral[1] (e.g., "It strikes me as the right thing to do"). Attention to multiple modes increases possibilities for coalescence (e.g., two participants may not agree on the logical validity of a claim, but they might find coalescence around a claim for which one finds logical warrant and the other finds emotional warrant). Gilbert (1997) also theorized that people engage in argumentation with multiple goals, including task goals and face goals (see Gilbert, p. 67-68). Thus, we understand resolution may be oriented toward a shared aim (e.g., the development of a curricular resource) *and* that participants have various other goals and priorities (e.g., maintaining relationships).

Engaging in participants' complex positions is a key to using the tools of coalescent argumentation (Gilbert, 1997). So, beyond modeling individual micro-arguments, coalescent argumentation can help us understand how participants relate micro-arguments to their diverse knowledge resources, institutional contexts, and sustained community engagement. We draw on Toulmin's model to support investigation of the topic and content of FOLC members' multiple arguments. When argumentation does not resolve in a single conclusion or involves more diffuse discursive engagement with elements of argumentation, we draw on Gilbert's (1997) theory of coalescent argumentation. Based primarily on these two theories, we developed an approach to support our analysis of participants' micro-arguments (argumentation as topic and content) and situating those micro-arguments in disciplinary, institutional, and community-specific contexts and relationships (argumentation as resource and process). This approach allows us to analyze participants' multiple, dynamic, and sometimes contradictory or unrelated positions and to understand a mode of argumentation in which members of the FOLC neither invalidate participants' starting positions nor disengagement from reasoning together. In our methods and results sections, we detail how we operationalized the theory of coalescent argumentation to interpret argumentation that initially included two oppositional positions—and out of which a third position emerged and coalesced them without invalidating their logic.

## Research Methodology

Our research occurs in the context of an FOLC that focuses on "Geometry for Teachers" (GeT) courses (i.e., college geometry courses required for prospective secondary mathematics teachers). Within the FOLC, 15 GeT instructors formed a working group to discuss student knowledge outcomes. Working group members' exchanges have included argumentation concerned with the state of the American college geometry course, which Grover and Connor (2000) had identified as having a "wide diversity" of course elements and a lack of consistent curriculum or instruction across institutions (p. 47).

While GeT courses are typically taught out of mathematics departments, these courses are usually taught by mathematicians or mathematics educators. The 15 members of the FOLC working group are a balance of instructors who identify as either mathematicians or mathematics educators. Some GeT courses are exclusively taken by future teachers, while others are electives

[1] Gilbert (1994) defined the kisceral mode as a "the mode of communication that relies on the intuitive, the imaginative, the religious, the spiritual, and the mystical" (p. 10). People often draw upon kisceral reasons in their everyday argumentation, and Gilbert noted (2011) that kisceral experiences can often be recast as strictly rational. Elaborating on the kisceral mode of argumentation, Gilbert (2011) gave the example of Euclid's Fifth postulate as a subject of knowledge that has often elicited "a *feeling* about what was right, what made sense, and what fit" (2011, p. 164).

for mathematics majors. In some cases, the GeT course is the only one that covers geometry content. Unlike many other college mathematics courses (e.g., Calculus), there is no broad consensus about what should be included in a GeT course, as there are many different types of GeT courses (Grover & Connor, 2000; Venema et al., 2015). To improve secondary geometry instruction, the FOLC working group realized it would be useful to identify a set of essential student learning objectives (SLOs, hereafter) for inclusion in GeT courses. *Essential* means the identifying content knowledge that all prospective secondary geometry teachers should learn in the course. They created this list of SLOs to help other GeT course instructors, especially new ones. The working group met monthly for the first year and a half and biweekly for the past six months to develop, elaborate, and steward the SLOs. The instructors' work is framed by their shared commitment to developing consensus—and SLOs that reflect that consensus—while navigating significant differences in their preparations, institutional and departmental affiliations, and disciplinary orientations. Our study of this group's work aims to understand the professional reasoning that occurs across such a situational landscape.

## Results: Operationalizing a Methodological Approach

In this section, we show how a methodological approach drawing on both Toulmin and Gilbert provides a basis for conducting argumentation analysis without a single conclusion and towards a common goal. The analytical process has two major parts: (1) modeling micro-arguments and (2) mapping and analyzing position interactions, including coalescence.

### Modeling Components of Micro-Arguments

One of the primary goals of the FOLC's working group has been to write SLO narratives that are detailed enough for GeT instructors outside the FOLC to use. The process of drafting these narratives starts with a subgroup of two to three people writing an initial draft narrative for each SLO and bringing it to the whole group for discussion and revision; this process repeats with the second draft and so on. The data in this section comes from a one-hour whole group meeting in March 2021, where the group reviewed a revised draft of the narrative of an SLO devoted to the role of definitions in mathematical discourse. As context: In a previous meeting about the first draft of the narrative, there had been a lively discussion about what constitutes non-Euclidean geometry, and whether they needed to define Euclidean and non-Euclidean geometry for their readers—as they themselves had varying definitions of these two types of geometries. Our example highlights exchanges between three participants: Miriam[2], Royce, and Michael. Over the course of the discussion, Miriam, Royce, and Michael engage in argumentation around issues related to teaching the nature of definitions in GeT courses.

To analyze the interactions and relationships between the arguments in the meeting(s), we first applied Toulmin's (2003) extended model of argumentation to meeting transcripts. We coded participants' turns of talk to identify claims (i.e., any conclusions they offered). Then, we coded turns of talk for contributions to arguments in service of a claim (or counter-claim). Each claim and its related elements (e.g., data, backing) represented a micro-argument that was putatively connected to the arc of the group's inquiry. Because we coded transcripts of verbal conversations, this process also involved looking for linguistic indicators of argumentation (e.g., "because"), including the use of modal qualifiers (e.g., "probably"). One example micro-argument consisted of the following elements, made by a participant called Miriam: (*claim*) the SLO narrative text uses too many examples of non-Euclidean geometry; (*data*)

[2] Pseudonyms are used for the participants in the study.

examples from the SLO text; (*qualifier)* the issue is context-specific, given the group's aims in writing the SLOs; (*warrants*) explaining that too many non-Euclidean examples might seem to promote a non-Euclidean approach to the course; and, finally, (*backings*) citing twice that she feels worried (a *kisceral* justification) and that her GeT course would show the need for the text to be less weighted toward non-Euclidean examples (a *logical* justification).

While completing the Toulmin (2003) diagramming of the transcript data helped us map the (quasi-)logic of the micro-argumentation, we also wanted to understand connections to the group's larger processes and individual and shared contexts for their work. We understood the group as sharing the goal of developing the SLOs. Gilbert (1997) detailed coalescent argumentation beginning from identified opposing sides (e.g., pro, con) of an "avowed disagreement" (p. 104). Thus, we identified disagreements participants had in direct relation to the SLO narrative, allowing us to thematically group elements of micro-arguments according to their indicated positions vis-a-vis a common dispute. Through this analysis, we identified three positions (i.e., thematic clusters of arguments): (1) the SLO should use examples of Euclidean geometry; (2) the SLO should use examples of non-Euclidean geometry; and (3) the SLO should explain the relevance of teaching the nature of definitions to pre-service teachers.

**Mapping & Analyzing Position Interactions, Including Coalescence**

In Gilbert's (1997) theory of coalescent argumentation, argumentation is not necessarily dialectical, because dialectical exchanges resolve in affirming the validity of some arguments and the invalidity of others. Toulmin's (2003) model of argumentation is well-suited for representing more dialectical exchanges, including collective participation (Krummheuer, 2007; Nardi et al., 2012). Diagramming micro-arguments illuminates participants' practical reasoning. Analyzing coalescence across the arguments illuminates the multi-modal rationales disparately maintained by various participants and that ultimately facilitate consensus. Figure 1 illustrates three parts of the argumentation occurring between Miriam, Royce, and Michael.

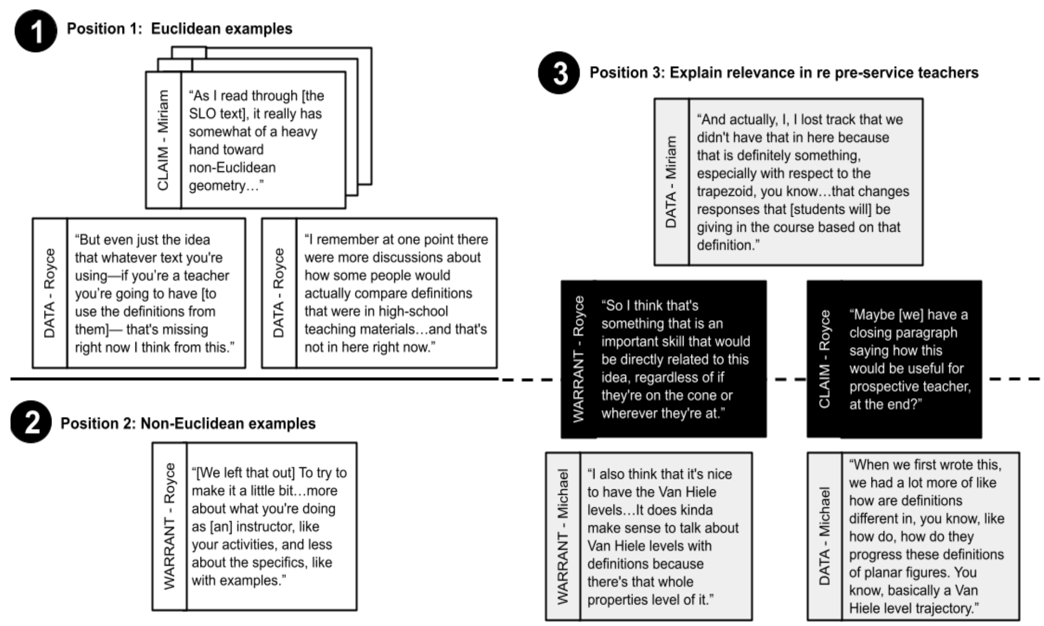


*Figure 1. Modeling Phases of Argumentation Leading Toward Coalescence*

Position 1 is constituted by micro-arguments advocating for using examples from Euclidean geometry. It includes Miriam's previously-described argumentation. It also includes two pieces of data that Royce offered in response to Miriam. Royce also offered a warrant that would support Position 2 (advocating for using non-Euclidean examples). However, he also gave a warrant that could support Position 1 *or* Position 2: "…that's something that is an important skill that would be directly related to this idea, regardless of if they're on the…." Here, Royce amplified Miriam's point that an example about defining a circle on the surface of a cone does not necessarily communicate the same set of skills she is saying a Euclidean geometry example might *and* offered justification for why non-Euclidean examples would still suffice. This warrant serves as the first indication of potential coalescence. Miriam responds to Royce's warrant by using it to agree with Royce's suggestion that they add additional text detailing the relevance of teaching the nature of definitions for pre-service teachers. This suggestion—a claim that represents Position 3—thus gains Miriam's support and allows her to maintain the rationale she employed in arguing Positions 1. It also allows Michael to accommodate his own rationale. Specifically, Michael identifies that Royce's data—a reference to structuring the narrative of examples in a way that would align with Van Hiele levels—is reason to support Royce's suggestion. Notably, Miriam and Michael's continued participation *could* align with the previous arguments made vis-a-vis Position 1 and 2, but they built coalescence around a third position that allowed them to maintain elements of the rationales underlying Positions 1 and 2.

## Applications to/Implications for Teaching Practice or Further Research

Thus, we claim that Gilbert's (1997) theory of coalescent argumentation is complementary to Toulmin's (2003) modelling of arguments. We identify coalescent argumentation's suitability for theoretical and empirical investigations of mathematics instructors' professional discourse in situations that support developing consensus without invalidating participants' individual rationales. We operationalize tenets of coalescent argumentation, including using the Toulmin framework to model multi-modal argumentation, bounding arguments by identifying positions (i.e., thematic clusters of micro-arguments), and analyzing interactions across positions to identify possible coalescence. We suggest that the theoretical implications of the three principles of coalescent argumentation—that argumentation is multi-modal, position-based, and goal-oriented—may have wide-reaching applications in mathematics education research.

Scholars in the RUME community have advocated for qualitative researchers to align their research questions and methods (Melhuish & Czocher, 2022), and for more research about cross-disciplinary knowledge resources in undergraduate STEM instruction (Speer et al., 2020). Lefcourt (2020) noted that, while there has been "increasing attention…on making undergraduate courses for mathematics teachers relevant to the work of teaching," it is uncertain how such efforts can occur with larger degrees of "scope, scale and impact" (p. 830). Even though we are explicit about our methodological approach's conceptual boundaries and limits, we share it to support conceptually-aligned research in undergraduate mathematics education that examines questions about cross- and trans-disciplinary knowledge resources and instructors' practical applications of these knowledge resources in their mathematics teaching.

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