Instructor-Student Interactions in Geometry Courses for Secondary Teachers: Results from A National Survey

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We report preliminary results of selected questions from a national survey of instructors of geometry courses for secondary teachers about the nature of instructor-student interactions. Survey responses (n= 118) are used to indicate six latent constructs describing aspects of instructor-student interaction that in turn quantify hypothesized characteristics of two didactical contracts, which we call inquiry in geometry and study of geometry. We found that instructors whose highest degree is in mathematics education are less likely to rely on a study of geometry contract than instructors whose highest degree is in mathematics. Also, instructors who have previously taught high school geometry are less likely to lecture.

Keywords: geometry, secondary teacher education, survey, didactical contract, instruction

Objectives

The work reported contributes to describing instruction in undergraduate mathematics education. Based on the responses from 118 instructors to 24 survey items, we describe how instructors relate to students in geometry courses taken by prospective secondary mathematics teachers (GeT courses, hereafter), including whether and how they incorporate students' input in lectures, how they handle student difficulties, and how they handle student contributions. After testing a measurement model of constructs that inform the extent to which instructors lead students in the study of geometry or in inquiry in geometry, we report on how indicators of these constructs relate to each other, and whether characteristics of the instructors (including whether their highest degrees are in mathematics or in mathematics education, and whether they have taught high school geometry in the past) predict scores in any of those latent variables.

Literature Review

The mathematical preparation of prospective secondary teachers (PST, hereafter) is an important area for investigation in the RUME community (e.g., Lai et al., 2023; Serbin & Bae, 2023). Whereas scholars and practitioners have written about the mathematics curriculum of teacher preparation for more than a century (Schubring, 1989), the empirical study of mathematics instruction in those courses has lagged for most of our field's history, along with the lag in the study of mathematics instruction at the undergraduate level. Speer et al. (2010) had noted how limited scholarship on mathematics instruction at the undergraduate level had been. Yet, a more recent review by Melhuish et al. (2022) updated that assertion, noting that the study of instruction at the undergraduate level has captured much more interest between 2010 and 2020. A variety of methods have been used in these studies, including, in particular, some instructor surveys of instructional practices (e.g., Johnson, et al., 2018, 2019). Though a main interest in the analysis has been to report on the incidence of lecture in instruction, researchers have also cautioned that the incidence of lecture is not necessarily an indicator of the absence of student-centered instruction (Smith et al., 2014). In advanced mathematics classes such as abstract algebra, however, studies of instructors' beliefs have suggested that mathematicians value lecture as an instructional method to prepare future mathematicians (Melhuish et al, 2022).

For the specific case of the preparation of PST, one might expect instruction could be different, especially considering the emphasis that has been given to inquiry-oriented instruction in the last couple of decades (Abell et al., 2018; Mahavier, 1999). Yoshinobu and Jones (2011) had singled out preservice teachers among those who could benefit the most from inquiry-based learning; and Laursen et al. (2016) documented important gains for PST who had learned mathematics through inquiry. Important questions to ask include: But to what extent do prospective secondary mathematics teachers participate in inquiry-oriented instruction?

GeT courses are salient locations where the incidence of lecture as well as of student-centered instructional practices could be inspected to answer that question. Though small-scale research has been done in GeT classes (e.g., Blanton, 2002), little research has looked at undergraduate geometry instruction at scale thus far. Wong (1970) was an early survey of institutions and the curriculum offered in GeT courses. Grover and Connor (2000) reported on a survey of ~100 GeT course instructors and included one question aimed at pedagogical practices. The responses showed that though only 7.1% of instructors described their courses as consisting of only lectures, only up to 34.3% included classroom discussions facilitated by instructor. Though responses to just one survey question are hardly enough to describe instruction, no other instructor survey has been conducted after Grover and Connor (2000) to expand or update what we know about instructional practice in GeT courses since. The answers from that one question, however, suggested that to understand instructional practice in more detail, we could use an analysis of the components of lecture and inquiry to develop an instrument that more accurately served for instructors to describe what they do in their classrooms.

An important precursor of the work reported here was Shultz's (2020) INQUIRE survey, which explored the extent to which undergraduate mathematics instructors engaged in practices that could be used to describe inquiry. Shultz's (2020) INQUIRE instrument defined latent constructs that could indicate various components of inquiry-oriented instruction described in the literature on inquiry-based learning. Shultz (2020) organized those constructs using the edges of the instructional triangle (Cohen et al., 2003). For example, *interactive lecture* and *hinting without telling* were two constructs identified to measure the extent to which instructor-student relationships (the instructor-student edge of the instructional triangle) were inquiry-oriented. Rather than relying on single questions to indicate a construct, the INQUIRE instrument included 5 items to indicate interactive lecture and 3 to indicate hinting without telling. Among important findings from Shultz (2020) are that the various constructs that can be associated with inquiry-based instruction portray a more complex distribution across instructors who claim to engage in inquiry. Shultz found evidence that lower-division undergraduate mathematics instructors might cluster in four different groups, depending on the scores on various of those constructs. Our GeT Instructor survey also used the instructional triangle to organize various aspects of instruction as latent constructs to be indicated by survey items. In this study we focus on the instructor-student edge of the instructional triangle, and we inquire on the incidence of constructs characteristic of inquiry as well as those which are characteristic of traditional study of geometry.

Theoretical Framework

We build on a theoretical framework about mathematics instruction that combines Cohen et al.'s (2003) instructional triangle and Brousseau's (1997) didactical contract. Specifically, Cohen et al. (2003) conceptualize instruction as a system of relationships among instructor, students, and content that take place in environments. The latter are institutional environments, namely mathematics departments and teacher education programs in colleges and universities. Herbst et al. (2023) further elaborate the content vertex of the instructional triangle to account for the fact that whereas students mainly relate to the content in terms of the work they are asked to do, instructors also relate to the content in terms of the instructional goals that such work is designed to support the acquisition of (Figure 1). This distinction is especially important in inquiry classrooms as the work students are asked to do may not too obviously disclose what the knowledge at stake is (e.g., Hitchman, 2017).

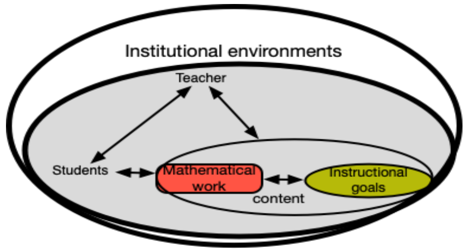

Figure 1. Elaboration of the instructional triangle

The specific ways in which those relationships are entertained call for the use of the notion of didactical contract. The literature has often used holistic names such as "school mathematics" and "inquiry classrooms" (Cobb et al., 1992) or "lecture-based" or "student-centered" to distinguish between types of teaching (Mesa et al., 2020). The notion of didactical contract (Brousseau, 1997), which Herbst et al. (2023) interpret as a system of norms that underpin how relationships among instructor, students, and content take place, serves us to operationalize those nominal distinctions into sets of possible norms that might characterize those relationships. Leading to the development of a survey that could help us elicit descriptive information about GeT instruction, we hypothesized that features of inquiry-oriented instruction could be considered possible norms of a didactical contract (inquiry in geometry) and that features of what often is called traditional or lecture-based instruction could also be identified to characterize a different didactical contract (the study of geometry). We did not expect that the didactical contract in any individual GeT class could be simply classified as either inquiry or study, but rather designed the survey so that we could measure instructors' recognition of each of the various norms that describe instructor-student relationship in both contracts. The present study reports on instructors' recognition of the various norms that characterize study and inquiry contracts. We hypothesized that the study contract would rely on norms such as LECTURE (the instructor is expected to introduce any new content), RIGHTANS (the instructor is expected to provide the right answers to students who have difficulties), and STALKTOINS (the instructor is expected to take students' public comments as directed to the instructor). And we hypothesized that inquiry contracts might rely on other norms including INTLECTURE (students are expected to participate in lectures), HINTNOTELL (the instructor is expected to hint without telling when students have difficulties, and STALKTOCLASS (the instructor is expected to take students public contributions as directed to the whole class). These norms were used to create the items in the survey with which we expected to answer three questions: (1) How likely is it that students participate when new knowledge is being installed? (2) How do instructors respond to individual student difficulties with class work? and (3) How do instructors make use of individual student contributions to the whole class? Further, we expected that constructs that describe a study contract would correlate with each other and the same would happen with variables that describe an inquiry contract. And we wondered the extent to which responses to those questions were predicted by individual characteristics of the instructors, specifically whether their highest degree was in mathematics or mathematics education and whether they had prior experience teaching high school geometry.

Methods

The GeT Instructor Survey

The GeT Instructor survey was designed to describe instruction across geometry courses for secondary teachers taught in mathematics departments across universities in the US. Broadly conceived, it aims to measure the incidence of various instructor-centered and student-centered practices as well as various types of students' engagement with content, including geometry and geometry knowledge for teaching. Some of those items ask instructors to report the extent to which they engage students in tasks of teaching geometry (such as providing feedback on students' written work). Data collection has been ongoing; the present report provides initial gleanings from the analysis of some of the constructs being measured.

Herbst et al. (2024) analyzed the GeT Instructor survey responses concerning students' interaction with content. In that analysis, we estimated the relationship among four factors that capture instructors' descriptions of the nature of their students engagement with content: (1) students *study* geometry (Study), (2) students *inquire* into geometry (Inquiry), (3) students engage in tasks of teaching geometry (ETT), and (4) students engage with dynamic geometry software (DGS). Analysis showed significant correlations between Inquiry and ETT, between Inquiry and DGS, and between ETT and DGS. A structural equation model showed that DGS fully mediates the relationship between Inquiry and ETT. The present report concentrates on another of the instructional triangle's edges: The instructor-student relationship.

In its initial design, the GeT Instructor survey included 24 items to indicate 7 constructs which could be used to describe the instructor-student relationship in the instructional triangle. Each of those items asked participants to indicate their level of agreement using a 6-point Likert scale (ranging from Strongly Disagree to Strongly Agree) in response to provided sets of statements that could describe the respondents' practice in the GeT class. For example, item 811204 presented the statement "While introducing new material, I called upon the students to ask questions about the material being covered" which we hypothesized would indicate the construct interactive lecture (INTLECTURE). Of the seven hypothesized constructs, only six could be measured with the designed items (items for the seventh construct did not meet standard requirements in a confirmatory factor analysis). The six constructs helped provide answers to the three first research questions. Besides, we expected that correlations among the six constructs might align with the different contracts: Study (constructs LECTURE, RIGHTANS and STALKTOINS) and Inquiry (constructs INTLECTURE, HINTNOTELL, and STALKTOCLASS). The GeT Instructor survey hypothesized other constructs as useful to answer questions about the other relationships represented in Cohen et al.'s (2003) instructional triangle. We do not report on those questions and constructs in the present report.

Sample

To reach widely across GeT Instructors in the US, we obtained lists of all the universities and colleges across the US and checked whether they had a secondary teacher preparation program and whether their mathematics departments offered a GeT course required for prospective teachers. This canvassing yielded (n=670) mathematics departments; emails were sent to department heads (or their secretaries) asking them to forward a link to the survey to the instructor who had taught the course last. By the time of this analysis, our effective sample size consisted of 118 GeT instructors who completed all items of a Qualtrics survey, including the GeT Instructor survey and a background questionnaire. Our sample participants confirmed they had taught a geometry course required for secondary mathematics teachers in the last ten years. The participants comprised approximately 55.9% male instructors and 39.8% female instructors. Approximately 72% had their highest degree in mathematics, while 25.4% had their highest degree in mathematics education. And 30.5% had prior teaching experience in high school geometry. A significant 85.6% of participants held either tenure or tenure-track faculty positions, while 10.2% occupied non-tenure roles including lecturers and graduate students.

Results

Descriptives of the raw scores for the seven hypothesized constructs are provided in Table 1. We performed a Confirmatory Factor Analysis (CFA) to evaluate how well the observed items measure each of the seven hypothesized constructs. Notably, items designed to indicate one construct, that instructor functions as an older peer (OLDERPEER), exhibited low correlations among them and small item loadings which we took as evidence that the items did not represent a single latent construct. After excluding these items, we re-evaluated CFA with the remaining items that had item loadings above the threshold of approximately 0.3 onto the six hypothesized constructs. The results indicate an acceptable model fit, with the Root Mean Square Error of Approximation (RMSEA) at 0.080, the Comparative Fit Index (CFI) at 0.854, and the Tucker-Lewis Index (TLI) at 0.818.

Table 1: Descriptive Statistics of the Seven Hypothesized Constructs

	N	Mean	Median	Min.	Max.	Std. Dev.
LECTURE	118	3.997	4.2	1	6	1.287
INTLECTURE	118	4.883	4.8	2.6	6	0.711
OLDERPEER	118	3.547	3.5	1.75	5.75	0.824
RIGHTANS	118	3.031	3	1	6	1.021
HINTNOTELL	118	3.723	3.7	1	6	0.925
STALKTOINS	118	2.989	3	1	4.7	0.879
STALKTOCLASS	118	4.381	4.5	1	6	1.039

The CFA analysis revealed notable patterns of correlation among the constructs, primarily distinguishing between constructs hypothesized as characteristic of the Study contract (LECTURE, RIGHTANS, and STALKTOINS) and those hypothesized as characteristic of the Inquiry contract (INTLECTURE, HINTNOTELL, and STALKTOCLASS). The highest correlations were observed between LECTURE and RIGHTANS (.550), RIGHTANS and STALKTOINS (.240), as well as LECTURE and STALKTOINS (.229). These findings suggest a strong connection between presenting traditional lectures, providing correct answers when students have difficulties, and taking student contributions as a dialogue between the student and the instructor. Conversely, in the realm of the inquiry contract, we observed significant correlations between HINTNOTELL and STALKTOCLASS (.207), and between INTLECTURE and STALKTOCLASS (.119) (see Table 2). These findings indicate relationships among delivering interactive lectures, fostering classroom discussions, and affording students opportunities to solve their own problems. Additionally, the correlation between LECTURE and INTLECTURE was significant (.161, $p < 0.05$). This may indicate instructors' inclination towards utilizing lectures, regardless of the specific type of lecture.

Table 2: Correlation Matrix of the Six Factors in CFA

	811100	811200	812100	812200	813100
811100					
811200	.161*				
812100	.550***	.037			
812200	-.111	.037	-.013		
813100	.229*	.004	.240**	-.054	
813200	-.198	.119*	-.177	.207*	-.069

* $p < 0.05$, **$p<0.01$, ***$p<0.001$

We also conducted a comparative analysis of latent variable means (LVM) among instructors from various demographic backgrounds to explore the potential association between these demographics and scores on specific constructs. Specifically, we focused on instructors' highest degree to answer whether having a highest degree in either mathematics (M) or mathematics education (ME) could predict whether the instructor's GeT class might follow more of a study or inquiry contract. To express the between-group differences, we set the LVM in the first group (mathematics) to zero and estimated the LVM in the mathematics education group (see Table 3).

Table 3: Latent Variable Mean (LVM) Difference -Between Demographics Groups

	Highest Degree in Mathematics (M) (N=85) or Mathematics Education (ME) (N=30)		Did Not Teach HS Geometry (N) (N=82) or Taught HS Geometry (Y) (N=36)	
	LVM Difference (LVM in ME after setting M to 0)	p-value	LVM Difference (LVM in Y after setting N to 0)	p-value
LECTURE	- .755***	.0009	-.462*	.04
INTLECTURE	.133	.18	.121	.19
RIGHTANS	-.543**	.002	-.285	.15
HINTNOTELL	.312	.08	.182	.25
STALKTOINS	-.334*	.02	-.171	.10
STALKTOCLASS	.390*	.04	-.054	.81

* p-value < 0.05, **$p<0.01$, ***$p<0.001$

A Wald test and chi-square difference test revealed significant differences in latent variable means between groups of instructors according to highest degree for constructs LECTURE, RIGHTANS, and STALKTOINS. Thus, instructors holding their highest degrees in mathematics education are less likely to manage a Study contract, in which traditional lectures are given, correct solutions are offered when students encounter difficulties, and student contributions are seen as one-on-one dialogues with the instructor, as compared to instructors with highest degrees in mathematics. There is some, but not enough evidence to say that instructors whose highest degree is in mathematics education are more likely than instructors with degrees in mathematics to engage in practices aligned with Inquiry. Furthermore, we examined instructors who had or had not taught high school geometry. After setting the LVM in the second group (had not taught high school geometry) to zero and estimating the LVM in the first (had taught) which represent between-group differences, we observed a significant mean difference in the LECTURE construct. Instructors with prior experience teaching high school geometry seem to be less prone to employing traditional lectures in their instruction.

Conclusion

A few observations about the contracts that we call Study and Inquiry can be made as regards how these contracts characterize the instructor-student relationship. The survey successfully deconstructs Study and Inquiry into six constructs (3 for study and 3 for inquiry) that are well indicated by several items. It thus can provide a more nuanced image of what the study and inquiry contracts mean. In particular, as related to the popular conflation of inquiry with no lecturing and the defense that some instructors have offered of the possibility to combine lecturing with inquiry (e.g., Alcock, 2018), the survey provides other well-indicated constructs that can be used to inspect the incidence of inquiry practices. Though the full survey is designed specifically for GeT courses, the specific constructs used to understand the instructor-student relationship are indicated with items that depend very little on the nature of the content being transacted (though they are about mathematics instruction); thus, researchers investigating instruction in other courses of study might be able to use same survey items.

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