Protecting Legacy Wireless Systems Against Interference using Massive MIMO

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***Abstract*—The growing demand for high-speed wireless communication has generated considerable interest in using frequency bands adjacent to those occupied by legacy wireless systems. Since legacy wireless systems were designed for past spectral usage, when bands were sparsely used, utilizing these new bands will lead to interference with the legacy users. Therefore, it is essential to develop signaling schemes that can protect legacy users from such interference. For many applications, legacy users are located within a geographically constrained region. In this paper, we use the knowledge of this region to limit the interference at legacy users. We achieve this by incorporating received power constraints termed as region constraints, in the massive multiple-input multiple-output (MIMO) system design. We perform a sum-rate analysis of the multi-user massive MIMO system with transmit power and region constraints.**

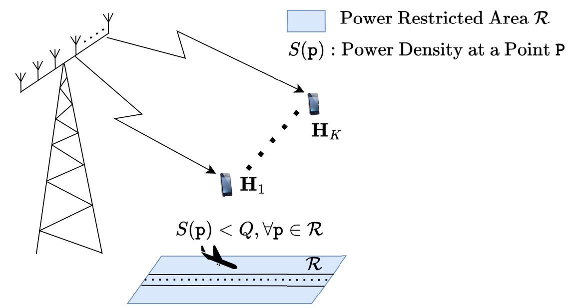

Fig. 1: Power restricted region using legacy wireless systems

I. Introduction

The demand for high-speed wireless communication has increased rapidly over the last decade. To address this, the Federal Communications Commission (FCC) and other spectrum authorities have considered several new sub-6 GHz bands suitable for wireless communication. However, the sub-6 GHz frequencies are heavily used by legacy systems essential to critical networks, e.g., airplane radar systems, global positioning systems, etc. The need to share spectrum with these legacy systems, which were deployed decades ago, has led to previously unexpected interference issues [1], [2]. This is primarily due to the lack of regulation for receivers in these legacy systems [3]. These receivers were designed when spectrum was lightly used, and they often used the assumption that there was little to no interference in bands 10s-100s of MHz away. The filters in these receivers may have sidebands that allow significant interference from transmissions in bands several hundred MHz away.

One example of such an interference scenario is at the radar altimeter receivers resulting from 5G C-band deployments [4]. It had been speculated that a 5G deployment might result in interference level that could potentially corrupt the radar altimeter measurements. Because of this issue, the deployment of 5G base stations using this band was delayed near airports, hospitals, and other areas of use. The Federal Aviation Administration (FAA) has stated that permanent changes to 5G systems must be made to guarantee the safety of aircraft [4]. Another example is the Ligado-GPS dispute, where Ligado proposed a wideband wireless network in the L-band [2], adjacent to band used by GPS. The GPS industry showed that allocating this band would lead to interference with GPS signals. Ligado later came up with a lower-power solution to mitigate this interference issue. The much discussed public policy solutions to such interference challenges are not viable in the long term due to the growing demand for additional spectrum. There is an urgent need for communication theory solutions that can incorporate the interference issue in the current system model.

Massive MIMO presents a promising solution to the increasing demand for high-speed wireless communication. However, most prior works on massive MIMO system design have not considered the interference issue with legacy systems. In this paper, we focus on regions (e.g., airports, hospitals, etc.) where strict received power requirements may apply, and we incorporate these regions into our system model. Fig. 1 illustrates an example of such a region containing legacy wireless users. To limit the received power in specific areas, we employ quadratic constraints in the design of massive MIMO precoders. Quadratically constrained optimization problems in MIMO systems have been explored in various contexts, some examples are in [5], [6]. Since many of these problems are convex, they can be solved using standard convex optimization techniques. We use convex optimization techniques in [7] to design precoders that maximizes the sum-rate under quadratic

This work is in part supported by the Office of Naval Research and the National Science Foundation under grants EEC1941529, CNS2212565 and CNS2225577.

constraints.

II. System Model

We consider a downlink multi-user massive MIMO setup with K users, where the base station is equipped with M_t transmit antennas and each user has M_r receive antennas. Assuming a narrow band channel, the input-output relationship for the k^{th} user is given by

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x} + \mathbf{n}_k, \ k = 1, ..., K, \tag{1}$$

where $\mathbf{y}_k \in \mathbb{C}^{M_r \times 1}$ is the received vector, $\mathbf{n}_k \in \mathbb{C}^{M_r \times 1}$ is the additive noise vector assumed to be distributed as $\mathbf{n}_k \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I})$, and $\mathbf{H}_k \in \mathbb{C}^{M_r \times M_t}$ is the channel matrix. We assume a precoding transmitter with the transmitted vector $\mathbf{x} \in \mathbb{C}^{M_t \times 1}$ is

$$\mathbf{x} = \sum_{k=1}^{K} \mathbf{F}_k \mathbf{s}_k, \tag{2}$$

where $\mathbf{F}_k \in \mathbb{C}^{M_t \times M}$ is the k^{th} user's linear precoder and $\mathbf{s}_k \in \mathbb{C}^{M \times 1}$ denotes the k^{th} user's transmit vector of M data streams, where $KM \leq M_t$. All transmit vectors are mutually independent and identically distributed, with $\mathbb{E}[\mathbf{s}_k] = \mathbf{0}$ and $\mathbb{E}[\mathbf{s}_k \mathbf{s}_k^H] = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. To satisfy the transmit power constraint, the precoder must be designed such that

$$\mathbb{E}[\mathbf{x}^H \mathbf{x}] = \sum_{k=1}^{K} \text{tr}(\mathbf{F}_k^H \mathbf{F}_k) \leq P, \tag{3}$$

where $\mathbb{E}[\cdot]$ denotes statistical expectation and $\text{tr}(\mathbf{A})$ is the trace of the matrix $\mathbf{A}$.

Let $\mathbf{G}_k \in \mathbb{C}^{M \times M_r}$ be the k^{th} user receiver such that the estimated transmit vector is

$$\hat{\mathbf{s}}_k = \mathbf{G}_k \mathbf{y}_k. \tag{4}$$

The mean squared error (MSE) matrix of the k^{th} user is [8]

$$\mathbf{E}_k = \mathbb{E}[(\hat{\mathbf{s}}_k - \mathbf{s}_k)(\hat{\mathbf{s}}_k - \mathbf{s}_k)^H] \tag{5}$$

$$\begin{aligned} &= \mathbf{I} + \mathbf{G}_k \mathbf{H}_k \left(\sum_{m=1}^{K} \mathbf{F}_m \mathbf{F}_m^H \right) \mathbf{H}_k^H \mathbf{G}_k^H - \mathbf{G}_k \mathbf{H}_k \mathbf{F}_k \\ &\quad - \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{G}_k^H + \sigma^2 \mathbf{G}_k \mathbf{G}_k^H. \end{aligned} \tag{6}$$

For fixed transmit precoders, the optimal minumum mean squared error (MMSE) receiver is [8]

$$\mathbf{G}_k^{\text{opt}} = \mathbf{F}_k^H \mathbf{H}_k^H \left(\mathbf{H}_k \left(\sum_{m=1}^{K} \mathbf{F}_m \mathbf{F}_m^H \right) \mathbf{H}_k^H + \sigma^2 \mathbf{I} \right)^{-1}. \tag{7}$$

Using (7), the corresponding MSE matrix is [8]

$$\mathbf{E}_k = \left(\mathbf{I} + \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{C}_k^{-1} \mathbf{H}_k \mathbf{F}_k \right)^{-1}, \tag{8}$$

where $\mathbf{C}_k$ is the effective noise covariance matrix given by

$$\mathbf{C}_k = \mathbf{H}_k \left(\sum_{m=1, m \neq k}^{K} \mathbf{F}_m \mathbf{F}_m^H \right) \mathbf{H}_k^H + \sigma^2 \mathbf{I}. \tag{9}$$

The sum-rate is [8]

$$\mathfrak{R}_{sum} = \sum_{k=1}^{K} \log \left| \mathbf{E}_k^{-1} \right|. \tag{10}$$

We assume that legacy wireless users are located at a confined geographic area, such as radar altimeters located at airports. The base station is assumed to be centered at the origin. We also assume that the base station is equipped with a uniform linear array (ULA). We consider a scenario with N distinct geographic regions, each containing legacy wireless users. These regions are represented by sets of points $\mathcal{R}_i \subset \mathbb{R}^2, i = 1, \ldots, N$.

The average interference power density in the i^{th} region is constrained such that $\mathbb{E}[S(\mathbf{p}_i, \mathbf{x})] \leq Q_i, \forall \mathbf{p}_i \in \mathcal{R}_i$, where $S(\mathbf{p}_i, \mathbf{x})$ denotes the instantaneous power density at point $\mathbf{p}_i \in \mathcal{R}_i$ due to the transmitted vector $\mathbf{x}$. Q_i represents the maximum permissible interference in the i^{th} region. If the region $\mathcal{R}_i$ is in the line-of-sight of the base station, the instantaneous received power density at any point $\mathbf{p}_i \in \mathcal{R}_i$ can be expressed as [9]

$$S(\mathbf{p}_i, \mathbf{x}) = \frac{1}{4\pi d_i^{\gamma}} |AF(\mathbf{p}_i, \mathbf{x})|^2, \tag{11}$$

where $AF(\mathbf{p}_i, \mathbf{x})$ denotes the array factor with the array elements illuminated by the transmit vector $\mathbf{x}$, γ represents the generalized path loss exponent, and d_i is the distance between the base station and the point $\mathbf{p}_i$.

The point $\mathbf{p}_i$ can be expressed either in Cartesian coordinates (c_1, c_2) or in polar coordinates (d_i, ϕ_i). In polar coordinates, the array response vector is denoted by $\mathbf{a}(\phi_i) \in \mathbb{C}^{M_t \times 1}$, where ϕ_i represents the azimuth angle of departure (AoD) corresponding to the point $\mathbf{p}_i$. We define the array factor illuminated by the transmit vector $\mathbf{x}$ as

$$AF(\mathbf{p}_i, \mathbf{x}) = \mathbf{a}(\phi_i)^H \mathbf{x}. \tag{12}$$

Using (12), the instantaneous power density at point $\mathbf{p}_i$ can be expressed as

$$S(\mathbf{p}_i, \mathbf{x}) = \mathbf{x}^H \mathbf{R}_i \mathbf{x}, \tag{13}$$

where $\mathbf{R}_i$ is a characteristic matrix given by

$$\mathbf{R}_i \triangleq \frac{1}{4\pi d_i^{\gamma}} \mathbf{a}(\phi_i) \mathbf{a}(\phi_i)^H. \tag{14}$$

The characteristic matrix is Hermitian positive semi-definite by construction. We want to constrain the average received power density in the region. The expected value of (13) is

$$\mathbb{E}[\mathbf{x}^H \mathbf{R}_i \mathbf{x}] = \sum_{k=1}^{K} \text{tr}(\mathbf{F}_k^H \mathbf{R}_i \mathbf{F}_k). \tag{15}$$

The characteristic matrix in (14) is inversely proportional to d_i^{γ}. As a result, by limiting the received power density at the region's boundary closest to the base station, the path loss will inherently satisfy the received power requirements throughout the rest of the region. The boundary of the i^{th} region can be represented by a set of points $\mathcal{B}_i$ that satisfy a curve equation.

The boundary $\mathcal{B}_i$ is represented as

$$\mathcal{B}_i = \{(c_1, c_2) \in \mathcal{R}_i : \kappa_c(c_1, c_2) = 0\}, \tag{16}$$

where $\kappa_c(c_1, c_2) = 0$ is a curve equation in Cartesian coordinates. The curve can be transformed from Cartesian coordinates to polar coordinates yielding the set

$$\mathcal{B}_i = \{(d, \phi) \in \mathcal{R}_i : \kappa_p(d, \phi) = 0\}, \tag{17}$$

where $\kappa_p(d, \phi) = 0$ is a curve equation in polar coordinates.

To define the region constraints, we consider a sampled constraint problem. By sampling the region's boundary, we can represent the interference problem using a finite number of constraints. The number of samples should be large enough to account for the worst-case interference. Let L_i denote the number of discrete samples for the i^{th} region. By obtaining L_i samples of the azimuth angle along the boundary curve nearest to the base station, denoted as (d_ℓ, ϕ_ℓ) for $\ell = 1, 2, \dots, L_i$, we can effectively limit the power density within the region $\mathcal{R}_i$ using L_i quadratic constraints.

To limit the interference within the region, the power density at all points $\mathbf{p}_i \in \mathcal{R}_i$ must stay below a given power threshold Q_i. This power density requirement in the i^{th} region $\mathcal{R}_i$ can be approximated using L_i quadratic inequality constraints. For N regions with specific received power requirements, we generalize these quadratic inequality constraints, known as region constraints, as

$$\sum_{k=1}^{K} \text{tr}(\mathbf{F}_k^H \mathbf{R}_{i,\ell} \mathbf{F}_k) \leq Q_i, i = 1, 2, .., N, \ell = 1, 2, .., L_i, \tag{18}$$

where $\mathbf{R}_{i,\ell} = \frac{1}{4\pi d_{i,\ell}^{\gamma}} \mathbf{a}(\phi_{i,\ell}) \mathbf{a}(\phi_{i,\ell})^H$ denotes the ℓ^{th} characteristic matrix used to constrain the interference in the i^{th} region. We assume that the channel $\mathbf{H}_k$ is perfectly known at both the transmitter and receiver.

III. Region Constraints

As introduced in the previous section, the legacy wireless users will be confined to N geographic regions represented by sets of points $\mathcal{R}_i \in \mathbb{R}^2, i = 1, \dots, N$. To restrict the received power density in these regions, we use multiple quadratic inequality constraints. The array steering vector for ULA is

$$\mathbf{a}(\psi) = [1, e^{-j\psi}, e^{-j2\psi}, \dots, e^{-j(M_t - 1)\psi}]^T, \tag{19}$$

where $\psi = 2\pi\delta \cos\phi / \nu$, δ is the distance between the antenna array elements, ν is the wavelength, and ϕ is the azimuth angle.

The region boundary can be represented using multiple line segments. Let $\mathcal{B}_i = \bigcup_{\eta} \Xi_{i,\eta}$, where $\Xi_{i,\eta}$ denotes the set of points representing the η^{th} line segment used to represent the i^{th} region boundary. The equation of a line segment is

$$\omega_{1,i,\eta} c_1 + \omega_{2,i,\eta} c_2 = \omega_{3,i,\eta}, \tag{20}$$

where the line segment is the set of points $\Xi_{i,\eta} = \{(c_1, c_2) : c_{1,i,\eta,\min} \leq c_1 \leq c_{1,i,\eta,\max}, c_{2,i,\eta,\min} \leq c_2 \leq c_{2,i,\eta,\max}, \text{and } (c_1, c_2) \text{ satisfies (20)}\}$. If we substitute $c_1 = d\cos(\phi)$ and $c_2 = d\sin(\phi)$, (20) leads to the distance in terms of angle ϕ as

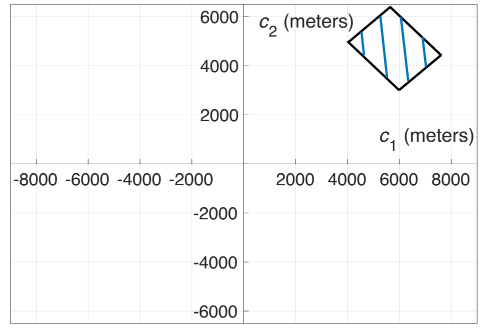

Fig. 2: Region boundary defined by two line segments

$$d(\phi) = \frac{\omega_{3,i,\eta}}{\omega_{1,i,\eta}\cos(\phi) + \omega_{2,i,\eta}\sin(\phi)}, \phi \in [\phi_{i,\eta,\min}, \phi_{i,\eta,\max}], \tag{21}$$

where $\phi_{i,\eta,\min} = \min(\Phi_{i,\eta})$, $\phi_{i,\eta,\max} = \max(\Phi_{i,\eta})$, and $\Phi_{i,\eta}$ is a set given by $\Phi_{i,\eta} = \left\{\phi : \phi = \arctan\frac{c_2}{c_1},\ \forall (c_1, c_2) \in \Xi_{i,\eta}\right\}$. Fig. 2 shows an example of a region defined by multiple line segments. The line segment closest to the base station is the curve equation $c_2 + c_1 = 9000$, where the end points are (4000,5000) and (6000,3000).

IV. Problem Formulation and Sum-Rate Analysis of Multi-user Massive MIMO with Region Constraint

In this section, we formulate the optimization problem for multi-user massive MIMO with transmit power and region constraints. We extend the precoder design framework in [10] and [8] to include region constraints. We formulate a multi-user MMSE problem with transmit power and region constraints and provide an optimal precoder structure. In what follows, we formulate a sum-rate maximizing problem with transmit power and region constraints, and provide an optimal precoder structure.

A. Multi-User MMSE Precoder With Region Constraints

The multi-user MMSE optimization problem with the transmit power and region constraints is

$$(\mathcal{T}1) \quad \min_{\{\{\mathbf{F}_k\},\{\mathbf{G}_k\}\}} \sum_{k=1}^{K} \text{tr}(\mathbf{E}_k),$$
$$s.t. \sum_{k=1}^{K} \text{tr}\left(\mathbf{F}_k^H \mathbf{R}_{i,\ell} \mathbf{F}_k\right) \leq Q_i, i = 1, .., N, \ell = 1, .., L_i,$$
$$\sum_{k=1}^{K} \text{tr}\left(\mathbf{F}_k^H \mathbf{F}_k\right) \leq P.$$

The Lagrangian of the problem $(\mathcal{T}1)$ is

$$\mathcal{L}(\{\mathbf{F}_k\},\{\mathbf{G}_k\},\mu,\{\lambda_{i,\ell}\}) = \sum_{k=1}^{K} \mathrm{tr}(\mathbf{E}_k) + \mu\left(\sum_{k=1}^{K} \mathrm{tr}\left(\mathbf{F}_k^H\mathbf{F}_k\right) - P\right) + \sum_{i=1}^{N}\sum_{\ell=1}^{L_i} \lambda_{i,\ell}\left(\sum_{k=1}^{K} \mathrm{tr}\left(\mathbf{F}_k^H\mathbf{R}_{i,\ell}\mathbf{F}_k\right) - Q_i\right), \quad (22)$$

where $\lambda_{i,\ell} \geq 0,\ i = 1,2,\ldots,N,\ \ell = 1,2,\ldots,L_i$ and $\mu \geq 0$ are, respectively, the dual variables for the region constraints and transmit power constraint. For fixed precoders, the optimal MMSE receiver structure is given by (7). Taking the gradient of (22) with respect to (w.r.t) the precoder $\mathbf{F}_k$ and setting it to 0, we get [10]

$$\mathbf{F}_k = \left(\sum_{m=1}^{K} \mathbf{H}_m^H\mathbf{G}_m^H\mathbf{G}_m\mathbf{H}_m + \sum_{i=1}^{N}\sum_{\ell=1}^{L_i} \lambda_{i,\ell}\mathbf{R}_{i,\ell} + \mu\mathbf{I}\right)^{-1} \mathbf{H}_k^H\mathbf{G}_k^H, k = 1,\ldots,K. \quad (23)$$

Since the optimal receivers are a function of the precoders and vice versa, [10] has proposed an iterative procedure to find the solution which is outlined in Algorithm 1. The problem $(\mathcal{T}1)$ is not jointly convex over all the precoder and receiver matrices, but it is convex when the receivers are designed with fixed precoders and vice versa. This ensures that the proposed algorithm converges to at least a local minimum [10]. In each iteration, the optimal dual variables are determined using sub-gradient search method, which is detailed in Section V.

B. Sum-Rate Maximizing Precoder With Region Constraints

The sum-rate maximization problem with the transmit power and region constraints is

$$(\mathcal{T}2) \quad \max_{\{\{\mathbf{F}_k\},\{\mathbf{G}_k\}\}} \sum_{k=1}^{K} \log\left|\mathbf{E}_k^{-1}\right|,$$
$$s.t. \sum_{k=1}^{K} \mathrm{tr}\left(\mathbf{F}_k^H\mathbf{R}_{i,\ell}\mathbf{F}_k\right) \leq Q_i, i = 1,..,N, \ell = 1,..,L_i,$$
$$\sum_{k=1}^{K} \mathrm{tr}\left(\mathbf{F}_k^H\mathbf{F}_k\right) \leq P.$$

The Lagrangian of the problem $(\mathcal{T}2)$ is

$$\mathcal{L}(\{\mathbf{F}_k\},\{\mathbf{G}_k\},\mu,\{\lambda_{i,\ell}\}) = -\sum_{k=1}^{K} \log\left|\mathbf{E}_k^{-1}\right| + \mu\left(\sum_{k=1}^{K} \mathrm{tr}\left(\mathbf{F}_k^H\mathbf{F}_k\right) - P\right) + \sum_{i=1}^{N}\sum_{\ell=1}^{L_i} \lambda_{i,\ell}\left(\sum_{k=1}^{K} \mathrm{tr}\left(\mathbf{F}_k^H\mathbf{R}_{i,\ell}\mathbf{F}_k\right) - Q_i\right), \quad (24)$$

where $\{\lambda_{i,\ell}\}$ and μ are the dual variables. For fixed precoders, the optimal MMSE receiver structure is given by (7). Taking the gradient of (24) w.r.t the precoder $\mathbf{F}_k$ and setting it to 0, and after some algebraic manipulation we get the precoder structure [8]

$$\mathbf{F}_k = \left(\sum_{m=1}^{K} \mathbf{H}_m^H\mathbf{G}_m^H\mathbf{E}_m^{-1}\mathbf{G}_m\mathbf{H}_m + \sum_{i=1}^{N}\sum_{\ell=1}^{L_i} \lambda_{i,\ell}\mathbf{R}_{i,\ell} + \mu\mathbf{I}\right)^{-1} \mathbf{H}_k^H\mathbf{G}_k^H\mathbf{E}_k^{-1}, k = 1,\ldots,K. \quad (25)$$

Algorithm 1 Iterative MMSE solution

1: **initialize**
$\left\{\mathbf{G}_k^{(0)}\right\}$, and τ = 1, where τ is the index of iteration.
2: **repeat**
3: Compute $\left\{\mathbf{F}_k^{(\tau)}\right\}$ for given $\left\{\mathbf{G}_k^{(\tau-1)}\right\}$ using (23) and the sub-gradient search method.
4: Compute $\left\{\mathbf{G}_k^{(\tau)}\right\}$ for given $\left\{\mathbf{F}_k^{(\tau)}\right\}$ using (7).
5: $\tau = \tau + 1$.
6: **until** $\left|\mathfrak{R}_{sum}^{(\tau-1)} - \mathfrak{R}_{sum}^{(\tau)}\right| \leq \epsilon$.

Similar to Section IV-A, the optimal receivers and precoders are interdependent. To find the solution, [8] has proposed an iterative approach outlined in Algorithm 2. Although the problem $(\mathcal{T}2)$ is not jointly convex with respect to both the precoder and receiver matrices, it becomes convex when either the receivers are designed for fixed precoders or vice versa. This guarantees that the proposed algorithm at least converges to a local maximum [8]. In each iteration, the optimal dual variables are computed using the sub-gradient search method, as detailed in Section V.

Algorithm 2 Iterative sum-rate maximizing solution

1: **initialize**
$\left\{\mathbf{F}_k^{(0)}\right\}$, and τ = 1.
2: **repeat**
3: Compute $\left\{\mathbf{G}_k^{(\tau)}\right\}$ for given $\left\{\mathbf{F}_k^{(\tau-1)}\right\}$ using (7).
4: Compute $\left\{\mathbf{E}_k^{(\tau)}\right\}$ for given $\left\{\mathbf{F}_k^{(\tau-1)}\right\}$ using (8).
5: Compute $\left\{\mathbf{F}_k^{(\tau)}\right\}$ for given $\left\{\mathbf{G}_k^{(\tau)}\right\}$ and $\left\{\mathbf{E}_k^{(\tau)}\right\}$, using (25) and the sub-gradient search method.
6: $\tau = \tau + 1$.
7: **until** $\left|\mathfrak{R}_{sum}^{(\tau-1)} - \mathfrak{R}_{sum}^{(\tau)}\right| \leq \epsilon$.

V. SIMULATION

This section presents the simulation results for the proposed precoder design methods. We performed Monte Carlo simulations to get the average sum-rate. We compare the performance of our proposed precoding methods, and precoding without the region constraints. We use the region boundary shown in Fig. 2. We take 16 equally spaced samples of the azimuth

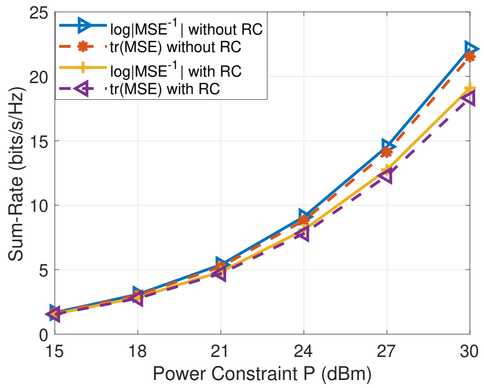

Fig. 3: Transmit power constraint P vs. sum-rate.

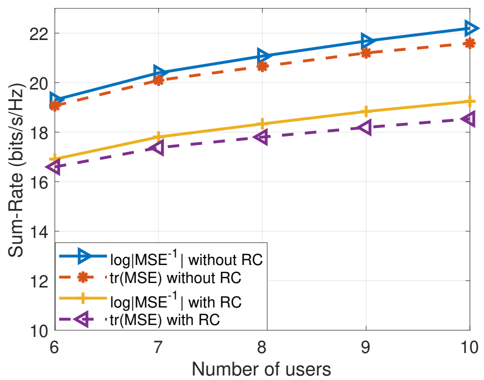

Fig. 4: Number of users vs. sum-rate.

angle span of the region, and we compute the distance in terms of ϕ using (21) to define the region constraints. In all simulations, the noise variance has a fixed value of $\sigma^2 = 1$, and the interference power threshold is $Q = -80$ dBm. We assume the channel follows an uncorrelated Rayleigh fading channel model, i.e., $\mathbf{H}_k \sim \mathcal{CN}(0, \mathbf{I})$. We assume half-wavelength array element spacing, i.e., $\delta = \nu/2$. In all the plots RC in the legend stands for region constraints. The legend '$\log|\text{MSE}^{-1}|$' refers to the precoder in Section IV-B, while the legend 'tr(MSE)' corresponds to the precoder in Section IV-A. We use a sub-gradient search method to find the optimal dual variables. The dual variables must satisfy the following KKT conditions: $\mu \left(\sum_{k=1}^{K} \text{tr}\left(\mathbf{F}_k^H \mathbf{F}_k\right) - P \right) = 0, \mu \geq 0$, and $\lambda_{i,\ell} \left(\sum_{k=1}^{K} \text{tr}\left(\mathbf{F}_k^H \mathbf{R}_{i,\ell} \mathbf{F}_k\right) - Q_i \right) = 0, \lambda_{i,\ell} \geq 0, \forall i, \ell$. Based on that we define the sub-gradient of the Lagrangaian w.r.t. μ as $\left(P - \sum_{k=1}^{K} \text{tr}\left(\mathbf{F}_k^H \mathbf{F}_k\right) \right)$, and the sub-gradient of the Lagrangian w.r.t. $\lambda_{i,\ell}$ as $\left(Q_i - \sum_{k=1}^{K} \text{tr}\left(\mathbf{F}_k^H \mathbf{R}_{i,\ell} \mathbf{F}_k\right) \right)$. We use the constant step-size method in [11] to iteratively update the dual variables and the precoders until the dual variables converges to a prescribed accuracy.

Fig. 3 shows transmit power vs. sum-rate for 32×2 massive MIMO, where $M_t = 32$ and $M_r = 2$. The number of users are $K = 10$. As P increases, the gap in sum-rate with and without region constraints widens because the impact of region constraints grows. For instance, at P = 30 dBm, the difference in sum-rate are 3 bits/s/Hz, and 3.2 bits/s/Hz for the $\log|\text{MSE}^{-1}|$ precoder, and tr(MSE) precoder, respectively. Additionally, the $\log|\text{MSE}^{-1}|$ precoder marginally outperforms the tr(MSE) precoder, at P = 30 dBm, the difference is 0.7 bits/s/Hz for simulation with the region constraints.

Fig. 4 shows the sum-rate vs. number of users for 32×2 massive MIMO with $P = 30$ dBm. Similar to the previous simulation, the $\log|\text{MSE}^{-1}|$ precoder marginally outperforms the tr(MSE) precoder. For example, with region constraints active and $K = 9$, the difference in sum-rate is 0.6 bits/s/Hz. The sum-rate increases as the number of users increases.

VI. Conclusion

The increased demand for wireless access and applications has led to the wireless spectrum becoming more congested. To protect legacy users from this new interference, we showed how signal processing techniques can be used to design waveforms that protect certain geographic regions. We incorporated quadratic constraints, referred to as region constraints, into the massive MIMO system model. We proposed MMSE precoder design and sum-rate maximizing precoder design approaches. Simulation results demonstrated the achievable sum-rate for multi-user massive MIMO systems with region constraints.

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